

## Linear Algebra – Homework 4 (corresponding to Quiz 4)

April 20, 2022

1. (30%) Let  $A$  be a 4 by 5 matrix with linearly independent column vectors  $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ , and whose remaining column vectors satisfy  $\mathbf{a}_4 = \mathbf{a}_1 - 5\mathbf{a}_3$  and  $\mathbf{a}_5 = 3\mathbf{a}_1 - 2\mathbf{a}_2 + 4\mathbf{a}_3$ .
  - (a) What is the dimension of  $N(A)$ ? Explain.
  - (b) Determine the reduced row echelon form of  $A$ .
2. (30%) Determine whether the following are linear operators on  $\mathbb{R}^2$ .
  - (a)  $L$  is the operator defined by  $L(\mathbf{x}) = (x_2, 2x_1)^T$ .
  - (b)  $L$  is the operator defined by  $L(\mathbf{x}) = (x_1 + x_2, x_1x_2)^T$ .
3. (20%) Let  $\mathbf{y}_1 = (1, 1, 1)^T$ ,  $\mathbf{y}_2 = (1, 1, 0)^T$ , and  $\mathbf{y}_3 = (1, 0, 0)^T$ , and let  $L$  be the linear operator on  $\mathbb{R}^3$ :
$$L(c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + c_3\mathbf{y}_3) = (c_1 + 2c_3)\mathbf{y}_1 + 2(c_1 + 2c_2 - c_3)\mathbf{y}_2 - (3c_3 - 2c_1)\mathbf{y}_3$$
Find a matrix representing  $L$  with respect to the ordered basis  $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ .
4. (20%) Let  $A$  and  $B$  be similar matrices. Show that  $A^T$  and  $B^T$  are similar.