Linear Algebra – Homework 6 (corresponding to Quiz 6)
June 1, 2022

1. (25%) Let $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ be an orthonormal basis for an inner product space V. If $\boldsymbol{x} = c_1\boldsymbol{u}_1 + c_2\boldsymbol{u}_2 + c_3\boldsymbol{u}_3$ is a vector with the properties $\|\boldsymbol{x}\| = 5$, $\langle \boldsymbol{u}_1, \boldsymbol{x} \rangle = 4$, and $\boldsymbol{x} \perp \boldsymbol{u}_2$, then what are the possible values of c_1, c_2, c_3?

2. Let

$$A = \begin{bmatrix} -2 & 6 \\ 1 & -6 \\ -2 & 3 \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

   (a) (25%) Use the Gram-Schmidt process to find an orthonormal basis for the column space of A.

   (b) (25%) Compute the QR factorization of the matrix A.

3. (25%) Show that the eigenvalues of a triangular matrix are the diagonal elements of the matrix.

4. (25%) Factor the matrix A into a product XDX^{-1}, where D is diagonal.

$$A = \begin{bmatrix} 2 & -8 \\ 1 & -4 \end{bmatrix}$$