1a. for $\varepsilon = \frac{1}{2}$. $\forall \delta > 0$. $\exists x_n = \frac{1}{2\pi n}$, $n \in \mathbb{N}$, $x_n \to 0$ as $n \to \infty$. $\exists M$. s.t. $|x_n - 0| < \delta$. $\forall n > M$.

but $\sin\frac{1}{x_n} = \sin(2\pi n) = 1 > \varepsilon$.

$\Rightarrow f$ doesn't conti. at 0.

let $a(x) = \sin x$, $b(x) = \frac{1}{x}$, they both continuous $\begin{cases} a \text{ conti on } \mathbb{R}. \\ b \text{ conti on } \mathbb{R}/\{0\}. \end{cases}$

$f(x) = a(b(x))$ for $x \neq 0$. by thm 4.7. f conti on $\mathbb{R}/\{0\}$.

$\forall x \neq 0$. $\exists f'(x) = (\cos\frac{1}{x})(-x^{-2}) = -x^{-2}\cos\frac{1}{x}$.

but f isn't differentiable at 0.

1b. $\forall \varepsilon > 0$. $\exists \delta = \varepsilon$. s.t $\forall |x-0| < \delta$, $|x\sin\frac{1}{x} - 0| = |x\sin\frac{1}{x}| \le |x| \cdot |\sin\frac{1}{x}| \le |x| < \delta = \varepsilon$

$\Rightarrow f$ conti at 0 --- ①.

by 1.a. $f(x) = \sin(\frac{1}{x})$ conti on $\mathbb{R}/\{0\}$ & $c(x) = x$ is conti on $\mathbb{R}$.

by thm 4.9. $g(x) = f(x) \cdot c(x)$ conti on $\mathbb{R}/\{0\}$ ... ②.

by ①.② g conti on $\mathbb{R}$.

$\exists g'(x) = \sin\frac{1}{x} - x^{-1}\cos\frac{1}{x}$ $\forall x \neq 0$.

1c. $\forall \varepsilon > 0$. $\exists \delta = \min\{0.9, \varepsilon\}$. $\forall |x-0| < \delta$, $|x^2\sin\frac{1}{x} - 0| \le |x^2| \cdot |\sin\frac{1}{x}| \le |x^2| < \varepsilon$.

$\begin{cases} \text{if } 0.9 < \varepsilon & x^2 < 0.9^2 < \varepsilon^2 < \varepsilon \\ \text{if } \varepsilon < 0.9 & x^2 < \varepsilon^2 < \varepsilon \end{cases}$ $\Rightarrow h$ conti at 0 ... ③

since $d(x) = x^2$ conti on $\mathbb{R}$ $\Rightarrow$ by thm 4.9, $h(x) = d(x) \cdot f(x)$ conti on $\mathbb{R}/\{0\}$ --- ④.

by ③.④ h conti on $\mathbb{R}$.

$\exists h'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$ $\forall x \neq 0$.

which isn't conti at 0. since $\lim_{n\to\infty} \cos(n)$ DNE $\Rightarrow \lim_{x\to 0} \cos\frac{1}{x}$ DNE.

1d. $\forall \varepsilon > 0$, $\exists \delta = \min\{0.9, \varepsilon\}$. $\forall |x-0| < \delta$, $|x^3\sin\frac{1}{x} - 0| \le |x^3| \cdot |\sin\frac{1}{x}| \le |x^2| < \varepsilon$ similar to ③ ... ⑥.

$e(x) = x^3$ conti on $\mathbb{R}$. by thm 4.9. $I(x) = e(x) \cdot f(x)$ conti on $\mathbb{R}/\{0\}$ ... ⑤ $\Rightarrow$ by ⑤.⑥ I on $\mathbb{R}$.

$\exists I'(x) = 3x^2\sin\frac{1}{x} - x\cos\frac{1}{x}$ $\forall x \in \mathbb{R}$.

conti at 0 since $3x^2\sin\frac{1}{x} = 3h(x)$ conti at 0 .. ⑦.

$\forall \varepsilon > 0$. $\exists \delta = \varepsilon$, s.t $|x\cos\frac{1}{x} - 0| \le |x| \cdot |\cos\frac{1}{x}| \le |x| < \varepsilon$.

$\Rightarrow \cos\frac{1}{x}$ conti at 0 --- ⑧.

by ⑦.⑧ thm 4.9. $I'(x) = 3h(x) - \cos\frac{1}{x}$ conti at 0.