Deadline : 2022/12/29, 17:00.

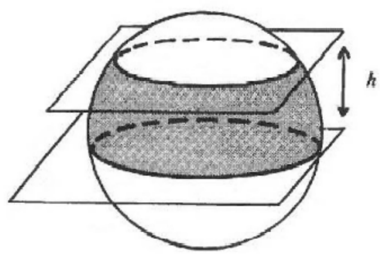
1. If the curve $y = f(x)$, $a \leq x \leq b$, is rotated about the horizontal line $y = c$, where $f(x) \leq c$, find the formula for the area of the resulting surface.

2. (**95' Calculus Exam**) Prove that the volume of the ellipsoid

$$\frac{(x-a)^2}{A^2} + \frac{(y-b)^2}{B^2} + \frac{(z-c)^2}{C^2} = 1$$

is $V = \frac{4}{3}\pi ABC$.

3. Prove that the area of the portion of the sphere shown in the below figure is $2\pi rh$.



Remark:

(i) This shows that the surface area of the band obtained depends only on the distance between two parallel planes, not on their location.

(ii) We can choose two parallel planes with distance $h = 2r$ apart such that the whole sphere is between these two planes. Hence, the area of the sphere is $4\pi r^2$.

4. (**83' Calculus Exam**) Let C be a curve with the parametric equation

$$\begin{cases} x(t) = t - \sin t, \\ y(t) = 1 - \cos t, \end{cases} \qquad 0 \leq t \leq 2\pi.$$

(a) Find the slope of the tangent line of the curve when $t = \dfrac{\pi}{4}$.

(b) Find the arc length of the curve.

(c) Find the area of the region which is bounded be the curve and x-axis.

5. (**93' Calculus Exam**) Find the arc length of the curve

$$\begin{cases} x(t) = \cos t + t\sin t, \\ y(t) = \sin t - t\cos t, \end{cases} \quad 0 \le t \le \pi.$$

6. Find an equation in x and y for the line(s) tangent to the curve

   (a) $x(t) = \frac{1}{t}$, $y(t) = t^2 + 1$ at $t = 1$.

   (b) $x(t) = e^t$, $y(t) = 3e^{-t}$ at $t = 0$.

   (c) $x(t) = t^3 - t$, $y(t) = t\sin(\frac{1}{2}\pi t)$ at the point $(0, 1)$.

7. Find the area of the surface obtained by rotating the curve

$$\begin{cases} x(t) = 3t - t^3, \\ y(t) = 3t^2, \end{cases} \quad 0 \le t \le 1.$$

   about the x-axis.

8. Sketch the curve with the given polar equation by first sketching the graph of r as a function of θ in Cartesian coordinates.

   (a) $r = 2(1 + \cos\theta)$.

   (b) $r = 2 + \sin 3\theta$.

   (c) $r^2 = \cos 4\theta$.

9. Let C be the polar curve $r = 2\sin\theta$, $0 \le \theta \le \pi$.

   (a) Find all vertical and horizontal tangent lines of the curve.

   (b) Find $\dfrac{d^2y}{dx^2}$ in terms of θ. (Use Chain Rule to deduce $\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{d\theta}(\frac{dy}{dx})}{\frac{dx}{d\theta}}$.)

   (c) Determine the concavity of the curve.

   (d) Sketch the curve.

10. Find the area between the curves

   (a) $r = 2\cos\theta$, $r = \cos\theta$ and the rays $\theta = 0$, $\theta = \frac{1}{4}$.

   (b) $r = \frac{1}{2}\sec^2(\frac{1}{2}\theta)$ and the vertical line through the origin.

   (c) $r = e^\theta$, $0 \le \theta \le \pi$; $r = e^\theta$, $2\pi \le \theta \le 3\pi$ and the rays $\theta = 0$, $\theta = \pi$.

(d) $r = \tan\theta$, $\frac{\pi}{6} \le \theta \le \frac{\pi}{3}$.

11. Find the area of the following regions

    (a) the region enclosed by one loop of the curve $r^2 = \sin 2\theta$.

    (b) the region lies inside the curve $r = 2 + \sin\theta$ and outside the curve $r = 3\sin\theta$.

    (c) the region lies inside both curves $r = a\sin\theta$, $r = b\cos\theta$, $a > 0$, $b > 0$.

    (d) the region between a large loop and the enclosed small loop of the curve $r = 1 + 2\cos 3\theta$.