2.

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(a)

Let E be the set of $x \in \mathbb{R}$ s.t. $a_{n_k} \to x$ for some subsequence $\{a_{n_k}\}$

$\Rightarrow \limsup_{n\to\infty} S_n = \sup E$, Let $\sup E$ denoted by S^*

① If $S^* = +\infty$, then E is not bounded (bdd) above, hence $\{a_n\}$ is not bdd above and $\exists$ subsequence $\{a_{n_k}\}$ s.t. $a_{n_k} \to +\infty$

② If $S^* \in \mathbb{R}$, then E is bdd above, and at least one subsequential limit exist, so that by thm 3.7 and thm 2.28, $\therefore S^* \in E$

③ If $S^* = -\infty$, then E contains only one element, namely $-\infty$, and there is no subsequential limit. Hence, $\forall M \in \mathbb{R}$, $a_n > M$ for at most a finite number of values of n, so that $a_n \to -\infty$.