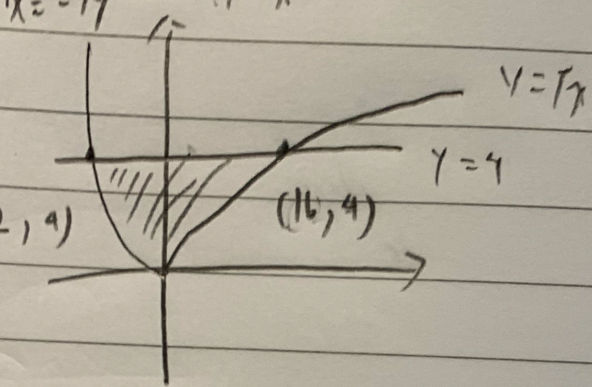
imply

1. $x = -\sqrt{y} \rightarrow y = x^2$

$D =$



$$\iint_D x \, dA$$

$$= \int_{-2}^{0}\int_{x^2}^{4} x \, dy \, dx + \int_{0}^{16}\int_{\sqrt{x}}^{4} x \, dy \, dx$$

$$= \int_{-2}^{0} x(4-x^2) \, dx + \int_{0}^{16} x(4-\sqrt{x}) \, dx$$

$$= \int_{-2}^{0} -x^3+4x \, dx + \int_{0}^{16} -x^{\frac{3}{2}}+4x \, dx$$

$$= \frac{-1}{4}x^4 + 2x^2 \Big|_{-2}^{0} + \frac{-2}{5}x^{\frac{5}{2}} + 2x^2 \Big|_{0}^{16}$$

$$= -\left(\frac{-1}{4}(-2)^4 + 2(-2)^2\right) + \frac{-2}{5}16^{\frac{5}{2}} + 2(16)^2$$

$$= -\left(\frac{-16}{4} + 8\right) + \frac{-2}{5}2^{10} + 2^9$$

$$= -4 + 2^9 \frac{1}{5}$$

$$= \frac{512-20}{5} = \frac{492}{5}$$

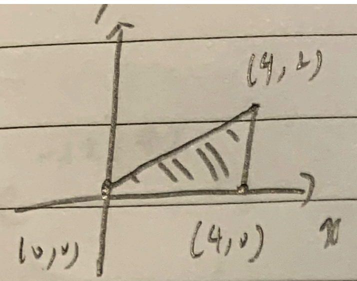
#

2.

$E =$

2.

$E =$



$0 \le x \le 4$

$0 \le y \le \frac{x}{2}$

$$\iint_E y\sqrt{x-2y}\, dA$$

$$= \int_0^4 \int_0^{\frac{x}{2}} y\sqrt{x-2y}\, dy\, dx$$

$$= \int_0^4 \frac{2}{15\times(-2)^2} (-6y-2x)(x-2y)^{\frac{3}{2}} \Big|_0^{\frac{x}{2}} dx$$

$$= \frac{1}{30}\int_0^4 -(-2x)\, x^{\frac{3}{2}}\, dx$$

$$= \frac{1}{30}\int_0^4 2x^{\frac{5}{2}}\, dx$$

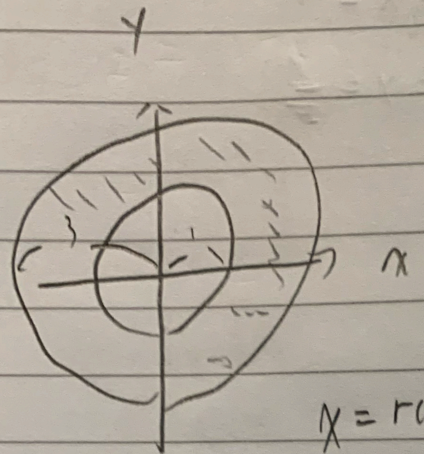
$$= \frac{1}{15}\, \frac{2}{7}\, x^{\frac{7}{2}} \Big|_0^4$$

$$= \frac{2}{105}\, 4^{\frac{7}{2}} = \frac{2^8}{105}$$

#

3.

D =



$1 \le r \le 3$

$0 \le \theta \le 2\pi$

$x = r\cos\theta$

$y = r\sin\theta$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} \qquad \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

$$A(S) = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

$$= \iint_D \sqrt{1+\frac{x^2+y^2}{x^2+y^2}}\, dA$$

$$= \iint_D \sqrt{2}\, dA$$

$$= \int_0^{2\pi}\int_1^3 \sqrt{2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{\sqrt{2}}{2} r^2 \Big|_1^3\, d\theta$$

$$= \int_0^{2\pi} 4\sqrt{2}\, d\theta$$

$$= 8\sqrt{2}\pi$$

#

4.

Date .

4.

$$\int_{-\infty}^{\infty} e^{-2x^2+4x}\,dx$$

$$= \int_{-\infty}^{\infty} e^{-2(x-1)^2+2}\,dx$$

$$= e^2 \int_{-\infty}^{\infty} e^{-2(x-1)^2}\,dx$$

$(u = \sqrt{2}(x-1) \quad du = \sqrt{2}\,dx)$

$$= e^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2}} e^{-u^2}\,du$$

$$= \frac{e^2}{\sqrt{2}} \int_{-\infty}^{\infty} e^{-u^2}\,du$$

$$= e^2 \sqrt{\frac{\pi}{2}}$$ #

Date . .

5.

D: $z \le 3-(x+y)$

$z \ge 0$

$x^2+y^2 \le 1$

$x = r\cos\theta$

$y = r\sin\theta$

$0 \le r \le 1$

$0 \le \theta \le 2\pi$

$\sin 2\pi - \cos 2\pi = 0-1 = -1$

$\sin 0 - \cos 0 = 0-1 = -1$

$E = \iiint_D dV$

$= \iiint_0^{3-(x+y)} dz\,dx\,dy$

$= \int_0^{2\pi}\int_0^1\int_0^{3-(x+y)} r\,dz\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^1 r(3-r(\cos\theta+\sin\theta))\,dr\,d\theta$

$= \int_0^{2\pi} \frac{3}{2}r^2 - \frac{1}{3}r^3(\cos\theta+\sin\theta)\Big|_0^1\,d\theta$

$= \int_0^{2\pi} \frac{3}{2} - \frac{1}{3}(\cos\theta+\sin\theta)\,d\theta$

$= 3\pi - \frac{1}{3}(\sin\theta-\cos\theta)\Big|_0^{2\pi} = 3\pi$ #

6.

E

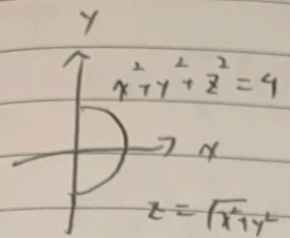
$\iiint_E z(x-y)\,dV =$

$=$

$=$

0-1 = -1

6. E is half a cone intersect half a ball



$2(x^2+y^2)=4$

$x^2+y^2=2$

$x=r\cos\theta$

$y=r\sin\theta$

$$\iiint_E z(x-y)\,dV=\int_0^{\pi}\int_0^{2}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z(x-y)\,r\,dz\,dr\,d\theta$$

$$=\int_0^{\pi}\int_0^{2}\int_r^{\sqrt{4-r^2}} r^2(\cos\theta-\sin\theta)\,z\,dz\,dr\,d\theta$$

$$=\int_0^{\pi}\int_0^{2} r^2(\cos\theta-\sin\theta)\,\frac{z^2}{2}\Big|_r^{\sqrt{4-r^2}}\,dr\,d\theta$$

$$=\int_0^{\pi}\int_0^{2} r^2(\cos\theta-\sin\theta)\,\frac{1}{2}(4-2r^2)\,dr\,d\theta$$

$$=\int_0^{\pi}\frac{1}{2}(\cos\theta-\sin\theta)\int_0^{2} -2r^4+4r^2\,dr\,d\theta$$

$$=\frac{1}{2}\int_0^{\pi}(\cos\theta-\sin\theta)\;\frac{-2}{5}r^5+\frac{4}{3}r^3\Big|_0^{2}\,d\theta$$

$$=\frac{1}{2}\left(\frac{-2}{5}2^5+\frac{4}{3}2^3\right)\;\sin\theta+\cos\theta\Big|_0^{\pi}$$

$$=\frac{1}{2}\left(\frac{-64}{5}+\frac{32}{3}\right)(-1-1)=-\left(\frac{-64}{5}+\frac{32}{3}\right)=\frac{192-160}{15}=\frac{32}{15}$$

DOW DEAN

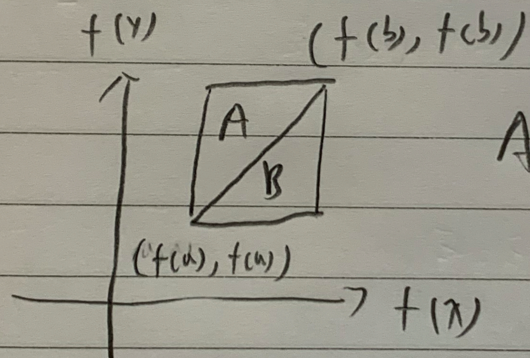
8.

$$2\int_a^b\int_x^b f(x)\, f(y)\, dy\, dx$$

$$=\int_a^b 2f(x)\int_x^b f(y)\, dy\, dx$$

∴

$$\int_a^b\int_a^x f(y)\, dy\, dx = \int_a^b\int_x^b f(y)\, dy\, dx$$



A = B

$$=\int_a^b f(x)\int_a^b f(y)\, dy\, dx$$

$$=\left(\int_a^b f(x)\, dx\right)^2$$

#