Calculus HW2

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Let $f(x) = \frac{1}{-x}$, $g(x) = sin^2(x)$ be two function from $\mathbb{R}^+$ to $\mathbb{R}$

$f'(x) = \frac{1}{x^2}$

$g'(x) = -2sin(x)cos(x) = -sin(2x)$

$\frac{sin^2(x)}{x^2} = f'(x)g(x)$

$f(x)g(x) = \frac{sin^2(x)}{-x}$

$f(x)g'(x) = \frac{sin(2x)}{x}$

Because $\forall x \in \mathbb{R}^+, f(x), g(x), f'(x), g'(x)$ all exists, so $\forall x \in \mathbb{R}^+, \frac{d}{dx}(f(x)g(x)) = [\frac{d}{dx}f(x)]g(x) + [\frac{d}{dx}g(x)]f(x)$

That is $\forall x \in \mathbb{R}^+, [f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

Then $\forall n_2, n_1 \in \mathbb{R}^+, \int_{n_1}^{n_2}[f(x)g(x)]'dx = \int_{n_1}^{n_2} f'(x)g(x)dx + \int_{n_1}^{n_2} f(x)g'(x)dx$

Because $f(x)g(x) = \frac{sin^2(x)}{-x}$ is differentiable on $\mathbb{R}^+$, so $\forall n_2, n_2 \in \mathbb{R}^+, \int_{n_1}^{n_2}[f(x)g(x)]'dx = f(x)g(x)|_{n_1}^{n_2}$

Change the order and put in the actual function we have:

$\forall n_2, n_1 \in \mathbb{R}^+, \int_{n_1}^{n_2}[f(x)g(x)]'dx = \int_{n_1}^{n_2} f'(x)g(x)dx + \int_{n_1}^{n_2} f(x)g'(x)dx$

$\forall n_2, n_1 \in \mathbb{R}^+, \int_{n_1}^{n_2} \frac{sin^2(x)}{x^2}dx = \frac{sin^2(x)}{-x}|_{n_1}^{n_2} - \int_{n_1}^{n_2} \frac{sin(2x)}{x}dx$

$\lim_{n_1 \to 0, n_2 \to \infty} \int_{n_1}^{n_2} \frac{sin^2(x)}{x^2}dx = \lim_{n_1 \to 0, n_2 \to \infty} \frac{sin^2(x)}{-x}|_{n_1}^{n_2} - \int_{n_1}^{n_2} \frac{sin(2x)}{x}dx$

We only have to prove $\lim_{n_1 \to 0, n_2 \to \infty} \frac{sin^2(x)}{-x}|_{n_1}^{n_2}$ and $\lim_{n_1 \to 0, n_2 \to \infty} \int_{n_1}^{n_2} \frac{sin(2x)}{x}dx$ exist, by directly calculating their value, then we have our result.

$\lim_{n_1 \to 0, n_2 \to \infty} \frac{sin^2(x)}{-x}|_{n_1}^{n_2} = \lim_{n_2 \to \infty} \frac{sin^2(x)}{-x}|_5^{n_2} + \lim_{n_1 \to 0} \frac{sin^2(x)}{-x}|_{n_1}^5 = 0 - \frac{sin^2(x)}{-x}|_5 + \frac{sin^2(x)}{-x}|_5 - 0$ **(notice :** $\lim_{x \to 0} \frac{sin(x)}{x} = 1$**)**

And $\lim_{n_1 \to 0, n_2 \to \infty} \int_{n_1}^{n_2} \frac{sin(2x)}{x}dx = 2\int_{n_1}^{n_2} \frac{sin(2x)}{2x}dx = 2\int_{n_1}^{n_2} L(2x)dx$, where $L(x) = \frac{sin(x)}{x}$

You should be capable of finishing the trivial part by your self.