- $m\frac{dv}{dt} = mg - \gamma v$ 速度　　　- $\frac{dp}{dt} = rp - k$ 人口

$$\boxed{\frac{dy}{dt} = ay - b}$$

ex. $\frac{dp}{dt} = 0.5p - 450$

$2\frac{dp}{dt} = p - 900 \Rightarrow \frac{2}{p-900}dp = dt \Rightarrow \int \frac{2}{p-900}dp = \int dt$

$\Rightarrow \ln|p-900| = \frac{1}{2}t + C \Rightarrow |p-900| = e^{\frac{1}{2}t+C} = e^{C}\cdot e^{\frac{1}{2}t}$

$\Rightarrow p = 900 \pm e^{C} e^{\frac{1}{2}t} = 900 + \tilde{C}\cdot e^{\frac{1}{2}t}$

If 加原始條件：$p(0) = 850 = 900 + \tilde{C}$

$\Rightarrow 50 = -\tilde{C} \Rightarrow p(t) = 900 - 50e^{\frac{1}{2}t}$

ex. $\frac{dy}{dt} = ay - b. \Rightarrow a(y - \frac{b}{a}) = \frac{dy}{dt} \Rightarrow \int \frac{1}{(y-\frac{b}{a})}dy = \int a\,dt$

$\Rightarrow \ln|y - \frac{b}{a}| = at + C \Rightarrow |y - \frac{b}{a}| = e^{at+C}$

$\Rightarrow y = \frac{b}{a} + \tilde{C}e^{at}$

If $y(0) = y_0 \Rightarrow y = y_0 = \frac{b}{a} + \tilde{C} \Rightarrow \tilde{C} = y_0 - \frac{b}{a} \Rightarrow y = \frac{b}{a} + (y_0 - \frac{b}{a})e^{at}$

If $y' = 0 \Rightarrow ay - b = 0,\ y = -\frac{b}{a} \Rightarrow$ equilibrium solution

If $y_0 = \frac{b}{a} \Rightarrow y = \frac{b}{a}$　If $y_0 > \frac{b}{a}$ $\begin{cases} a>0 \Rightarrow y \nearrow \infty \\ a<0 \Rightarrow y \searrow \frac{b}{a} \end{cases}$

If $y_0 < \frac{b}{a}$ $\begin{cases} a>0 \Rightarrow y \searrow -\infty \\ a<0 \Rightarrow y \nearrow \frac{b}{a} \end{cases}$

$\Rightarrow$ a的正負、初始值會影響

ex. $\begin{cases} \frac{dv}{dt} = 9.8 - \frac{v}{5} \\ V(0) = 0 \end{cases} \Rightarrow -\frac{1}{5}(v - 49) = \frac{dv}{dt} \Rightarrow \int \frac{1}{v-49}dv = -\frac{1}{5}\int dt$

$\Rightarrow \ln|v - 49| = -\frac{1}{5}t + C$

$\Rightarrow |v - 49| = e^{-\frac{1}{5}t} + \tilde{C} \Rightarrow V = 49 + e^{-\frac{1}{5}t}\cdot\tilde{C}$

$V(0) = 49 + \hat{C} = 0.\ \hat{C} = -49 \Rightarrow V = 49 - 49e^{-\frac{1}{5}t}$

ex. $\frac{dy}{dt}-5y=12-t$

同乘 e^{-5t}: $e^{-5t}\frac{dy}{dt}-5e^{-5t}y=12e^{-5t}-te^{-5t}$

$\frac{d}{dt}(e^{-5t}y)=12e^{-5t}-te^{-5t}$

$\Rightarrow e^{-5t}y=-\frac{12}{5}e^{-5t}+(\frac{1}{5}te^{-5t}+\frac{1}{25}e^{-5t}+C)$

$\Rightarrow y=-\frac{12}{5}+\frac{1}{5}t+\frac{1}{25}+Ce^{5t}$

· 一般的線性 O.D.E: $\frac{dy}{dt}+p(t)y=g(t)$

同乘 $\mu(t)$: $\mu(t)\frac{dy}{dt}+\boxed{\mu(t)p(t)y}=\mu(t)g(t)$ 跟y有關的在左邊

→ Find $\mu(t)$: $\frac{d}{dt}(\mu(t)y)=\mu(t)\frac{dy}{dt}+\boxed{\mu'(t)y}$ → 讓他們一樣

$\mu'(t)=\mu(t)p(t)$, $\boxed{\frac{\mu'(t)}{\mu(t)}}=p(t)$

$\frac{d}{dt}\ln|\mu(t)|=p(t)\Rightarrow \ln|\mu(t)|=\int p(t)\,dt$

$\Rightarrow \mu(t)=e^{\int p(t)\,dt}$

$\mu(t)y=\int\mu(t)g(t)\,dt+C$

$\Rightarrow y=\frac{1}{\mu(t)}\int\mu(t)g(t)\,dt+\frac{C}{\mu(t)}$

ex. $ty'+2y=4t^2$ 因「ty'」係數不是1, 不能直接把 $2y$ 當 $p(t)y$

同除t. $y'+\frac{2}{t}y=4t \Rightarrow p(t)=\frac{2}{t}$, $\int p(t)\,dt=2\ln t$

$\Rightarrow e^{2\ln t}=t^2$

$\frac{d}{dt}(t^2y)=t^2y'+2ty=4t^3$

$\Rightarrow t^2y=t^4+C \Rightarrow y=t^2+Ct^{-2}$

If 給初始值 $y(1)=2$, $2=y(1)=1+C$, $C=1$.

$y=t^2+t^{-2}$

		order	degree
ex.	$\frac{d^5y}{dt^5} + \left(\frac{dy}{dt}\right)^2 + y(t) = 0$	5	2
	$\left(\frac{d^4y}{dt^4}\right)^2 + \left(\frac{dy}{dt}\right)^6 + \sin(y(t)) = 0$	4	2 (看最高order)
	$\left(\frac{d^3y}{dt^3}\right)^6 + \left(\frac{d^2y}{dt^2}\right)^4 + \frac{dy}{dt} + y(t) = 0$	3	6

· An ODE $F(t, y, y', y'' \dots y^{(n)}) = 0$ is linear if F is linear in the variables $y, y' \dots y^{(n)}$ $\Rightarrow$ $a_0(t)y^{(n)} + a_1(t)y^{(n-1)} + \dots a_n(t)y = y(t)$

ex. (1) $y' + 3y = 0$ (2) $y'' + 3e^y y' - 2t = 0$ ⇒ 係數跟y有關

(3) $y'' + 3y' - 2t^2 = 0$ (4) $\frac{d^4y}{dt^4} - t\frac{d^2y}{dt^2} + 1 = t^2$

(5) $U_{xx} + UU_y = \sin t$ (6) $U_{xx} + \sin(U)U_y = \cos t$

⇒ 係數跟U有關 ⇒ 跟U有關

· A solution $\phi(t)$ to an ODE $y^{(n)}(t) = f(t, y, y' \dots y^{(n-1)})$ satisfies $\phi^{(n)}(t) - f(t, \phi(t) \dots \phi^{(n-1)}(t))$

ex. $y'' + y = 0$, $y_1(t) = -\cos t \Rightarrow \cos t - \cos t = 0$

$y'' = -y$ $\quad y_2(t) = 2\sin t \Rightarrow -2\sin t + 2\sin t = 0$

$y_3(t) = \sin 2t \Rightarrow \times$

· Three important questions in the study of diff eqs

I: Is there a solution? (Existence)

II: If there is a solution, is it unique? (Uniqueness)

III: If there is a solution, how to find it?

(Analytical sol, Neumerical Approximation, etc)

- ODE : When the unknown fn depends on a single independent variable, only ordinary derivatives appear in the equation

ex. $\frac{dv}{dt} = 9.8 - \frac{v}{5}$, $\frac{dp}{dt} = \frac{p}{2} - 450$. $\frac{d^2Q(t)}{dt^2} + R \cdot \frac{dQ(t)}{dt} + \frac{1}{C}Q(t) = E(t)$

- PDE : When the unknown fn depends on several independent variables, partial derivations appear in the equation

ex. $a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}$ (heat eq)

$a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}$ (wave eq)

- System of differential eqs

ex. $\begin{cases} \frac{dx}{dt} = ax - \alpha xy \\ \frac{dy}{dt} = -cy + \gamma xy \end{cases}$

- The order of a differential eq is the order of the highest derivative that appears in the eq

ex. $F(t, y, y' \cdots y^{(n)}) = 0 \rightarrow$ n th order

ex. $y''' + 2e^t y'' + yy' = t^4 \Rightarrow$ order 3

$y^{(n)} = f(t, y, y' \dots y^{n-1})$

- The degree of a diff eq is the exponent of the highest order derivative that appear in the eq

ex. $F(t, y', (y')^2 \cdots (y')^m) = 0 \Rightarrow$ degree m

ex. $F(t, y, (y')^2 \cdots (y')^m, y'', (y'')^2 \cdots (y'')^r) = 0$

$\Rightarrow$ order 2, degree r

1-1 A linear 1st order ODE

- Def. $\frac{dy}{dt} = f(t,y)$

ex. $\frac{dy}{dt} = -ay+b$; $\frac{dy}{dt} + p(t)y = g(t)$

ex. $(5+3t^2)\frac{dy}{dt} + 6ty = 2t$

$\frac{d}{dt}[(5+3t^2)y] = 2t \Rightarrow \int_{t_0}^{t} \frac{dy}{dt}[(5+3t^2)y]\,dt = \int_{t_0}^{t} 2t\,dt$

$\Rightarrow (5+3t^2)y - (5+3t_0^2)y(t_0) = t^2 - t_0^2$

$\Rightarrow (5+3t^2)y = t^2+C$, $C = (5+3t_0^2)y(t_0) - t_0^2$

ex. $\frac{dy}{dt} + \frac{2}{3}y = \frac{1}{3}e^{\frac{t}{7}}$

$\mu(t)\frac{dy}{dt} + \frac{2}{3}\mu(t)y = \frac{1}{3}\mu(t)e^{\frac{t}{7}}$

$\Rightarrow \frac{d}{dt}[\mu(t)y] = \frac{1}{3}\mu(t)e^{\frac{t}{7}} \Rightarrow \mu'(t)y + \mu(t)\frac{dy}{dt} = \frac{1}{3}\mu(t)e^{\frac{t}{7}}$

$\Rightarrow \mu'(t) = \frac{2}{3}\mu(t) \Rightarrow \frac{\mu'(t)}{\mu(t)} = \frac{2}{3} \Rightarrow \frac{d}{dt}[\ln|\mu(t)|] = \frac{2}{3}$

$\Rightarrow \ln|\mu(t)| = \frac{2}{3}t + C \Rightarrow \mu(t) = \tilde{c}e^{\frac{2}{3}t}$

$e^{\frac{2}{3}t}\frac{dy}{dt} + \frac{2}{3}e^{\frac{2}{3}t}y = \frac{1}{3}e^{\frac{17}{21}t} \Rightarrow \frac{d}{dt}[e^{\frac{2}{3}t}y] = \frac{1}{3}e^{\frac{17}{21}t}$

$\Rightarrow e^{\frac{2}{3}t}y = \frac{7}{17}e^{\frac{17}{21}t} + C \Rightarrow y = \frac{7}{17}e^{\frac{1}{7}t} + \tilde{c}$

ex. $\frac{dy}{dt} + ay = g(t)$

$\mu(t)\frac{dy}{dt} + a\mu(t) = \mu(t)g(t)$

$\Rightarrow e^{at}\frac{dy}{dt} + ae^{at}y = e^{at}g(t) \Rightarrow \frac{d}{dt}[e^{at}y] = e^{at}g(t)$

$\Rightarrow e^{at}y = \int_{t_0}^{t} e^{as}g(s)\,ds + C \Rightarrow y = e^{-at}\int_{t_0}^{t} e^{as}g(s)\,ds + Ce^{-at}$

ex. $y' + 4y = t + e^{-2t}$

$\mu(t)y' + 4\mu(t)y = \mu(t)t + \mu(t)e^{-2t} \Rightarrow \frac{d}{dt}(\mu(t)\cdot y) = \mu(t)t + \mu(t)e^{-2t}$

$\Rightarrow \frac{d\mu(t)}{dt} = 4\mu(t) \Rightarrow \frac{1}{\mu(t)}\cdot d\mu(t) = 4dt \Rightarrow \int\frac{1}{\mu(t)}\,d\mu(t) = \int 4\,dt$

$\Rightarrow \ln|\mu(t)| = 4t + C_1 \Rightarrow \mu(t) = e^{4t}\cdot C_2$. Choose $C_2 = 1$. $\mu(t) = e^{4t}$

$\Rightarrow e^{4t}\frac{dy}{dt} + 4e^{4t}y = e^{4t}(t+e^{-2t}) \Rightarrow \frac{d}{dt}(e^{4t}y) = e^{4t}(t+e^{-2t})$

ex. $\begin{cases} 2y' + ty = 2 \\ y(0) = 1 \end{cases}$

$y' + \frac{t}{2}y = 1 \Rightarrow p(t) = \frac{t}{2}, \quad \int p(t)\,dt = \frac{t^2}{4}, \quad e^{\frac{t^2}{4}}$

$e^{\frac{t^2}{4}}y' + \frac{t}{2}e^{\frac{t^2}{4}}y = e^{\frac{t^2}{4}}$

$\frac{d}{dt}\left(e^{\frac{t^2}{4}}y\right) = e^{\frac{t^2}{4}}y' + \frac{t}{2}e^{\frac{t^2}{4}}y = e^{\frac{t^2}{4}}$

$\Rightarrow e^{\frac{t^2}{4}}y = \int_0^t e^{\frac{s^2}{4}}\,ds + C$

$y(0) = 1 \Rightarrow 1 = C$

$\Rightarrow e^{\frac{t^2}{4}}y = \int_0^t e^{\frac{s^2}{4}}\,ds + 1 \Rightarrow y = e^{-\frac{t^2}{4}}\int_0^t e^{\frac{s^2}{4}}\,ds + e^{-\frac{t^2}{4}}$

2-2 $\frac{dy}{dx} = f(x,y) \iff \frac{dy}{dx}\ \frac{M(x,y)}{N(x,y)} \iff M(x,y)\,dx = N(x,y)\,dy$

If $M(x,y) = M(x)$ and $N(x,y) = N(y)$ then $M(x)\,dx = N(y)\,dy$

$\Rightarrow \int M(x)\,dx = \int N(y)\,dy$

ex. $\frac{dy}{dx} = \frac{x^2}{1-y^2}$

$\int x^2\,dx = \int (1-y^2)\,dy \Rightarrow \frac{x^3}{3} = y - \frac{y^3}{3} + C$

ex. $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1$

$\int 3x^2+4x+2\,dx = \int 2y-2\,dy \Rightarrow x^3+2x^2+2x+C = y^2-2y$

$y^2-2y-(x^3+2x^2+2x+C) = 0$

$y = \frac{2 \pm \sqrt{4+4(x^3+2x^2+2x+C)}}{2} = 1 \pm \sqrt{1+(x^3+2x^2+2x+C)}$

代入 $y(0) = -1$, $-1 = 1-\sqrt{1+C}$, $C = 3$

$y = 1-\sqrt{x^3+2x^2+2x+4}$

ex. $\begin{cases} \frac{dy}{dx} = \frac{4x-x^2}{4+y^3} \\ y(0)=1 \end{cases}$

$\int 4x-x^2\,dx = \int 4+y^3\,dy \Rightarrow -\frac{x^3}{3}+2x^2 = \frac{y^4}{4}+4y+C$

$\frac{y^4}{4}+4y = -\frac{x^3}{3}+2x^2+\tilde{C}$

代入 $y(0)=1$, $4+\frac{1}{4}=\tilde{C}=\frac{17}{4}$

$4y+\frac{1}{4}y^4 = 2x^2-\frac{1}{4}x^4+\frac{17}{4}$

2-5 ??? $\frac{dy}{dt} = r(1-\frac{y}{k})y$, $r.k>0$

↳ Logistic equation 沒時間慢慢講

考

$\frac{dy}{dt}=0 \Rightarrow r(1-\frac{y}{k})y=0 \Rightarrow y=0$ or $y=k$

If $0<y_0<k$, then $0<y<k$

$\frac{dy}{dt} = r(1-\frac{y}{k})y \Rightarrow \int \frac{1}{(1-\frac{y}{k})y}\,dy = \int r\,dt = rt+C$

$\frac{1}{(1-\frac{y}{k})y} = \frac{A}{(1-\frac{y}{k})}+\frac{B}{y}$,

$1 = Ay+B(1-\frac{y}{k}) = Ay+B-\frac{B}{k}y \Rightarrow \begin{cases} B=1 \\ A=\frac{1}{k} \end{cases}$

$\therefore \frac{1}{(1-\frac{y}{k})y} = \frac{1}{k-y}+\frac{1}{y}$

$\int \frac{1}{(1-\frac{y}{k})y}\,dy + \frac{1}{y}\,dy = -\ln|k-y|+\ln|y| = \ln\frac{y}{k-y} = rt+c$

$\frac{y}{k-y} = \tilde{C}e^{rt} \Rightarrow y = k\tilde{C}e^{rt}-y\tilde{C}e^{rt}$

$\Rightarrow (1+\tilde{C}e^{rt})y = k\tilde{C}e^{rt} \Rightarrow y = \frac{\tilde{C}ke^{rt}}{1+\tilde{C}e^{rt}}$

2-6 $M(x,y)+N(x,y)y'=0$, Find $\varphi(x,y)$ s.t $\varphi_x=M(x,y)$, $\varphi_y=N(x,y)$

$\Leftarrow$ $\frac{d\varphi}{dx}=\varphi_x+\varphi_y y'=M(x,y)+N(x,y)y'=0 \Rightarrow \varphi(x,y)=C$

ex. $6x+2y^2+4xyy'=0$

$M(x,y)=6x+2y^2$, $N(x,y)=4xy$ $\quad \int N(x,y)\,dy=2xy^2$

$\int M(x,y)\,dx=3x^2+2xy^2 \quad \varphi(x,y)=3x^2+2xy^2=C$

(If $M_y(x,y)=\varphi_{xy}=\varphi_{yx}=N_x(x,y)$.就可以解嘍)

ex. $3x^2y-\cos x+(x^3-\sin y)y'=0$

$M(x,y)=3x^2y-\cos x$, $N(x,y)=x^3-\sin y$

$\int N(x,y)\,dy=x^3y+\cos y$. $\int M(x,y)\,dx=x^3y-\sin x$

$\varphi(x,y)=x^3y-\sin x+\cos y+C$

ex. $(6xy+y^2)+(2x^2+xy)y'=0$

$M(x,y)=6xy+y^2$, $N(x,y)=2x^2+xy$

$\int N(x,y)\,dy=2x^2y+\frac{1}{2}xy^2$. $\int M(x,y)\,dx=3x^2y+xy^2$

$M(x,y)\,dx+N(x,y)\,dy=0 \; (M_y\neq N_x) \xrightarrow{\times\mu} \mu M\,dx+\mu N\,dy=0$

$(\mu M)_y=(\mu N)_x \Rightarrow (\mu_y M+\mu M_y=\mu_x N+\mu N_x)$

Find $\mu=\mu(x)$. $N\mu_x=\mu M_y-\mu N_x \Rightarrow \mu_x=\frac{\mu(M_y-N_x)}{N}$

$\Rightarrow \frac{\mu_x}{\mu}=\frac{M_y-N_x}{N}$

$\Rightarrow \ln|\mu|=\int\frac{M_y-N_x}{N}dx \Rightarrow \mu=e^{\int\frac{M_y-N_x}{N}dx}$

ex. $(6xy+y^2)+(2x^2+xy)y'=0$

$\frac{M_y-N_x}{N}=\frac{6x+2y-(4x+y)}{2x^2+xy}=\frac{2x+y}{2x^2+xy}=\frac{1}{x} \Rightarrow \mu=e^{\int\frac{1}{x}dx}=x$

$(6x^2y+xy^2)\,dx+(2x^3+x^2y)\,dy=0 \Rightarrow 2x^3y+\frac{1}{2}x^2y^2=C$

2-8 Thm: If f and $\frac{\partial f}{\partial y} \in C(D)$, $D = \{(t,y) \in \mathbb{R}^2 . |t| \leq a . |y| = l\}$

then there is same interval $|t| \leq h \leq a$ in which there exists a unique solution $y = \phi(t)$ of $\begin{cases} y' = f(t,y) \\ y(0) = b \end{cases}$

$\int_0^t y'(s)\,ds = \int_0^t f(s,y)\,ds \Rightarrow y(t) = y(0) + \int_0^t f(s,y)\,ds = b + \int_0^t f(s,y)\,ds$

Find $y = \phi(t) \in C(R)$, $R = \{(t,y) : |t| \leq h . |y| \leq l\}$

s.t $\phi(t) = b + \int_0^t f(s,\phi(s))\,ds$

Picard method. $\phi_0 = b$, $\phi_1(t) = b + \int_0^t f(s,\phi_0(s))\,ds$

$$\phi_2(t) = b + \int_0^t f(s,\phi_1(s))\,ds$$
$$\vdots$$
$$\phi_n(t) = b + \int_0^t f(s,\phi_{n-1}(s))\,ds$$

$\phi = \lim_{n\to\infty} \phi_n$, $\phi(t) = \lim_{n\to\infty} b + \int_0^t f(s,\phi_{n-1}(s))\,ds = b + \int_0^t f(s.\phi(s))\,ds$

$\phi_n(t) = \phi_0 + \phi_1 - \phi_0 + \cdots + \phi_n - \phi_{n-1}$, $|\phi_n(t)| \leq |\phi_1| + |\phi_2 - \phi_1| + \cdots |\phi_n - \phi_{n-1}|$

ex. $\begin{cases} y'(t) = -y \\ y(0) = 1 \end{cases}$ $f(t,y) = -y$, $\phi_0 = 1$

$\phi_1(t) = 1 + \int_0^t -1\,ds = 1 - t$

$\phi_2(t) = 1 + \int_0^t -(1-s)\,ds = 1 - \int_0^t (1-s)\,ds = 1 - t + \frac{t^2}{2}$

$\phi_3(t) = 1 - \int_0^t (1 - s + \frac{s^2}{2})\,ds = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!}$

$\vdots$

$\phi_{n-1}(t) = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} \cdots (-1)^{n-1} \frac{t^{(n-1)}}{(n-1)!}$

$\phi_n(t) = 1 - \int_0^t \phi_{n-1}(s)\,ds = 1 - t + \frac{t^2}{2!} + \cdots (-1)^{n-1} \frac{t^{(n-1)}}{(n-1)!} + (-1)^n \frac{t^n}{n!}$

$\phi(t) = \lim_{n\to\infty} \phi_n(t) = e^{-t}$

下禮拜四會出一題這個、考7題

ex. $\begin{cases} y' = 2t(1+y) \\ y(0) = 0 \end{cases}$, $f(s,\phi(s)) = 2s(1+\phi(s))$

$\phi_0 = 0$, $\phi_1 = \int_0^t 2s(1-\phi_0(s))\,ds = \int_0^t 2s\,ds = t^2$

$\phi_2 = \int_0^t 2s(1+s^2)\,ds = t^2 + \frac{1}{2}t^4$

每個考2題 $\phi_3 = \int_0^t 2s(1+s^2+\frac{1}{2}s^4)\,ds = t^2 + \frac{1}{2}t^4 + \frac{1}{6}t^6 \cdots$

$\vdots$

Picard Method

$\phi_n(t) = t^2 + \frac{1}{2!}t^4 + \cdots \frac{1}{n!}(t^2)^n = \sum_{k=1}^{n} \frac{(t^2)^k}{k!} = \sum_{k=0}^{n} \frac{(t^2)^k}{k!} - 1$

$+1$ $\phi(t) = e^{t^2} - 1$, $\phi'(t) = 2te^{t^2} = 2t(\phi+1) = 2t\phi + 2t$

Uniqueness : Suppose ϕ and Ψ satisfy

$\phi(t) = b + \int_0^t f(s,\phi(s))\,ds$, $\Psi(t) = b + \int_0^t f(s,\Psi(s))\,ds$

$(\phi - \Psi)(t) = \int_0^t f(s,\phi(s)) - f(s,\Psi(s))\,ds$

$= \int_0^t \frac{\partial f}{\partial y}(\phi(s) - \Psi(s))\,ds$

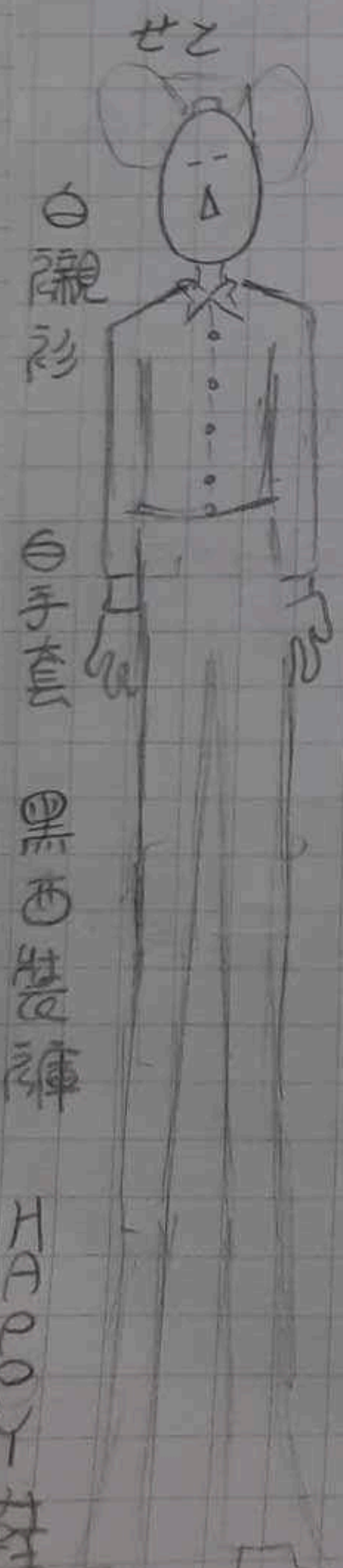
$|(\phi-\Psi)(t)| = \int_0^t |\frac{\partial f}{\partial y}||(\phi-\Psi)(s)|\,ds = A\int_0^t |(\phi-\Psi)(s)|\,ds$

Define $U(t) = \int_0^t |(\phi-\Psi)(s)|\,ds \geq 0$ $(*)$

$U'(t) = |(\phi-\Psi)(t)|$, From $(*)$, $U'(t) \leq AU(t)$

$\Rightarrow U'(t) - AU(t) \leq 0$, $\frac{d}{dt}[e^{-At}U(t)] \leq 0$

$e^{-At}U(t) - U(0) \leq 0 \rightarrow U(t) = 0 \rightarrow \phi = \Psi$



2x ① 積分因子

2x ② 分離系數 (隱函數)(前前前頁)

2x ③ $N_y = M_x$, (1題直接算、1題 $\times \mu(t)$)

1x ④ Picard Method