Subject ▸ Date ▸

1.

$$\begin{bmatrix} A & I \\ 0 & A^T \end{bmatrix} \begin{bmatrix} \hat{x} \\ r \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

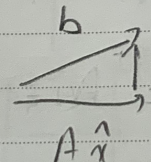
$$\begin{cases} A\hat{x} + r = b \\ A^T r = 0 \end{cases}$$

$N(A^T) = R(A)^\perp$

$r \in R(A)^\perp$

$\therefore (A\hat{x}) \perp r$

Then



satisfy $A\hat{x} + r = b$ #

$\hat{x}$ Least square solution

Subject ▸

2.

$\|x\|$

A

$\|x\|_A = \|$

2.

$$\|x\|_A = \|Ax\|_2$$

$$Ax = a_1x_1 + a_2x_2 + \cdots a_nx_n$$

$\|x\|_A = \|Ax\|_2 \ge 0$ clearly --- ①

$$\|\alpha x\|_A = \|\alpha Ax\|_2 = \alpha\|Ax\|_2 = \alpha\|x\|_A \quad \text{--- ②}$$

$$\|x+y\|_A = \|A(x+y)\|_2 = \|Ax+Ay\|_2 \le \|Ax\|_2 + \|Ay\|_2$$

$$= \|x\|_A + \|y\|_A$$

--- ③

$\|x\|_A = 0$ iff $x=0$, for A is nonsingular --- ④

#

3.

Say, $u = u_1 x_1 + u_2 x_2 + \cdots u_k x_k \in S_1$

$v = v_1 x_{k+1} + v_2 x_{k+2} + \cdots v_{n-k} x_n \in S_2$

$\forall u, v \ (u \cdot v = 0)$

$S_1 \perp S_2$ #

4.

$$\begin{cases} x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} & x_3 = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \\ x_2 = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \end{cases}$$

$$y_1 = x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$y_2 = x_2 - \frac{y_1^T x_2}{y_1^T y_1} y_1$$

$$= \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} - \frac{7}{6} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{6} \\ \frac{4}{6} \\ \frac{7}{6} \end{bmatrix}$$

$$y_3 = x_3 - \frac{y_1^T x_3}{y_1^T y_1} y_1 - \frac{y_2^T x_3}{y_2^T y_2} y_2$$

$$= \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} - \frac{6}{6} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - \frac{0}{\|y_2\|} \begin{bmatrix} -\frac{1}{6} \\ \frac{4}{6} \\ \frac{7}{6} \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$$u_1 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$u_2 = \frac{1}{\sqrt{66}} \begin{bmatrix} -1 \\ 4 \\ 7 \end{bmatrix}$$

$$u_3 = \frac{1}{\sqrt{11}} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

5.

Subject ▶

Date ▶

5.

last term of $p(\lambda) = 3^2 (-2)^2 (1)$

$= 36 = \det(A)$

$\det(A^{-1}) = \frac{1}{36}$

#

Subject ▸

Date ▸

6.

$$A = \begin{bmatrix} a_{11} & a_{12} & & \\ a_{21} & & & \\ & & \ddots & \\ & & & a_{nn} \end{bmatrix} \quad \text{where } \sum_{i=1}^{n} a_{im} = 1 \quad \forall m$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & \\ a_{12} & \ddots & & \\ & & & a_{nn} \end{bmatrix}$$

$$A^T e = \begin{bmatrix} \Sigma a_{i1} \\ \Sigma a_{i2} \\ \vdots \\ \Sigma a_{in} \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = e$$

$$A^T e = e$$

e is an eigenvector of A^T

Subject ▸

Date ▸

7.

$$f(x,y)=\frac{y}{x^2}-\frac{x}{y^2}+xy$$

$$\frac{\partial f(x,y)}{\partial x}=-2yx^{-3}-y^{-2}+y$$

$$\frac{\partial f(x,y)}{\partial y}=x^{-2}+2xy^{-3}+x$$

$f_{xx}=6yx^{-4}$ $\qquad f_{xx}(1,-1)=-6$

$f_{xy}=(-2x^{-3}+1)+2y^{-3}$, $\qquad f_{xy}(1,-1)=1$

$f_{yy}=-6xy^{-4}$ $\qquad f_{yy}(1,-1)=-6$

$$D=(-6)(-6)-1^2=35$$

$$f_{xx}(1,-1)=-6<0$$

$(1,-1)$ is a local minimum #

Subject ▸ Date ▸

8.

$a_{11} = (L_{11})^2$

$a_{21} = L_{21} L_{11}$

$a_{31} = L_{31} L_{11}$

$$\begin{bmatrix} 1 & -3 & 0 \\ -3 & 11 & -2 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} L_{11} & L_{21} & L_{31} \\ 0 & L_{22} & L_{32} \\ 0 & 0 & L_{33} \end{bmatrix}$$

$L_{11} = 1$

$L_{21} = -3$

$L_{31} = 0$

$(L_{22})^2 + (L_{21})^2 = 11$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -3 & \sqrt{2} & 0 \\ 0 & -\sqrt{2} & \sqrt{2} \end{bmatrix}$$

$L_{22} = \sqrt{2}$

$L_{31} L_{21} + L_{22} L_{32} = -2$

$L_{32} = -\sqrt{2}$

$(L_{33})^2 + (L_{31})^2 + (L_{32})^2 = 4$

$L_{33} = \sqrt{2}$

력

Subject ▶ Date ▶

9.

$$\sigma_i^2(A) = \lambda_i(A\overline{A}^T)$$

$$= \lambda_i(A\overline{A})$$

$$\sigma_i(A) = \lambda_i(A)$$

#