Representation Theory of finite groups

Assignment Set 5

Due Day: May 16 (no extension using latex this time)

G is always a finite group. Any vector space V is finite dimensional over $\mathbb{C}$. $Z(G)$ means the center of G.

Problem A. (5pts) Decompose the quaternion group Q_8 into a disjoint union of its conjugacy classes.

Problem B. (5pts) Find the character table of the Klein 4-group $V_4 \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.

Problem C. Let V be an irreducible $\mathbb{C}[G]$-module and let χ be its character.

1. (5pts) Prove that if $g \in Z(G)$, then $|\chi(g)| = \dim V$.
2. (10pts) Prove that $(\dim V)^2 \leq [G : Z(G)] = \frac{|G|}{|Z(G)|}$.

Problem D. (10pts) Suppose G is a group of order 12 with 6 conjugacy classes. Each class has a representative element g_i and the size of each class is known. With the given information and some known entries, complete the following character table of G; that is, find all those unknowns α_i, β_i.

Size of conjugacy class	12	12	6	6	4	4
Representative	$e = g_1$	g_2	g_3	g_4	g_5	g_6
χ_1	1	1	1	1	1	1
χ_2	1	-1	-1	1	i	$-i$
χ_3	1	1	1	1	-1	-1
χ_4	1	-1	-1	1	$-i$	i
χ_5	α_1	α_2	α_3	α_4	α_5	α_6
χ_6	β_1	β_2	β_3	β_4	β_5	β_6