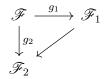
Proposition ([H] Prop 1.2 page 64) Given a presheaf \mathscr{F} then there exists a sheaf \mathscr{F}^+ and a unique morphism $g:\mathscr{F}\to\mathscr{F}^+$ with the following property: For any sheaf \mathscr{G} and morphism $f:\mathscr{F}\to\mathscr{G}$, there exits a unique morphism $\varphi:\mathscr{F}^+\to\mathscr{G}$ such that $f=\varphi\circ g$ i.e. the diagram

$$\mathscr{F} \xrightarrow{g} \mathscr{F}^+ \downarrow^{\varphi}_{\mathscr{G}}$$

commute. And such \mathscr{F}^+ is unique up to isomorphism. The sheaf \mathscr{F}^+ is called the sheaf associated to the presheaf \mathscr{F} .

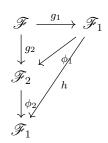
Proof. We prove the uniqueness first. Suppose we have two sheaves (\mathscr{F}_1, g_1) , (\mathscr{F}_2, g_2) satisfies the condition above. Then we have a commutative diagram



so we get a morphism $\phi_1: \mathscr{F}_1 \to \mathscr{F}_2$ such that $g_2 = \phi_1 \circ g_1$. On the other hand, we have

$$\begin{array}{ccc} & & \mathscr{F} & \\ & \swarrow_{g_2} & & \downarrow_{g_1} \\ \mathscr{F}_2 & \longrightarrow & \mathscr{F}_1 & \end{array}$$

Thus we also get a morphism $\phi_2: \mathscr{F}_2 \to \mathscr{F}_1$ such that $g_1 = \phi_2 \circ g_2$. Now we have a commutative diagram



where h can be chosen by id or $\phi_2 \circ \phi_1$. Since $\phi_2 \circ g_2 = \mathrm{id} \circ g_1 = g_1$, then by uniqueness $\mathrm{id} = \phi_2 \circ \phi_1$. We can do the same thing to $\phi_1 \circ \phi_2$. This proved the uniqueness.

Now we prove the existence. We constructed \mathscr{F}^+ by for each open set U, $\mathscr{F}^+(U)$ is the collection of the function $s: U \to \coprod_{P \in U} \mathscr{F}_P$ with the following properties:

- (1) $s(P) \in \mathcal{F}_P$ for all $P \in U$
- (2) For each $P \in U$ there is an open nbd. $V_P \subset U$ of P and a section $t \in \mathscr{F}(U)$ such that for all $Q \in U$ $s(Q) = t_Q \in \mathscr{F}_Q$

The restriction map of \mathscr{F}^+ is the restriction map of fuction. This makes \mathscr{F}^+ into a sheaf.

For each $s \in \mathscr{F}(U)$ we define $g(s) = s^* : U \to \coprod \mathscr{F}_P$ by $Q \mapsto s_Q$ then $s^* \in \mathscr{F}^+(U)$. May verify g is a morphism of pesheaves imediately. Now given a sheaf \mathscr{G} and a morphism $f : \mathscr{F} \to \mathscr{G}$, we define $\varphi : \mathscr{F}^+ \to \mathscr{G}$ as follows: For each open set $U, s^* \in \mathscr{F}^+(U)$, if s^* is come forem $\mathscr{F}(U)$ then we define $\varphi(s^*) = f(s)$, if not, we use the condition (2), there is an open cover V_P of U such that $s^*|_{V_P}$ is come from $\mathscr{F}(V_P)$ for some $t^P \in \mathscr{F}(V_P)$, then we define $\varphi(s^*)$ to be the glueing section of the family $\{f(t^P)\}$.

Remark By the condition (2) we see that every section in \mathscr{F}^+ is locally comes from \mathscr{F} . In particular, every global section $s \in \Gamma(X, \mathscr{F}^+)$ can be represent by $\{(s_i, U_i)\}$, $s_i \in \mathscr{F}(U_i)$ and $s|_{U_i} = g(s_i) = s_i^* : U_i \to \coprod \mathscr{F}_P$, $s|_{U_i}(P) = (s_i)_P \in \mathscr{F}_P$ for all $P \in U_i$. This notion we be used to represent *Cartier divisor* of a scheme X, also note that if s is already come from \mathscr{F} , then s_i can be chosen by $s_i = s|_{U_i}$.