Deadline : 2022/09/28, 17:00.

1. Use ε-δ to prove that
   (i) $\lim_{x\to a} x = a$.
   (ii) $\lim_{x\to a} c = c$

2. Find the limit $\lim_{x\to 0^+} \frac{\sin x}{x}$. Hint: Do not use L'Hôpital's rule.Use Squeeze theorem.

3. Use ε-δ to prove that $\lim_{x\to 2} x^4 = 16$.

4. The Heaviside function is defined as
$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x \le 0. \end{cases}$$
   Use ε-δ to prove that $H(x)$ does not have limit at $x = 0$.

5. Let $f(x) = \sin(\frac{1}{x})$. Use ε-δ to prove that $f(x)$ does not have limit at $x = 0$.

6. Prove that the following three triangle inequalities are equivalent.
   (i) $|a + b| \le |a| + |b|$
   (ii) $|a| - |b| \le |a - b|$
   (iii) $\big||a| - |b|\big| \le |a - b|$

   Hint: Prove (i) $\iff$ (ii) $\iff$ (iii) or prove (i) $\to$ (ii) $\to$ (iii) $\to$ (i).

7. (Uniqueness of limit) Use ε-δ to prove that if the limit of a function exists as x approaches a, then it is unique. That is, if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} f(x) = M$, then $L = M$.

8. Use ε-δ to prove that $\lim_{x\to a} f(x) = 0$ if and only if $\lim_{x\to a} |f(x)| = 0$.

9. (Product rule of limit law) Use ε-δ to prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then $\lim_{x\to a} (fg)(x) = LM$.

10. (Quotient rule of limit law) Use ε-δ to prove that if $\lim_{x\to a} g(x) = M$ provided $M \ne 0$, then $\lim_{x\to a} \frac{1}{g(x)} = \frac{1}{M}$.