

Quiz 9, Advanced Calculus I, Yung Fu Fang

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Name:

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Group

Riemann-Stieltjes integral of bounded function f with respect to α over the interval $[a, b]$

Let α be a function on $[a, b]$.

Let $\mathcal{P} =$ be a partition of $[a, b]$.

$\Delta\alpha_j =$

Upper Sum $U(\mathcal{P}, f, \alpha) =$, where $M_j =$.

Lower Sum $\mathcal{L}(\mathcal{P}, f, \alpha) =$, where $m_j =$.

Upper Integral $\overline{\int}_a^b f(x) d\alpha =$.

Lower Integral $\underline{\int}_a^b f(x) d\alpha =$.

f is called Riemann-Stieltjes integrable if $=$, which is called

the Riemann-Stieltjes integral of f with respect to α over $[a, b]$ and denoted by .

The partition \mathcal{P}^* is a refinement of the partition \mathcal{P}_2 if .

The partition \mathcal{P}^* is a common refinement of the partitions \mathcal{P}_1 and \mathcal{P}_2 if .

If \mathcal{P}^* is a refinement of \mathcal{P} , then the relations for Upper Sums, Lower Sums, Upper Integral, and Lower

Integral of f are

$$\boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{} \leq \boxed{}.$$

If f is Riemann-Stieltjes integrable with respect to α over $[a, b]$, then for every , there is a

partition \mathcal{P} such that .