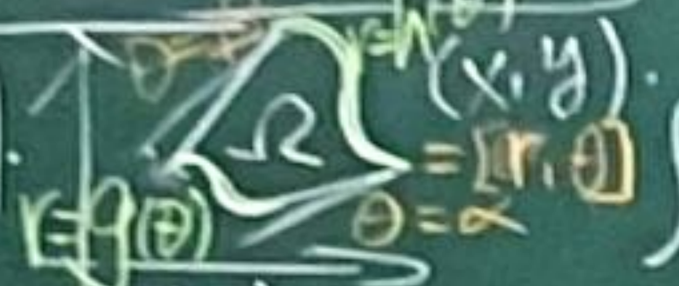
May 01, 2023, Midterm II
0810 ~ 11:00

Recall $\iint_{\Omega} f\,dA = \iint_{\Omega_1} f\,dA + \iint_{\Omega_2} f\,dA$

$\Omega = \Omega_1 \cup \Omega_2,\ \Omega_1 \cap \Omega_2 = \phi$

Recall

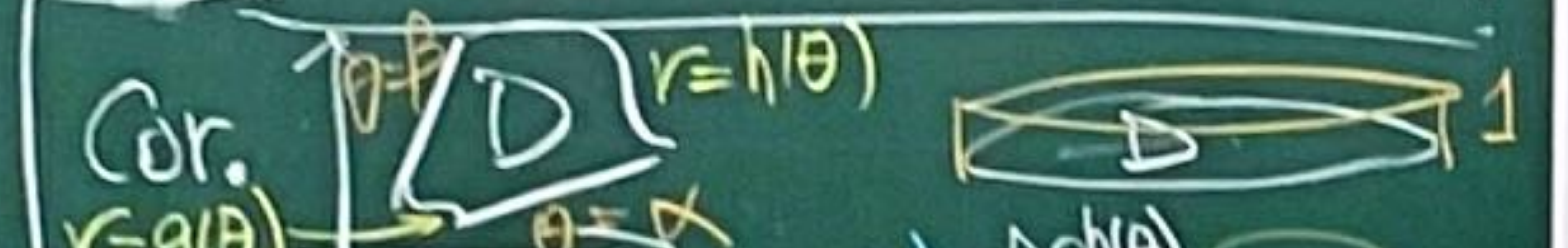


$\iint_{\Omega} f(x,y)\,dA$

$= \int_{\alpha}^{\beta}\int_{g(\theta)}^{h(\theta)} f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta$ (*)

$\in \mathbb{R}^2$ Ω $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Cor.



$A = |D| = \iint_D 1\,dA \overset{(*)}{=} \int_{\alpha}^{\beta}\int_{g(\theta)}^{h(\theta)} 1\cdot r\,dr\,d\theta$

$= \int_{\alpha}^{\beta} \frac{1}{2} r^2 \Big|_{g(\theta)}^{h(\theta)} d\theta = \int_{\alpha}^{\beta} \frac{1}{2}[(h(\theta))^2 - (g(\theta))^2]\,d\theta$

Total mass $m \approx \sum_{i=1}^{k}\sum_{j=1}^{\ell} \rho(x_i^*, y_j^*)\Delta A$ (Riemann Sum)

$\Rightarrow m = \lim_{k,\ell\to\infty} \sum_{i=1}^{k}\sum_{j=1}^{\ell} \rho(x_i^*, y_j^*)\Delta A$

$\equiv \iint_D \rho(x,y)\,dA$

$\sigma(x,y)$ = charge density at $(x,y) \in D$

$\Rightarrow$ Total charge $Q = \iint_D \sigma(x,y)\,dA$ ※

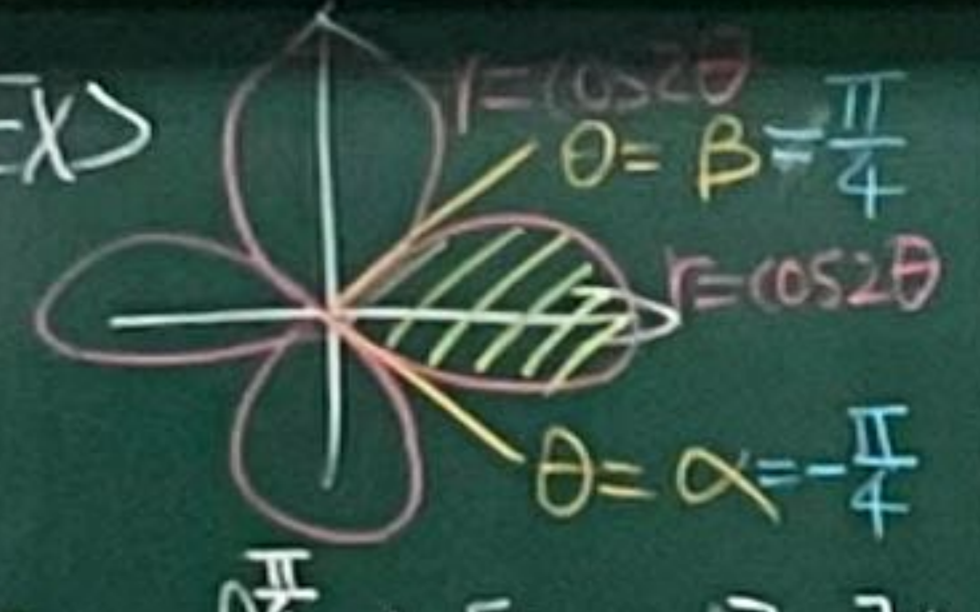
Moments and Center of Mass.

1) 矩 = mass × distance.

mass of $R_{ij} \approx \rho(x_{ij}^*, y_{ij}^*)\Delta A$

Moment of R_{ij} w.r.t. the x-axis $\approx [\rho(x_{ij}^*, y_{ij}^*)\Delta A]\, y_{ij}^*$

Ex)



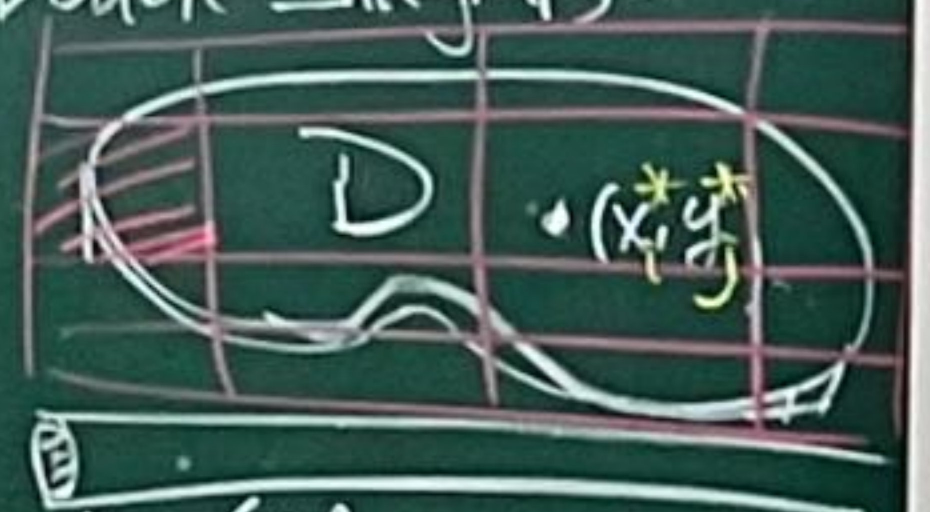
$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}[(\cos 2\theta)^2 - 0]\,d\theta$

$=$ ※

§15.5 Applications of Double Integrals.

Density and Mass.

Consider a thin plate



density $\rho(x,y)$ = conti. on D (if ρ = const. $\Rightarrow$ $\Delta M = \rho\Delta A$)

$\Rightarrow \rho(x,y) \approx \lim_{\Delta A\to 0} \frac{\Delta M}{\Delta A}$

Total moment about the x-axis.

$M_x \equiv \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} [\rho(x_{ij}^*, y_{ij}^*)\Delta A]\, y_{ij}^*$

(Riemann Sum)

$\equiv \iint_R \rho(x,y)\, y\, dA$

Similarly $M_y = {}'' = \iint_R \rho(x,y)\, x\, dA$ ※

Moment of Inertial (2nd Moment)

Moment of Inertial $I = mr^2$

$I_x \equiv \iint_D y^2 \rho(x,y)\,dA$

$I_y = \iint_D x^2 \rho(x,y)\,dA$

Moment of inertia about the origin

$$I_0 = I_x + I_y = \iint_R (x^2+y^2)\rho(x,y)\,dA$$

Radius of gyration $\bar{\bar{y}}$ w.r.t. the x-axis

$$\boxed{m\bar{\bar{y}}^2 = I_x}$$

" " " $\bar{\bar{x}}$ " " y-

$$\boxed{m\bar{\bar{x}}^2 = I_y}$$

Probability p.d.f.

$f(x)$ = probability density fn of a conti. random variable X

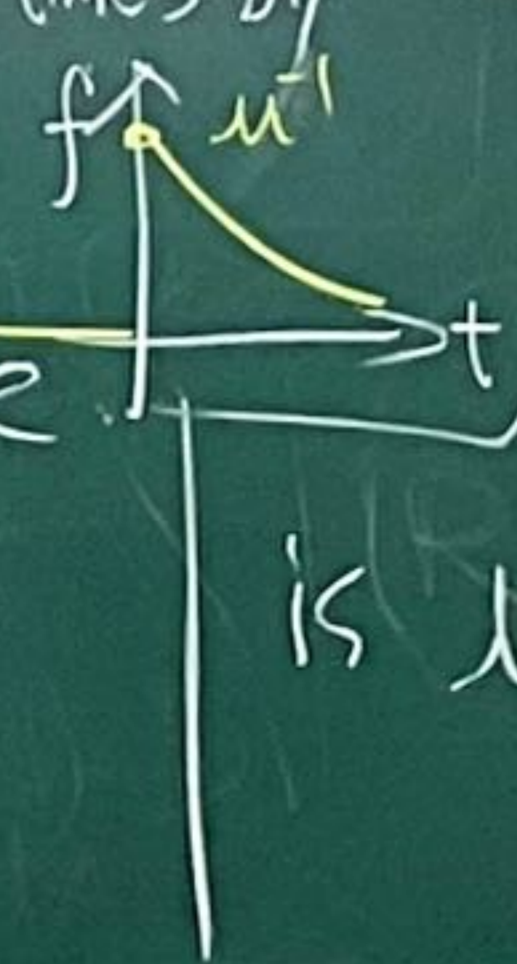
$$\left(f(x) \geq 0,\ \int_{-\infty}^{\infty} f(x)\,dx = 1 \right)$$

the probability that $a \leq X \leq b$ is

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

In §8.5 we modeled waiting times by

$$f(t) = \begin{cases} 0 & t \leq 0 \\ \mu^{-1} e^{-\frac{t}{\mu}}, & t \geq 0 \end{cases}$$



where μ = mean waiting time

(EX) next time

Expected Values. In §8.5

if X = R.V. w.r.t. p.d.f f

$\Rightarrow$ its mean (Expected value)

is $\mu = \int_{-\infty}^{\infty} x f(x)\,dx$

Consider a pair of conti. R.V. X & Y

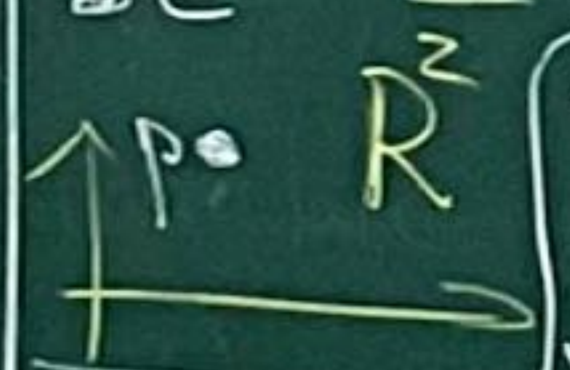
Def f = joint density fn of X and Y

if $P((X,Y) \in D) = \iint_D f(x,y)\,dA$

In particular,

$$P(a \leq X \leq b,\ c \leq Y \leq d) = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Also $\iint_{R^2} f(x,y)\,dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$



Suppose.

X = R.V. with pdf $f_1(x)$

Y = " " $f_2(y)$

Def X, Y = independent R.V.

if their joint density fn $f(x,y) = f_1(x) f_2(y)$

If X, Y = R.V. with joint pdf. f

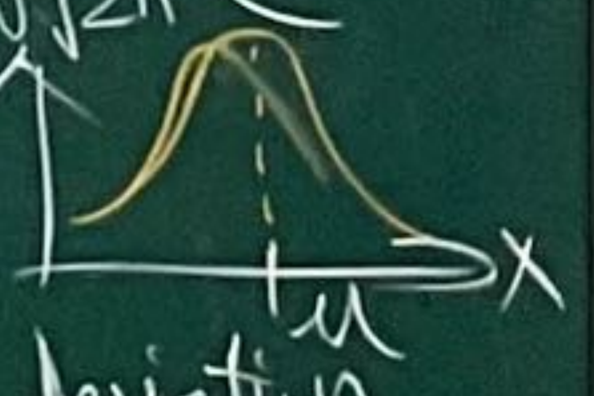
X-mean (expected value of X) $\mu_1 \equiv \iint_{R^2} x f(x,y)\,dA$

Y " " Y $\mu_2 \equiv \iint_{R^2} y f(x,y)\,dA$

Expected values (μ_1, μ_2) ~ Moments (M_x, M_y)

Def normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



μ = mean

σ = standard deviation