$$N(A) = N([a_1, a_2, a_3, a_4, a_5])$$

$$= N([a_1, a_2, a_3, a_1 - 5a_3, 3a_1 - 2a_2 + 4a_3])$$

$$= N(EA) = N(a_1, a_2, a_3, 0, 0)$$

$$\text{rank}(a_1, a_2, a_3, 0, 0) = 3$$

$$N(a_1, a_2, a_3, 0, 0) = 5 - 3$$

$$= 2 \quad \#$$

---

b) rank $(A) = 3$ and it has to be redweil

echelon form

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & \boxed{[illegible]} & \boxed{[illegible]} \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & & \end{bmatrix}$$

$\therefore a_4 = a_1 - 5a_3, \quad a_5 = 3a_1 - 2a_2 + 4a_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.

a)

$$L(\alpha x) = L((\alpha x_1, \alpha x_2)^T)$$

$$= (\alpha x_2, 2\alpha x_1)^T$$

$$= \alpha (x_2, 2x_1)^T$$

$$= \alpha L(x)$$

$$L(x+y) = L((x_1+y_1, x_2+y_2)^T)$$

$$= (x_2+y_2, 2x_1+2y_1)^T$$

$$= (x_2, 2x_1)^T + (y_2, 2y_1)^T$$

$$= L(x) + L(y)$$

Yes

即.

b)

$$L(\alpha x) = L((\alpha x_1, \alpha x_2)^T)$$
$$= (\alpha(x_1 + x_2), \alpha^2 x_1 x_2)$$
$$\neq \alpha L(x) = (\alpha(x_1 + x_2), \alpha x_1 x_2)$$

No. #

3.

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & 4 & -2 \\ 2 & 0 & -3 \end{bmatrix} \quad A$$

$$L(Y_1 + 0 + 0) = Y_1 + 2Y_2 + 2Y_3$$
$$L(0 + Y_2 + 0) = 4Y_2$$
$$L(0 + 0 + Y_3) = 2Y_1 - 2Y_2 - 3Y_3$$

$\downarrow$

with respect of a basis mean you can express absolute numbers, vectors through addition and scaler multiplication of the basis

4.

A similar to B.

exist a S, such that $S^{-1}AS = B$

$(S^{-1}AS)^T = B^T$

$S^T A^T (S^{-1})^T = B^T$

If $(S^{-1})^T S^T = I$,

A^T similar to B^T

$S \cdot S^{-1} = I$ (clearly)

$(S \cdot S^{-1})^T = I^T$

$(S^{-1})^T S^T = I^T = I$

#