Representation Theory of finite groups

Assignment Set 2

Due Day: Mar 22 (or Mar 25 if you are using latex)

Problem A. (9pts) Suppose that (ϕ,V) and (τ,V) are equivalent representations of the same group G. Show that

(a) If ϕ is faithful, then so is τ.

(b) If ϕ is irreducible, then so is τ.

(c) If ϕ is of degree 1, then $\phi = \tau$. (They are exactly the SAME, not just being equivalent)

Problem B. (6pts) Let V be a H-module and let $\theta : G \to H$ be a group homomorphism. Show that V is also a G-module by the following action

$$g \cdot v := \theta(g) \cdot v,$$

for all $g \in G, v \in V$.
In particular, if $N \lhd G$ and V is a G/N-module, then V is a G-module due to the canonical quotient homomorphism $\pi : G \to G/N$. This module is called the pullback of π.

Problem C. Recall the quaternion group $Q_8 =< a, b \,|\, a^4 = b^4 = 1, a^2 = b^2, ba = a^{-1}b >$. Define the map $\psi : Q_8 \to GL_4(\mathbb{C})$ by

$$\psi(a) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \psi(b) = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

(1) (5pts) Show that ψ gives a representation for Q_8.

(2) (5pts) Show that ψ is faithful.