Proposition ([H] Prop 1.2 page 64) Given a presheaf $\mathscr{F}$ then there exists a sheaf $\mathscr{F}^+$ and a unique morphism $g : \mathscr{F} \to \mathscr{F}^+$ with the following property: For any sheaf $\mathscr{G}$ and morphism $f : \mathscr{F} \to \mathscr{G}$. there exits a unique morphism $\varphi : \mathscr{F}^+ \to \mathscr{G}$ such that $f = \varphi \circ g$ i.e. the diagram

$$\begin{tikzcd}
\mathscr{F} \arrow[r, "g"] \arrow[dr, "f"'] & \mathscr{F}^+ \arrow[d, "\varphi"] \\
& \mathscr{G}
\end{tikzcd}$$

commute. And such $\mathscr{F}^+$ is unique up to isomorphism. The sheaf $\mathscr{F}^+$ is called the sheaf associated to the presheaf $\mathscr{F}$.

Proof. We prove the uniqueness first. Suppose we have two sheaves $(\mathscr{F}_1, g_1)$, $(\mathscr{F}_2, g_2)$ satisfies the condition above. Then we have a commutative diagram

$$\begin{tikzcd}
\mathscr{F} \arrow[r, "g_1"] \arrow[d, "g_2"] & \mathscr{F}_1 \arrow[dl] \\
\mathscr{F}_2
\end{tikzcd}$$

so we get a morphism $\phi_1 : \mathscr{F}_1 \to \mathscr{F}_2$ such that $g_2 = \phi_1 \circ g_1$. On the other hand, we have

$$\begin{tikzcd}
& \mathscr{F} \arrow[dl, "g_2"] \arrow[d, "g_1"] \\
\mathscr{F}_2 \arrow[r] & \mathscr{F}_1
\end{tikzcd}$$

Thus we also get a morphism $\phi_2 : \mathscr{F}_2 \to \mathscr{F}_1$ such that $g_1 = \phi_2 \circ g_2$. Now we have a commutative diagram

$$\begin{tikzcd}
\mathscr{F} \arrow[r, "g_1"] \arrow[d, "g_2"] & \mathscr{F}_1 \arrow[dl, "\phi_1"] \arrow[ddl, "h"] \\
\mathscr{F}_2 \arrow[d, "\phi_2"] & \\
\mathscr{F}_1 &
\end{tikzcd}$$

where h can be chosen by id or $\phi_2 \circ \phi_1$. Since $\phi_2 \circ g_2 = \mathrm{id} \circ g_1 = g_1$, then by uniqueness id$= \phi_2 \circ \phi_1$. We can do the same thing to $\phi_1 \circ \phi_2$. This proved the uniqueness.
Now we prove the existence. We constructed $\mathscr{F}^+$ by for each open set U, $\mathscr{F}^+(U)$ is the collection of the function $s : U \to \coprod_{P \in U} \mathscr{F}_P$ with the following properties:
(1) $s(P) \in \mathscr{F}_P$ for all $P \in U$
(2) For each $P \in U$ there is an open nbd. $V_P \subset U$ of P and a section $t \in \mathscr{F}(U)$ such that for all $Q \in U$ $s(Q) = t_Q \in \mathscr{F}_Q$
The restriction map of $\mathscr{F}^+$ is the restriction map of fuction. This makes $\mathscr{F}^+$ into a sheaf.

For each $s \in \mathscr{F}(U)$ we define $g(s) = s^* : U \to \coprod \mathscr{F}_P$ by $Q \mapsto s_Q$ then $s^* \in \mathscr{F}^+(U)$. May verify g is a morphism of pesheaves imediately. Now given a sheaf $\mathscr{G}$ and a morphism $f : \mathscr{F} \to \mathscr{G}$, we define $\varphi : \mathscr{F}^+ \to \mathscr{G}$ as follows: For each open set U, $s^* \in \mathscr{F}^+(U)$, if s^* is come forem $\mathscr{F}(U)$ then we define $\varphi(s^*) = f(s)$, if not, we use the condition (2), there is an open cover V_P of U such that $s^*|_{V_P}$ is come from $\mathscr{F}(V_P)$ for some $t^P \in \mathscr{F}(V_P)$, then we define $\varphi(s^*)$ to be the glueing section of the family $\{f(t^P)\}$. $\square$

Remark By the condition (2) we see that every section in $\mathscr{F}^+$ is locally comes from $\mathscr{F}$. In particular, every global section $s \in \Gamma(X, \mathscr{F}^+)$ can be reprensent by $\{(s_i, U_i)\}$, $s_i \in \mathscr{F}(U_i)$ and $s|_{U_i} = g(s_i) = s_i^* : U_i \to \coprod \mathscr{F}_P$, $s|_{U_i}(P) = (s_i)_P \in \mathscr{F}_P$ for all $P \in U_i$. This notion we be used to represent *Cartier divisor* of a scheme X, also note that if s is already come from $\mathscr{F}$, then s_i can be chosen by $s_i = s|_{U_i}$.