(1) Give an example of a smooth manifold. (Verify all the details in the definition.)

(2) Let X be a point set equipped with the following:
- a collection of subsets $\{U_\alpha\}_{\alpha\in I}$ covering X: that is, $U_\alpha \subset X$ and $\cup_{\alpha\in I} U_\alpha = X$ (where I is simply an index set)
- for each α, a bijection $\varphi_\alpha : U_\alpha \to \varphi_\alpha(U_\alpha) \subset \mathbb{R}^n$ onto an open set $\varphi_\alpha(U_\alpha)$ in $\mathbb{R}^n$
- for each $\alpha, \beta \in I$, $\varphi_\alpha(U_\alpha \cap U_\beta)$ is open in $\mathbb{R}^n$
- for each $\alpha, \beta \in I$, $\varphi_\beta \circ \varphi_\alpha^{-1} : \varphi_\alpha(U_\alpha \cap U_\beta) \to \varphi_\beta(U_\alpha \cap U_\beta)$ is C^∞ with C^∞ inverse.

Give X a topology (i.e., say which subset of X is open) so that X is a smooth manifold, ignoring Hausdorff and second countability.

(3) Let $\mathbb{R}$ be the real line with the differentiable structure given by the maximal atlas of the chart $(\mathbb{R}, \varphi = id : \mathbb{R} \to \mathbb{R})$, where id is the identity map, and let $\mathbb{R}'$ be the real line with the differentiable structure given by the maximal atlas of the chart $(\mathbb{R}, \psi : \mathbb{R} \to \mathbb{R})$, where $\psi(x) = x^{1/3}$.

(a) Show that these two differentiable structures are distinct.

(b) Show that there is a diffeomorphism between $\mathbb{R}$ and $\mathbb{R}'$. (Hint: The identity map $\mathbb{R} \to \mathbb{R}$ is not the desired diffeomorphism.)