Quiz 7, Advanced Calculus I, Yung Fu Fang

Nov. 27, 2023 Show All Work Name: Id: *Group*

4.8 Theorem *let f be a mapping from a metric space X into a metric space Y.*

Then $\boxed{f \text{ is continuous on } X} \iff \boxed{f^{-1}(V) \text{ is open in } X}$ *for every open set V in Y.*

Proof. For " $\Longrightarrow$ ",

Suppose that $\boxed{}$ and V is an open set in Y.

To show $f^{-1}(V)$ is open in X $\iff$ $\boxed{}$.

Suppose that $p \in f^{-1}(V) \subset X$, then $\boxed{}$.

Since V is open in Y, there is an $\varepsilon > 0$ such that the ball $\boxed{}$.

Since f is continuous at p, given $\varepsilon > 0$, there is a $\delta > 0$ such that $\boxed{}$ implies that $\boxed{}$,

thus $f(x) \in V$, that is $x \in f^{-1}(V)$. Hence the ball $\boxed{}$ $\subset f^{-1}(V)$, that is

p is an $\boxed{}$. Finally $\boxed{}$ is open in X.

For " $\Longleftarrow$ ", suppose that $\boxed{}$.

To show $\boxed{}$ $\iff$ Given an $\varepsilon > 0$, there is a $\delta > 0$ such that $\boxed{}$

implies that $\boxed{}$.

Let us fix a $p \in X$ and $\varepsilon > 0$. Denote the ball $V = \{y \in Y : d_Y\big(f(p), y\big) < \varepsilon\}$ which is $\boxed{}$.

Following the assumption, $\boxed{}$ and $\boxed{}$.

Then there is a $\delta > 0$ such that $\boxed{}$.

Thus $d_X(x, p) < \delta \Rightarrow$ $\boxed{}$ $\Rightarrow f(x) \in V \Rightarrow$ $\boxed{}$. Hence f is continuous at p.

The argument works for $\boxed{}$. Finally f is $\boxed{}$. $\square$