3.2 HW2

Question 25

Consider hear flow in a long circular cylinder where the temperature depends only on t and on the distance r to the axis of the cylinder. Here $r = \sqrt{x^2 + y^2}$ is the cylindrical coordinate. From the three dimensional hear equation derive the equation $u_t = k(u_{rr} + \frac{u_r}{r})$

Proof. Write the three dimensional hear equation by

$$u_t = k\Delta u$$

Note that the Laplacian Δu when written in cylindrical coordinate is

$$\Delta u = u_{rr} + \frac{u_r}{r} + \frac{u_{\theta\theta}}{r^2} + u_{zz}$$

Because the premise says that u is constant in z and θ, we know $u_{\theta\theta} = u_{zz} = 0$

$$\Delta u = u_{rr} + \frac{u_r}{r}$$

This give us

$$u_t = k(u_{rr} + \frac{u_r}{r})$$

■