DE Sec 2.1

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Definitions and Theorems

Definition 1. *A differential equation $f(t, y, y', \cdots, y^{(n)}) = 0$ is a linear equation if f is linear to $y, y', y'', \cdots, y^{(n)}$, and the order of f equals to n.*

Theorem 1. *All first-order linear equations $f(t, y, y') = 0$, if solvable, can be solved as following.*

Proof. Let $0 = f(t, y, y') = py' + q_0 y + r_0$, where p, q_0, r_0 are all functions of t.

If we exclude points such $p_0 = 0$ from domain, $y' + \frac{q_0}{p} y + \frac{r_0}{p} = 0$ share the same solutions with $f = 0$.

Let μ be a function of x defined by $\mu = exp(\int q dt + c)$, where c can be arbitrary number.

Let $\frac{q_0}{p} = q, -\frac{r_0}{p} = r$. Then

$$y' + qy = r$$

$\implies$

$$\mu y' + \mu q y = r\mu$$

Because $\mu' = (q) exp(\int q dx + c) = q\mu$

$$\frac{d}{dt} \mu y = \mu y' + \mu q y = r\mu$$

Integrating with respect to t from both side, we obtain

$$\mu y = \int r\mu dt$$

Noted μ is an exponential functions always greater than 0, so

$$y = \frac{1}{\mu} \int_{t_0}^{t} r\mu dt$$

where t_0 can be specific by other conditions.

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Exercises

1.

Proof. ■