1 Notes from elementary group theory to ring theory

For any group to be a ring, the second binary operation defined on it must follows the axioms:

R1: The group is abelian.
an axiom only about the first operation of the ring.

R2: The second operation is associative.
an axiom only about the second operation of the ring

R3: The second operation is distributive to the first operation on **both side**.
an axiom connect the first and second operation.

We **conjecture** that none of these 3 axioms are dependent to any combination of the others, that is, with any one or two axioms above, we can also build some ring-like structures.
The examples of algebraic structure that only satisfy the proper subset of three axioms will be given in **TBD**.

Generally, ring is not a group on group, that is, the second operation do not form a group.

However, we can still describe the second operation with the name, semi-group with a both-side zero, where the both-side zero is exactly the additive identity, which we will prove later in *Fact 1*.

R3 allow us to deduce some facts and raise some question.

Fact 1:

Let 0 denote the identity of first operation/addition.
$\forall x \in R, 0x = x0 = 00 = 0.$

Pf:

$\forall x, y \in R$
$0x = (y - y)x = yx - yx = 0$
$x0 = x(y - y) = xy - xy = 0$

Fact 2:

$\forall x, y \in R, -(xy) = (-x)y = x(-y)$ and $0 = -0$

Pf:

$\forall x, y \in R$
$xy + (-x)y = (x + (-x))y = 0y = 0$
$xy + x(-y) = x(y + (-y)) = x0 = 0$

Fact 3: