1.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

Find $\min_{\hat{c}} \| Y - A\hat{c} \|$

$$\hat{C} = (A^T A)^{-1} A^T Y$$

$$\hat{C_1} = \frac{4}{3}$$

$$\hat{C_2} = 1$$

2.

a) $\langle x, y \rangle$

$$= \langle 6u_1 - 3u_2 + 2u_3, 5u_1 + 8u_2 - 3u_3 \rangle$$

Say $m_1 = 6u_1, \ m_2 = -3u_2, \ m_3 = 2u_3$

$\qquad n_1 = 5u_1, \ n_2 = 8u_2, \ n_3 = -3u_3$

$\langle x, y \rangle$

$$= \sum_{i, k} \langle m_i, n_k \rangle$$

$$= \sum_{i=1} \langle m_i, n_i \rangle \qquad \left(\because \langle u_1, u_2 \rangle = 0 \right. \qquad \textcircled{1}$$
$$\langle u_1, u_3 \rangle = 0$$
$$\left. \langle u_2, u_3 \rangle = 0 \quad \text{given} \right)$$

$$= \langle 6u_1, 5u_1 \rangle + \langle -3u_2, 8u_2 \rangle + \langle 2u_3, -3u_3 \rangle$$

$$= 30 - 24 - 6 = 0$$
$$\#$$

b) $\| x \| = \sqrt{\langle x, x \rangle}$

$$= \sqrt{\sum_{i=1} \langle m_i, m_i \rangle} \quad (\because \textcircled{1})$$

$$= \sqrt{36 + 9 + 4} = \sqrt{49} = 7 \qquad \#$$

3.

$$\|u\| = \|(u-x)+x\| \leq \|u-x\| + \|x\|$$

$$\|u\| - \|x\| \leq \|u-x\|$$

$$\|x\| = \|(x-u)+u\| \leq \|x-u\| + \|u\|$$

$$\|x\| - \|u\| \leq \|x-u\|$$
$$\downarrow \times (-1) \qquad\qquad \downarrow \times (-1)$$

$$\|u\| - \|x\| \geq -\|u-x\|$$

$$\left| \|u\| - \|x\| \right| \leq \|u-x\|$$

Say, $x = -v$

$$\left| \|u\| - \|v\| \right| \leq \|u+v\|$$
$$\#$$