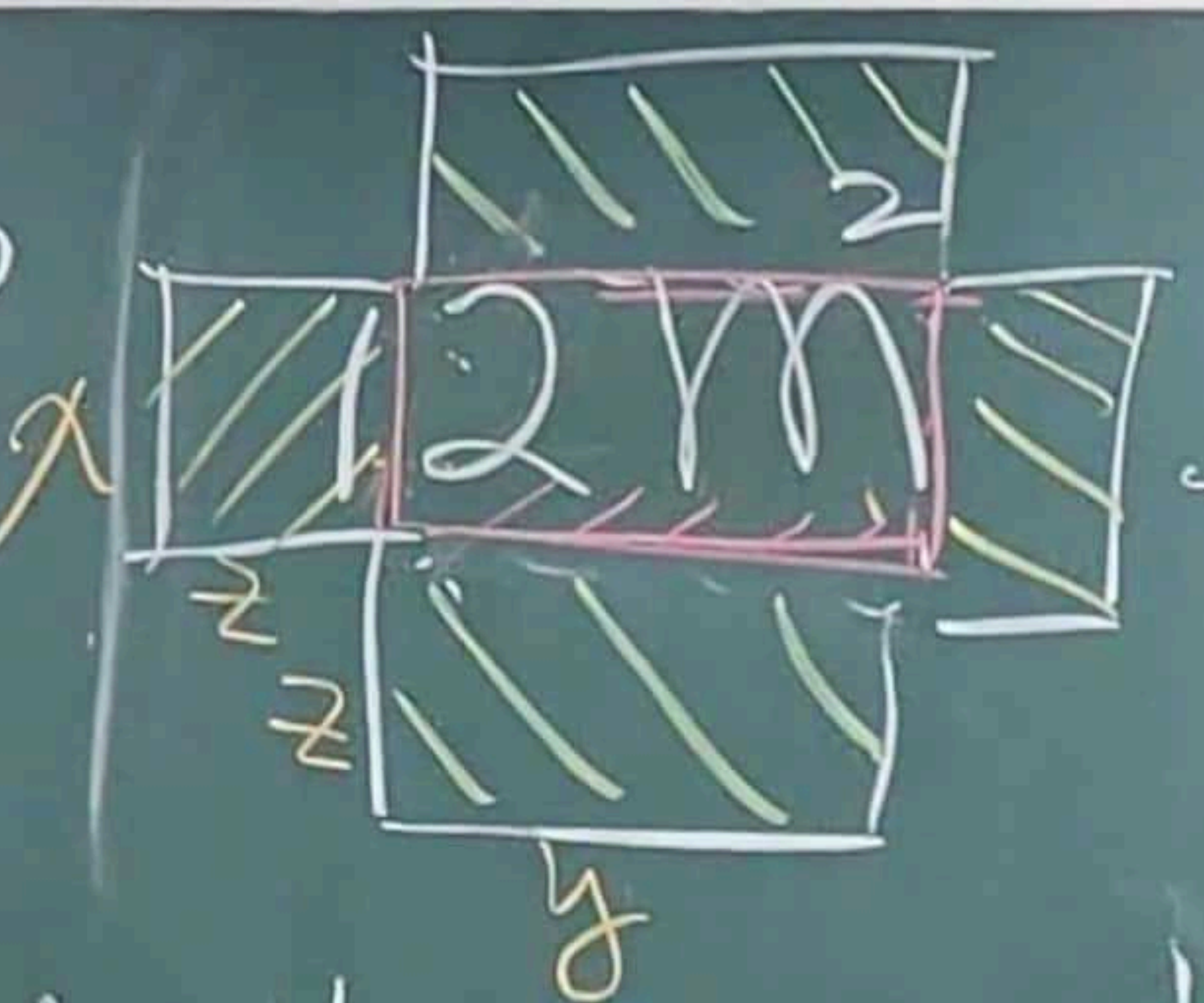
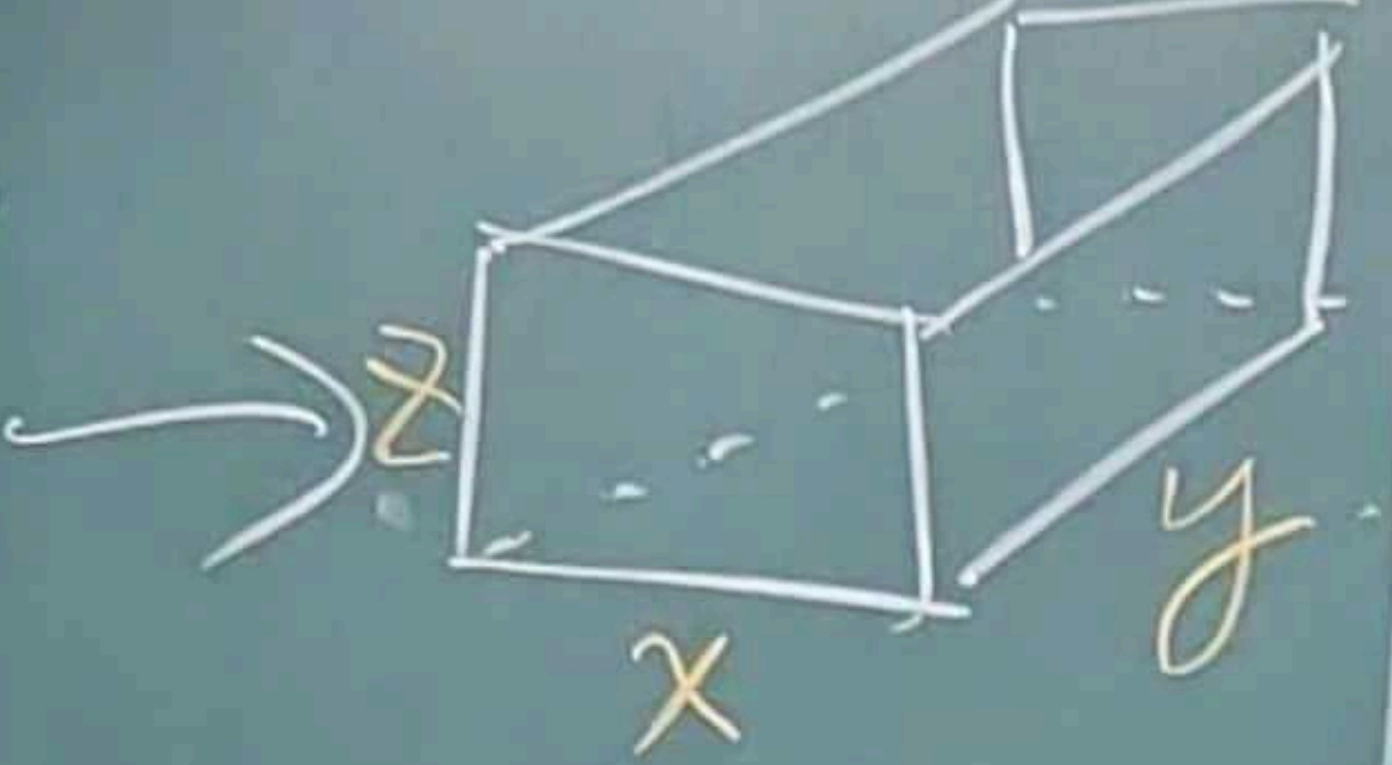
<Ex>





Find the max volume of box.

Sol: Area: $\underline{2xz}+\underline{2yz}+\underline{xy}=12$ (A)

$\Rightarrow z=\dfrac{12-xy}{2x+2y}$

Volume: $V=xyz=\dfrac{xy(12-xy)}{2(x+y)}$

$\Rightarrow V_x=0=V_y \Rightarrow \cdots\cdots$

$\Rightarrow \begin{cases} y^2(12-2xy-x^2)=0 \\ x^2(12-2xy-y^2)=0 \end{cases}$

assume $(x\neq 0,\ y\neq 0)$

$\Rightarrow \begin{cases} 12-2xy-x^2=0 \\ 12-2xy-y^2=0 \end{cases} \Rightarrow x^2=y^2 \Rightarrow x=\cancel{\pm}y$

$\Rightarrow 12-2x^2-x^2=0 \Rightarrow x=y=2 \Rightarrow z=1$

$\Rightarrow V = xyz = 2 \cdot 2 \cdot 1 = 4$

By 2nd Derivative Test:

$D = \begin{vmatrix} V_{xx} & V_{xy} \\ V_{yx} & V_{yy} \end{vmatrix}_{\substack{x=2 \\ y=2}} \overset{\text{Check}}{>} 0$

$V_{xx}\Big|_{\substack{x=2 \\ y=2}} \overset{\text{Check}}{<} 0 \quad \therefore V(2,2,1) = \text{local max}$

(B) Area $= 2xz + 2yz + xy = 12$.

$\Rightarrow z = \dfrac{12 - xy}{2x+2y} \Rightarrow V = xy\dfrac{12-xy}{2x+2y}$

Compute $V_x = 0,\ V_y = 0 \Rightarrow$ $x>0,\ y>0$ $x=a,\ y=b$.

$\Rightarrow V = xyz\Big|_{\substack{x=a \\ y=b \\ z=c}} = abc$. $z = c$.

※ By 2nd Derivative Test, max $V = abc$. #

(c) Graph $V(x,y) = xy\,\dfrac{12-xy}{2x+2y}$

Recall $y=f(x)$ Extreme Value Thm

If f = conti. on $[a,b]$ $\Rightarrow$ f has an absolute $\begin{matrix}\max\\ \min\end{matrix}$

closed interval

and $\begin{matrix}\min(f)\\ \max(f)\end{matrix} \in \{ f(a), f(b), f(c) \mid c = \text{critical pts of } f \}$

Def: D = closed set in $\mathbb{R}^2$ if D contains all its boundary pts

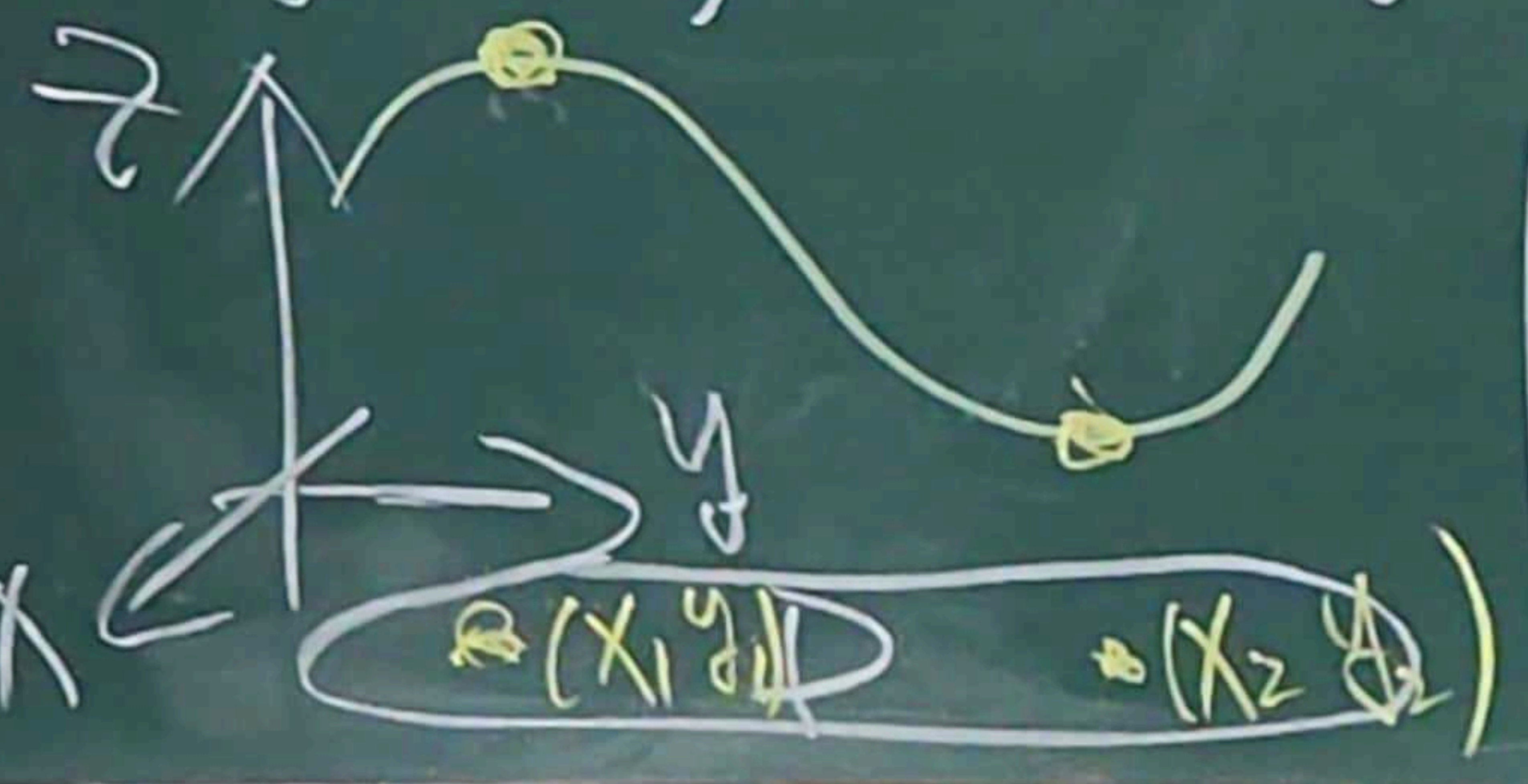
Def D = bounded set in $\mathbb{R}^2$ if

Extreme Value Thm for $z=f(x,y)$

If f is conti on a closed, bdd set $D \subset \mathbb{R}^2 \Rightarrow$

$\exists\ (x_1, y_1), (x_2, y_2) \in D$ s.t.

$f(x_1, y_1) = \max \quad f(x_2, y_2) = \min$



To find absolute max & min values of a conti. fn f on D (= closed & bdd)

1° Find all critical pts, $\begin{cases} f_x = 0 \\ f_y = 0 \end{cases}$ and f_x, f_y DNE

2° Find the extreme value of f on ∂D.

On ∂D, $x = x(t)$, $y = y(t) \Rightarrow f(x, y)\big|_{\partial D} = f(x(t), y(t)) \equiv g(t)$

3° largest one in 1° & 2° = max value

smallest one in 1° & 2° = min value

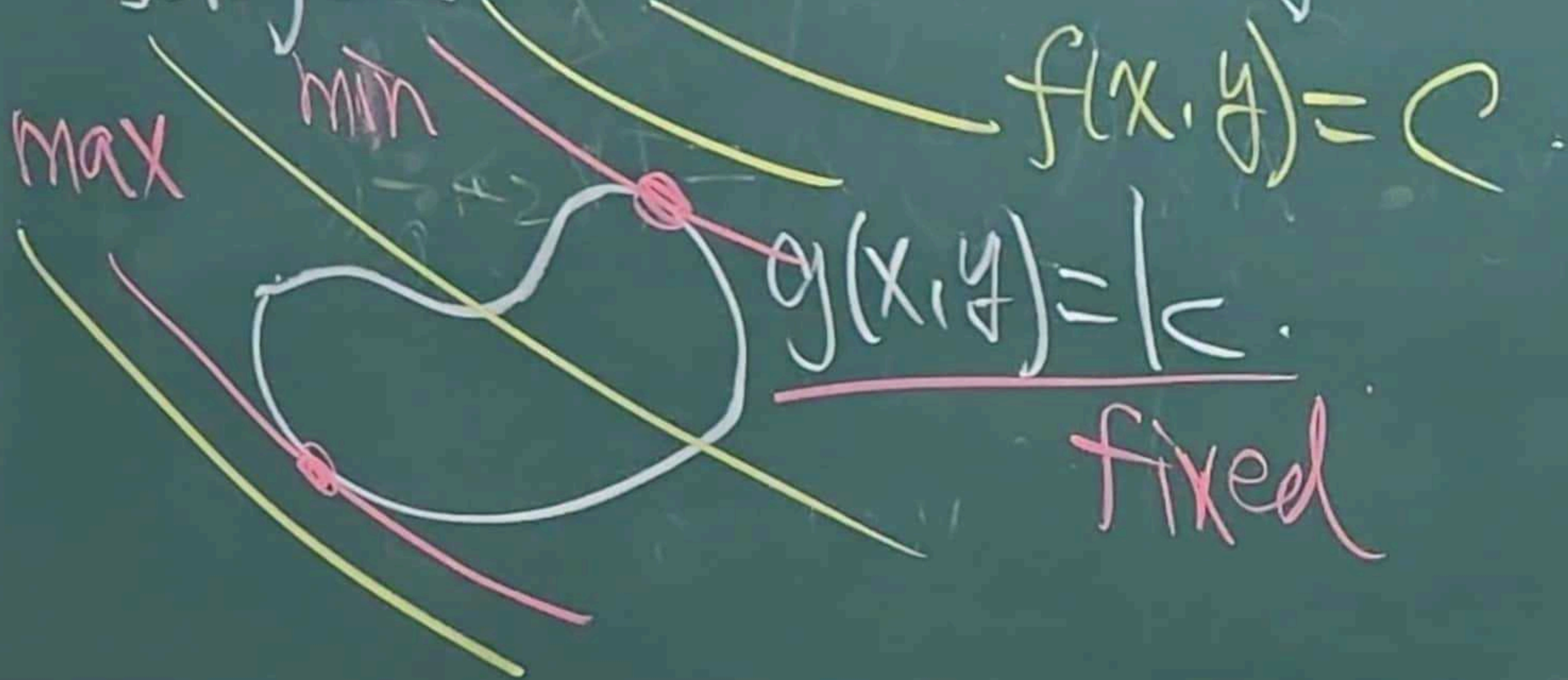
§14.8 Lagrange Multipliers

level curve

Find the extreme value of $\underline{f(x,y) = C_1}$

subject to a constraint $\underline{g(x,y) = k}$.

$\Rightarrow \nabla f \parallel \nabla g$



Solve $\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y) = k \end{cases}$

Find $\lambda = \lambda_0$, $x = a$, $y = b$

$\Rightarrow f(a,b) = \begin{matrix} \text{max ?} \\ \text{min ?} \end{matrix}$

$$\begin{cases} x-3=\lambda x \\ y-1=\lambda y \\ z+1=\lambda z \\ x^2+y^2+z^2=4 \end{cases} \Rightarrow \begin{cases} x^2=\left(\dfrac{3}{1-\lambda}\right)^2 \\ y^2=\left(\dfrac{1}{1-\lambda}\right)^2 \\ z^2=\left(\dfrac{-1}{1-\lambda}\right)^2 \end{cases}$$

$$+ \quad = 4$$

$$\Rightarrow (x,y,z)=\left(\frac{6}{\sqrt{11}},\frac{2}{\sqrt{11}},\frac{-2}{\sqrt{11}}\right)$$ closed pt

and $$\left(\frac{-6}{\sqrt{11}},\frac{-2}{\sqrt{11}},\frac{2}{\sqrt{11}}\right)$$ farthest pt

$$\Rightarrow \lambda = 1 \pm \frac{\sqrt{11}}{2}$$

Method of Lagrange Multipliers.

To find max or min of $f(x,y,z)$.

subject to $g(x,y,z)=k$

Assume $\nabla g \neq 0$

(a) Solve $\begin{cases} \nabla f(x,y,z)=\lambda \nabla g(x,y,z) \leftarrow 3 \text{ eq'ns} \\ g(x,y,z)=k \leftarrow 1 \text{ eq'n} \end{cases}$

say $(x_1,y_1,z_1,\lambda_1),(x_2,y_2,z_2,\lambda_2),\ldots$

(b) Evaluate $f(x_1,y_1,z_1), f(x_2,y_2,z_2)\cdots$

The largest one = max

smallest one = min

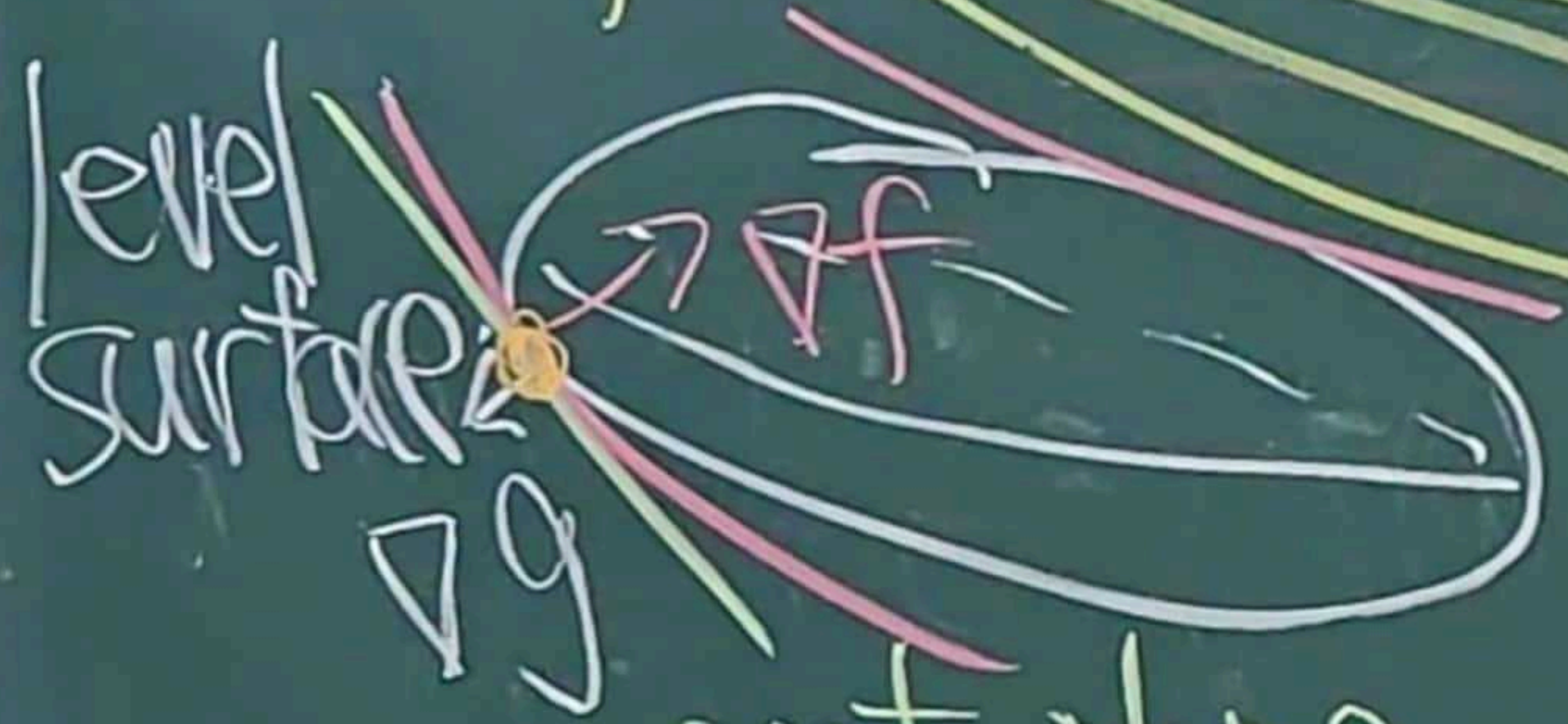
$\lambda \equiv$ Lagrange Multiplier

乘子

Midterm II on 2023-05-01
08:10 ~ 12:00

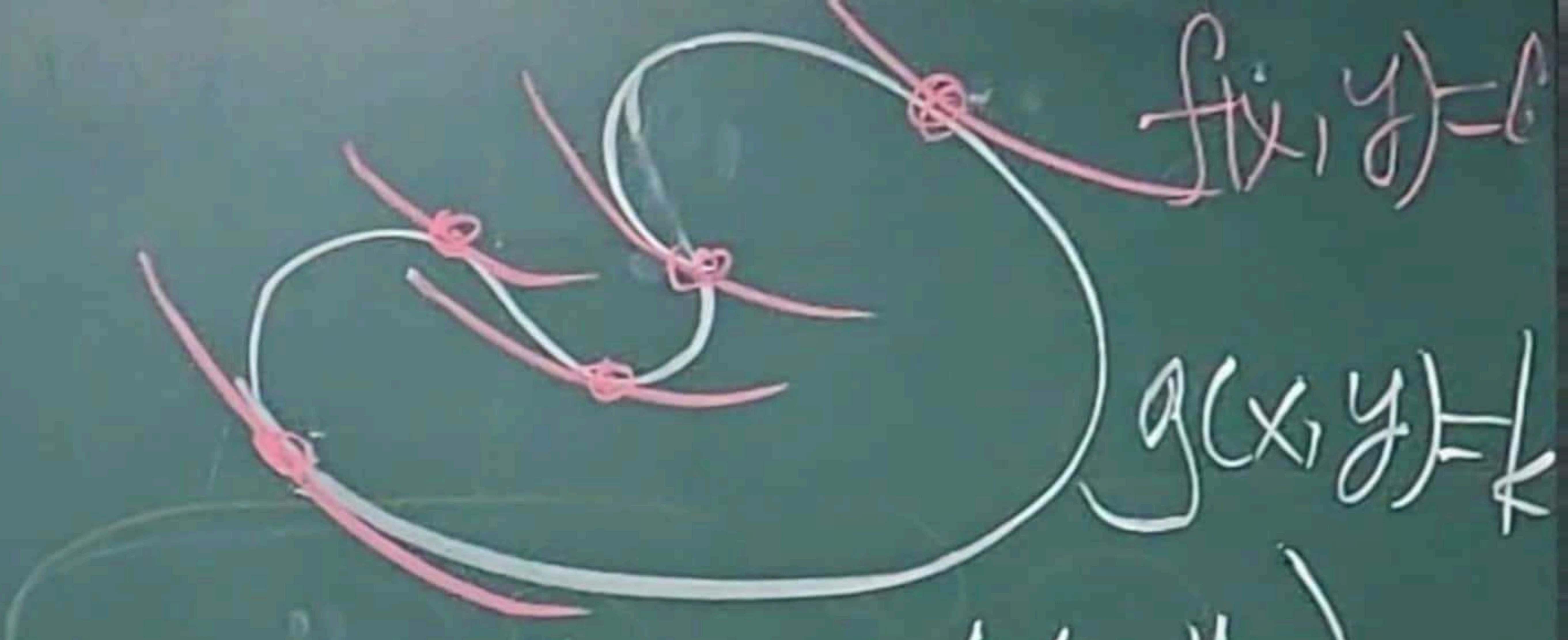
借教室 36173 (黃頎数)

Level surface $\Longrightarrow f(x,y,z)=C$.



tangent plane

$g(x,y,z)=k$. solve $\begin{cases} \nabla f = \lambda \nabla g(x,y,z) \\ g(x,y,z)=k \end{cases}$

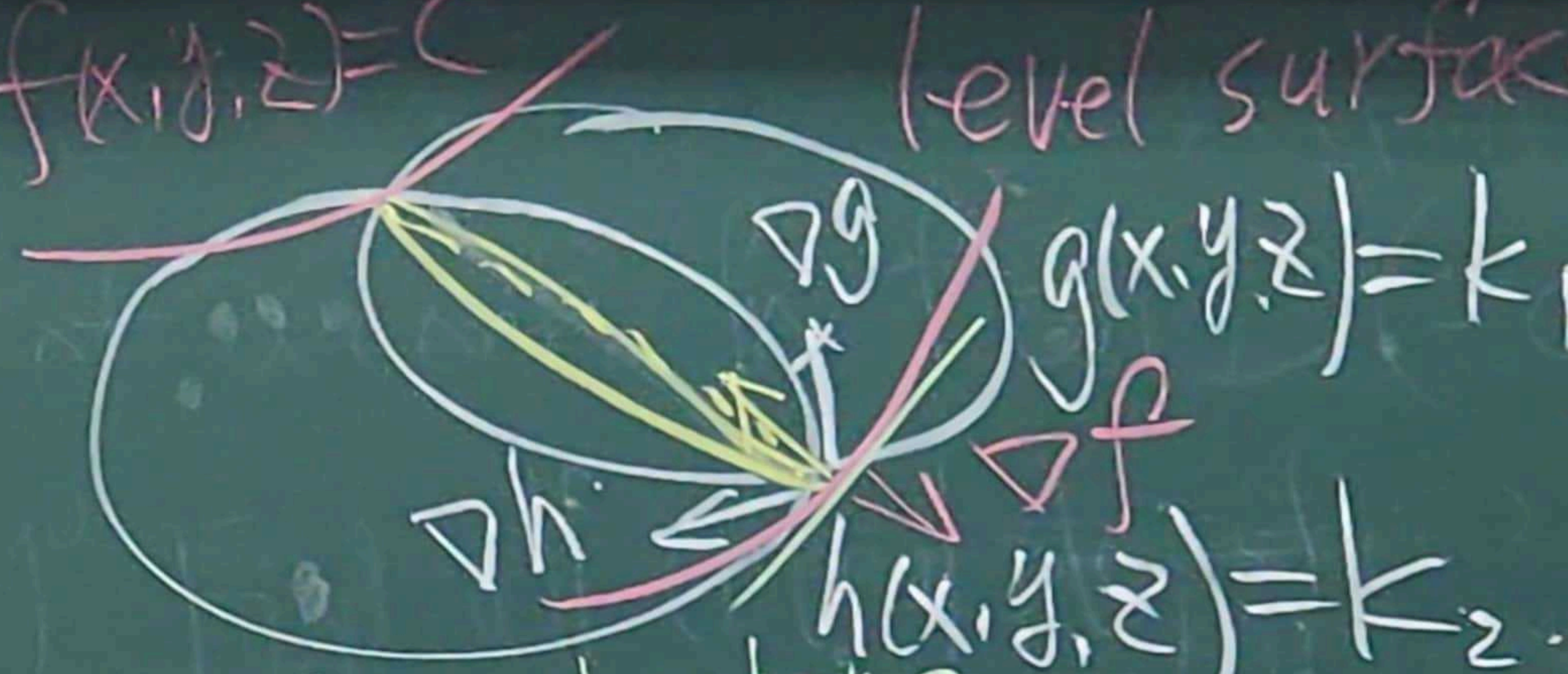


$(x_1,y_1), \ldots\ldots (x_5\ y_5)$.

compute $f(x_1,y_1) \ldots\ldots f(x_5\ y_5)$

To find max or min of $f(x,y,z)$

subject to $\left.\begin{matrix} g(x,y,z)=k_1 \\ h(x,y,z)=k_2 \end{matrix}\right\}$ given



1° solve $\begin{cases} \boxed{\nabla f = \lambda \nabla g + \mu \nabla h} \\ g(x,y,z)=k_1 \\ h(x,y,z)=k_2. \end{cases}$

tangent plane

$\nabla f \parallel \boxed{\lambda \nabla g + \mu \nabla h}$

$\Rightarrow$ solve $\begin{cases} \nabla f = \lambda \nabla g + \mu \nabla h \\ g(x,y,z)=k_1 \\ h(x,y,z)=k_2 \end{cases}$

(EX) Find the pts on $x^2+y^2+z^2=4$ that are closed to and farthest from the pt $(3,1,-1)$.

(x,y,z)

distance from (x,y,z) to $(3,1,-1)$

$$d^2=\sqrt{(x-3)^2+(y-1)^2+(z+1)^2}^{\,2}$$

$$f(x,y,z)\equiv = (x-3)^2+(y-1)^2+(z+1)^2$$

$$g(x,y,z)=x^2+y^2+z^2=4$$

$$\Rightarrow \begin{cases} \nabla f=\lambda\nabla g \\ g(x,y,z)=k \end{cases}$$

$$\Rightarrow \begin{cases} \nabla\left((x-3)^2+(y-1)^2+(z+1)^2\right) = \lambda\nabla(x^2+y^2+z^2) \\ x^2+y^2+z^2=4 \end{cases}$$

$$\Rightarrow \begin{cases} \langle \cancel{2}(x-3), \cancel{2}(y-1), \cancel{2}(z+1)\rangle = \lambda\langle \cancel{2}x, \cancel{2}y, \cancel{2}z\rangle \\ x^2+y^2+z^2=4 \end{cases}$$