

Consider a network as defined in (3.1) and (3.2).

Assume that $n_L = 1$, find an algorithm to calculate $\nabla A^{[L]}(x)$.

Some element: $\{n_{\bar{x}}\}_{\bar{x}=1}^L$ where $n_{\bar{x}} \in \mathbb{N}$ for all $\bar{x} = 1, 2, \dots, L$ and $n_L = 1$. $W^{[\bar{x}]} \in \mathbb{R}^{n_{\bar{x}} \times n_{\bar{x}-1}}$, $b^{[\bar{x}]} \in \mathbb{R}^{n_{\bar{x}}}$.

$$A^{[1]} = x \in \mathbb{R}^{n_1} \quad \text{and} \quad A^{[\bar{x}]} \in \mathbb{R}^{n_{\bar{x}}} \quad \text{for all } \bar{x} = 1, 2, \dots, L.$$

by (3.2), we have $A^{[L]} = \sigma(W^{[L]} A^{[L-1]} + b^{[L]}) \in \mathbb{R}^{n_L}$ for $L = 1, 2, \dots, L$.

Then we can construct a algorithm by following to get $\nabla A^{[L]}(x)$.

Input: $\{n_{\bar{x}}\}_{\bar{x}=1}^L$ in \mathbb{N} , $x \in \mathbb{R}^{n_1}$, $\{W^{[\bar{x}]}\}_{\bar{x}=2}^L$ the collection of weight, $\{b^{[\bar{x}]}\}_{\bar{x}=2}^L$ the collection of bias, $\sigma(x)$.

Output: $\nabla A^{[L]}(x)$.

$$A[1] = x.$$

for \bar{x} from 2 to L ;

$$Z[\bar{x}] = W[\bar{x}] A[\bar{x}-1] + b[\bar{x}]$$

$$A[\bar{x}] = \sigma(Z[\bar{x}]).$$

$$\delta[L] = \sigma'(Z[L])$$

for \bar{x} from $L-1$ to 2;

$$\delta[\bar{x}] = (W[\bar{x}+1])^T \delta[\bar{x}+1] \cdot \sigma'(Z[\bar{x}])$$

$$\nabla A^{[L]}(x) = (W^{[2]})^T \delta^{[2]} \quad \text{return.}$$

