Make a suitable title: my paper on the cosmic microwave background and formation of structures in our Universe

AST5220: Cosmology II

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ABSTRACT

Context. Something about a CAMB solver
Aims. Cosmological simulations
Methods. Calculations are done
Results. Results are resulted
Conclusions. An abstract for the paper. Describe the paper. What is the paper about, what are the main results, etc.

Key words. cosmic microwave background – cosmic background radiation – large-scale structure of Universe – recombination

1. Introduction

Write an introduction here. Give context to the paper. Citations to relevant papers. You only need to do this in the end for the last milestone.

Study the fundamental cosmology of our Universe and its early evolution, from right after inflation to the present day at very large scales. Doing this predicts the existence and behaviour of a Cosmic Microwave Background (CMB), which we can study the physical properties of.

The goal of this whole project is to be able to predict the CMB (and matter) fluctuations - described by the so-called power-spectrum - from first principles and learn about all the different physical processes that goes on to be able to explain the results.

Use SI units in project, equations need to have actual units

2. Milestone I

Some introduction about what it is all about.

Cite: Baumann (2017), Dodelson (2003) and (Callin 2006; Winther 2024; Hu et al. 1998)

2.1. Theory

This study is baselined on the common Lambda-CDM model (chap. 1 Dodelson & Schmidt 2021, sec. 1.6), with a Friedmann-Robertson-Walker metric for spacetime.

The scale factor a is used as the main time-ish variable - a is not a time but since time, size of the universe, and cosmic distances are all closely related it can fulfill the role of a time variable nonetheless, and describe the evolution of our simulated universe (Winther 2024). The scale factor is a dimensionless quantity.

For numerical work we use the variable $x \equiv \log a$, this is for numerical stability over large timescales with highly variable quantities.

We also consider the cosmic time t, and in theory a is dependent on t, $a(t)$, but in practice we derive the cosmic time t from x (and thus a) and not the other way around, see eq. 5.

We define a_0 as the value of the scale factor today, such that $a_0 \equiv a(t_{\text{today}}) \equiv 1$. Note that $x_0 = 0$. In general, subscript $_0$ indicates the value of a parameter as measured in the current day.

Another related quantity is the redshift z, this is also $z_0 = 0$ in the present day. As observers we must by definition be at zero redshift, after all. Redshift is given by $1 + z = a_0/a(t) = 1/a$.

Fiducial cosmology and initial parameter values taken from the Planck 2018 results (Collaboration et al. 2020). Input parameters are listed at 1.

$$\begin{aligned}
h &= 0.67, \\
T_{\mathrm{CMB0}} &= 2.7255\,\mathrm{K}, \\
N_{\mathrm{eff}} &= 3.046, \\
\Omega_{b0} &= 0.05, \\
\Omega_{\mathrm{CDM0}} &= 0.267, \\
\Omega_{k0} &= 0, \\
\Omega_{\nu 0} &= N_{\mathrm{eff}} \cdot \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \Omega_{\gamma 0}, \\
\Omega_{\Lambda 0} &= 1 - (\Omega_{k0} + \Omega_{b0} + \Omega_{\mathrm{CDM0}} + \Omega_{\gamma 0} + \Omega_{\nu 0}), \\
n_s &= 0.965, \\
A_s &= 2.1 \cdot 10^{-9}, \\
Y_p &= 0.245, \\
z_{\mathrm{reion}} &= 8, \\
\Delta z_{\mathrm{reion}} &= 0.5, \\
z_{\mathrm{Hereion}} &= 3.5, \\
\Delta z_{\mathrm{Hereion}} &= 0.5.
\end{aligned} \tag{1}$$

2.1.1. Friedmann equation

Rather than dealing with the full Einstein equations directly, it is possible to derive the Friedmann equation in order to describe the expansion of the universe, this is eq. 2 (Winther 2024).

$$\boxed{H = H_0 \sqrt{\Omega_{M0} a^{-3} + \Omega_{R0} a^{-4} + \Omega_{k0} a^{-2} + \Omega_{\Lambda 0}}}, \tag{2}$$

where the Ω_X are density parameters describing relative density of their respective form of energy contributing to the expansion of the universe. Density parameters are dimensionless. Subscript $_0$ indicates a value for the universe of today, since we as observers are by definition at $x = z = 0$.

$\Omega_M = (\Omega_b + \Omega_{\mathrm{CDM}})$ is a composite density parameter describing non-relativistic matter (baryons and cold dark matter), and $\Omega_R = (\Omega_\gamma + \Omega_\nu)$ is a composite density term for radiation (photons and neutrinos). Ω_Λ is the density parameter for dark energy. Ω_k is a curvature term, and not properly an energy density. However, it contributes to the Friedmann equation as if it were a normal matter fluid with equation of state $\omega = -1/3$. This term prescribes negative curvature when < 0, positive curvature when > 0, and a spatially flat universe when $= 0$. Our universe is observationally confirmed to be very close to flat (Bennett et al. 2013), so this term should be close to 0.

The unit of H is slightly ambiguous, as in SI units both (km/s)/Mpc and the simplified 1/s are used. The first way of writing the unit is more intuitive, as it relates to the change in velocity of distant galaxies based on their distance from the observer (us), aka Hubble's law. 1/s is technically correct but interpreting the Hubble parameter as a frequency is not helpful. For the Friedmann equation and derived quantities, we mostly skip this problem by keeping H in "units of H", using the value of the Hubble parameter today (H_0) as a constant which gives the right units to any H that pops up.

For numerical work, we calculate the constant H_0 by adding the right units to a dimensionless constant h (eq. 3), which is commonly used and reported in the literature (Croton 2013). h is one of the input observables for our numerical simulation, so we use the 0.67 value reported by Planck 2018 (Collaboration et al. 2020).

$$H_0 = 100 * h \;\; \mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{3}$$

Equation for critical density of the universe today 4

$$\rho_{c0} \equiv \frac{3H_0^2}{8\pi G} \;\; \mathrm{kg\,m^{-3}} \tag{4}$$

2.1.2. More stuff

Paper with supernova fitting data Betoule et al. (2014).

$$t(x) = \int_0^a \frac{da}{aH} = \int_{-\infty}^{x} \frac{dx}{H(x)} \quad \text{[unit s, can convert to Gyr etc.]} \tag{5}$$

2.2. *Implementation details*

Something about the numerical work.

2.3. *Results*

Show and discuss the results.

See figs. 1, 2, 3, 4, 5.

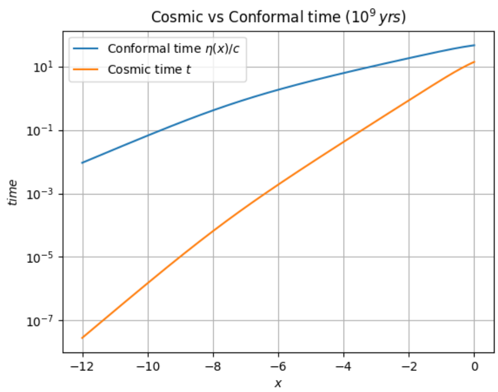

Fig. 1: Caption

3. Milestone II

With fundamental cosmology established in sec. 2, we can now describe the baseline or so-called "background" behaviour of our universe, with a relatively simple evolution of cosmological parameters as the universe expands (refer fig. 4). Now we wish to look backwards from our current time, and compute the path of photons travelling towards a current-day observer from the early universe.

In order to study the behaviour of photons and thermal evolution of the early universe, we consider it to be a large continuous fluid, specifically a hot plasma. The thermodynamics and statistical mechanics for this are described in Baumann (2017, chap. 3) and Dodelson (2003, chap. 3,4), while the specific Boltzmann formalism utilized is that of Winther (2024), available here.

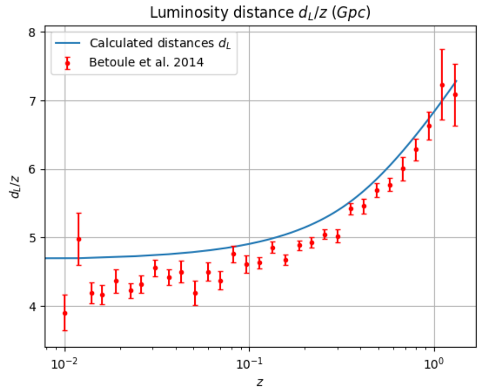

Fig. 2: Caption

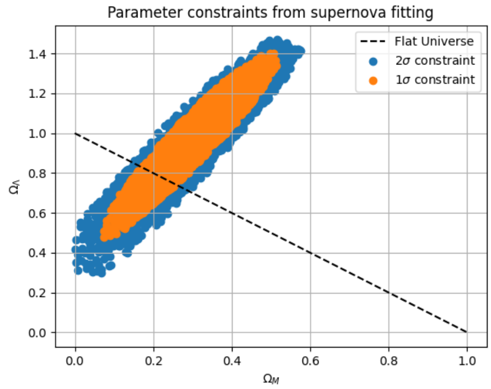

Fig. 5: Caption

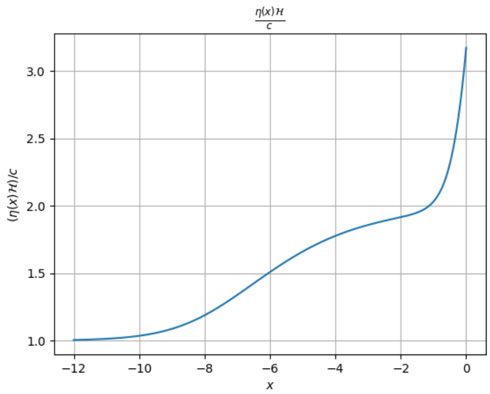

Fig. 3: Caption

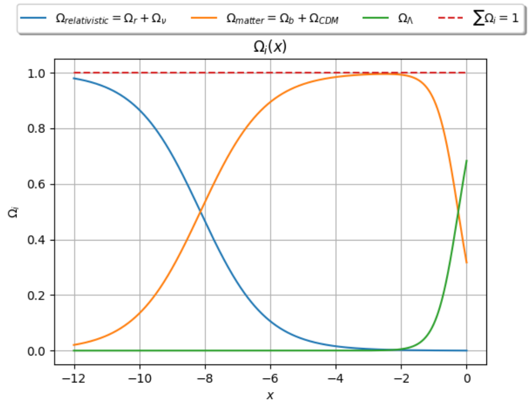

Fig. 4: Caption

3.1. Theory

Considering the early universe, there are three main interactions between particle species of interest, Couloumb scattering (6), Thomson scattering (7), and the formation/ionization of Hydrogen (8).

$$e^- + p^+ \rightleftharpoons e^- + p^+ \tag{6}$$

$$e^- + \gamma \rightleftharpoons e^- + \gamma \tag{7}$$

$$e^- + p^+ \rightleftharpoons H + \gamma \tag{8}$$

Start with eqs. 9 (Dodelson 2003, sec. 4.4) and 10.

$$\tau(\eta) = \int_\eta^{\eta_0} n_e \sigma_T a d\eta' \quad \text{[dimensionless]} \tag{9}$$

$$\boxed{\tau' = \frac{d\tau}{dx} = -\frac{c n_e \sigma_T}{H}} \quad \text{[dimensionless]} \tag{10}$$

Also the visibility function 11.

$$\tilde{g}(x) = \frac{d}{dx} e^{-\tau} = -\tau' e^{-\tau}, \quad \text{by def.} \int_{-\infty}^0 \tilde{g}(x)dx = 1 \tag{11}$$

We wish to compute the fractional electron density given by 12, where we assume all baryons are protons and there are no heavier elements. This approximation is acceptable for getting a simple reionization with clear falloff of electrons as they get absorbed into hydrogen atoms. By including ionization into Helium, our resulting ionization plot would have multiple bumps for ionization into different states of Hydrogen+Helium.

$$\boxed{X_e \equiv n_e/n_H}, \text{ with } n_H = n_b \approx \frac{\rho_b}{m_H} = \frac{\Omega_{b0}\rho_{c0}}{m_H a^3} \tag{12}$$

In order to calculate the electron density, we can use the Saha approximation given in eq. 13. This is valid for large temperatures T, early in the universe. However, as the temperature falls the Saha approximation continues to predict a simple exponential decay of free electrons as they all become bound to hydrogen (and heavier elements), but this assumes the interaction 8 continues to be perfectly efficient, which is not the case in practice. See fig. 3.8 in Baumann (2017, sec. 3.3.3). We seek to recreate this figure with a numerical simulation.

$$\boxed{\frac{X_e^2}{1 - X_e} = \frac{1}{n_b}\left(\frac{m_e T_b}{2\pi}\right)^{3/2} e^{-\frac{\epsilon_0}{k_b T_b}}} \quad [X_e \text{ dimensionless}] \tag{13}$$

In order to properly simulate the change in electron density, we can switch to the more accurate Peebles equation given as eq. 14 (Peebles 1968; Zel'dovich et al. 1969), with supporting definitions in eq. C. This equation is numerically unstable when solved for very large T (early on), but is perfectly appropriate around when the Saha approximation stops being accurate.

$$\boxed{\frac{dX_e}{dx} = \frac{C_r(T_b)}{H}\left[\beta(T_b)(1 - X_e) - n_H\alpha^{(2)}(T_b)X_e^2\right]} \quad [X_e \text{ dimensionless}] \tag{14}$$

We will also calculate the so-called "sound horizon at decoupling", the total distance a sound-wave in the primordial photon-baryon plasma can propagate from the Big Bang until photons decouple. Pure photons have a sound speed $c/\sqrt{3}$, while sound-waves in the primordial plasma follow the slightly lower $c_s = c\sqrt{\frac{R}{3(1+R)}}$, with $R = \frac{4\Omega_{\gamma 0}}{3\Omega_{b0}a}$. Thus we end up with the sound-horizon given in eq. 15.

$$\boxed{s(x) = \int_0^a \frac{c_s dt}{a} = \int_{-\infty}^x \frac{c_s dx}{\mathcal{H}} \rightarrow \frac{ds(x)}{dx} = \frac{c_s}{\mathcal{H}}}, \quad \text{with } s(x_{\text{ini}}) = \frac{c_s(x_{\text{ini}})}{\mathcal{H}(x_{\text{ini}})} \quad [\text{unit m}] \tag{15}$$

3.2. Implementation details

Something about the numerical work.

3.3. Results

Show and discuss the results.

4. Conclusions

Write a short summary and conclusion in the end.

1. Conclusive statement 1
2. Conclusive statement 2
3. Conclusive statement 3

Acknowledgements. Thank

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Appendix A: Code repository

All code used for this project is available at this Github repository. Description of code folders and places to look here.

Appendix B: Milestone I, extra plots

Evolution of some physical quantities in figs. B.1 and B.2.

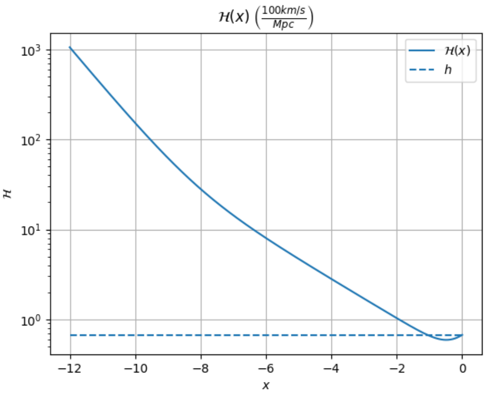


Fig. B.1: Caption

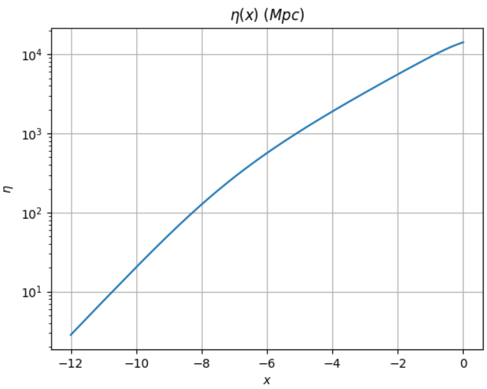


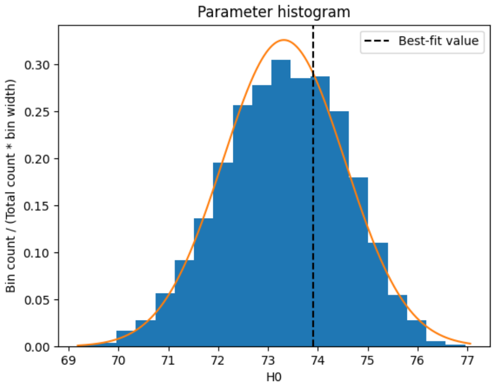
Fig. B.2: Caption

Histograms of parameter distribution in fig. B.3.

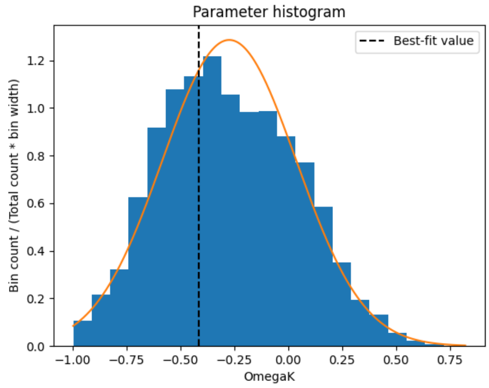
Appendix C: Milestone II, extra math

Definitions to support Peebles equation (14):

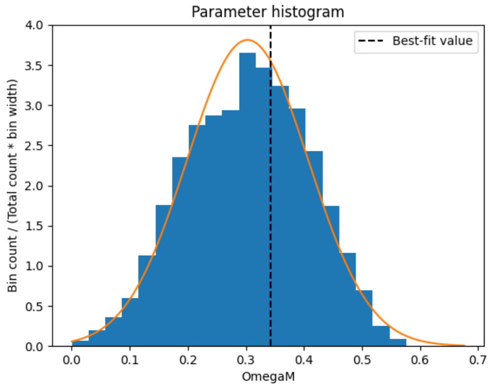
$$
\begin{aligned}
C_r(T_b) &= \frac{\Lambda_{2s\to 1s} + \Lambda_\alpha}{\Lambda_{2s\to 1s} + \Lambda_\alpha + \beta^{(2)}(T_b)}, \text{ (dimensionless)}\\
H&, \text{ (dimension } 1/\mathrm{s})\\
\Lambda_{2s\to 1s} &= 8.227, \text{ (dimension } 1/\mathrm{s})\\
\Lambda_\alpha &= H\frac{(3\epsilon_0)^3}{(8\pi)^2 n_{1s}}, \text{ (dimension } 1/\mathrm{s})\\
n_{1s} &= (1-X_e)n_H, \text{ (dimension } 1/\mathrm{m}^3)\\
n_H &= (1-Y_p)\frac{3H_0^2\Omega_{b0}}{8\pi G m_H a^3}, \text{ (dimension } 1/\mathrm{m}^3)\\
\beta^{(2)}(T_b) &= \beta(T_b)e^{3\epsilon_0/4T_b}, \text{ (dimension } 1/\mathrm{s})\\
\beta(T_b) &= \alpha^{(2)}(T_b)\left(\frac{m_e T_b}{2\pi}\right)^{3/2} e^{-\epsilon_0/T_b}, \text{ (dimension } 1/\mathrm{s})\\
\alpha^{(2)}(T_b) &= \frac{64\pi}{\sqrt{27\pi}}\frac{\alpha^2}{m_e^2}\sqrt{\frac{\epsilon_0}{T_b}}\phi_2(T_b), \text{ (dimension } \mathrm{m}^3/\mathrm{s})\\
\phi_2(T_b) &= 0.448\ln(\epsilon_0/T_b), \text{ (dimensionless)}\\
\alpha &\simeq \frac{1}{137.0359992}, \text{ (dimensionless, fine-structure constant)}
\end{aligned}
$$



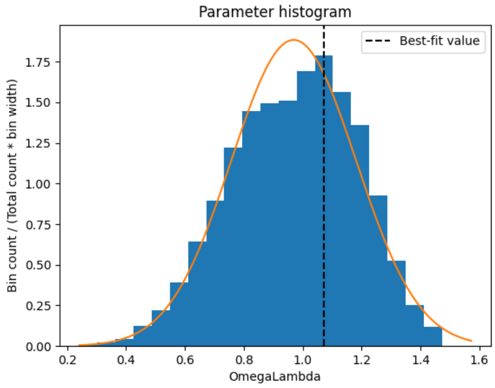
(a) Caption 1



(b) Caption 2



(c) Caption 3



(d) Caption 4

Fig. B.3: Whole figure caption

Appendix D: Milestone X, extra