Astronomy & Astrophysics templated numerical project
March 21, 2025

Make a suitable title: my paper on the cosmic microwave background and formation of structures in our Universe

AST5220: Cosmology II

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ABSTRACT

Context. Something about a CAMB solver
Aims. Cosmological simulations
Methods. Calculations are done
Results. Results are resulted
Conclusions. An abstract for the paper. Describe the paper. What is the paper about, what are the main results, etc.

Key words. cosmic microwave background – cosmic background radiation – large-scale structure of Universe – recombination

1. Introduction

Write an introduction here. Give context to the paper. Citations to relevant papers. You only need to do this in the end for the last milestone.

Study the fundamental cosmology of our Universe and its early evolution, from right after inflation to the present day at very large scales. Doing this predicts the existence and behaviour of a Cosmic Microwave Background (CMB), which we can study the physical properties of.

The goal of this whole project is to be able to predict the CMB (and matter) fluctuations - described by the so-called power-spectrum - from first principles and learn about all the different physical processes that goes on to be able to explain the results.

2. Milestone I

Some introduction about what it is all about.

Cite: Baumann (2017) or Dodelson & Schmidt (2021) (Callin 2006; Winther 2024; Hu et al. 1998)

2.1. Theory

The theory behind this milestone. See Friedmann equation 1

Fiducial cosmology and initial parameter values taken from Planck 2018 results (Collaboration et al. 2020).

$$\boxed{H = H_0 \sqrt{\Omega_{M0} a^{-3} + \Omega_{R0} a^{-4} + \Omega_{k0} a^{-2} + \Omega_{\Lambda 0}}}, \tag{1}$$

where the Ω_X are density parameters describing relative density of their respective form of energy contributing to the expansion of the universe. Subscript $_0$ indicates a value for the universe of today, since we as observers are by definition at $x = a = z = 0$.

$\Omega_{M0} = (\Omega_{b0} + \Omega_{\mathrm{CDM}0})$ is a composite density parameter describing non-relativistic matter, and $\Omega_{R0} = (\Omega_{\gamma 0} + \Omega_{\nu 0})$ is a composite density term for radiation.

Paper with supernova fitting data Betoule et al. (2014)

Equation for critical density of the universe today 2

$$\rho_{c0} \equiv \frac{3H_0^2}{8\pi G} \tag{2}$$

2.2. Implementation details

Something about the numerical work.

2.3. Results

Show and discuss the results.

See figs. 1, 2, 3, 4, 5.

3. Milestone II

With fundamental cosmology established in sec. 2, we can now describe the baseline or so-called "background" behaviour of our Universe, with a relatively simple evolution of cosmological paremeters as the universe expands (refer fig. 4). Now we wish to look backwards from our current time, and compute the path of photons travelling towards a current-day observer from the early universe.

In order to study the behaviour of photons and thermal evolution of the early universe, we consider it to be a large continuous fluid, specifically a hot plasma. The thermodynamics and statistical mechanics for this are described in Baumann (2017, chap. x) and Dodelson (2003, chap. x), while the specific Boltzmann formalism utilized is that of Winther (2024), which can be found here.

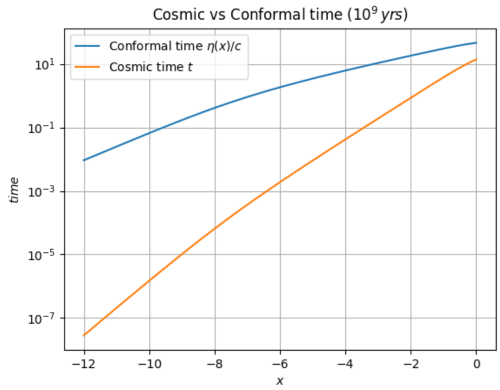

Fig. 1: Caption

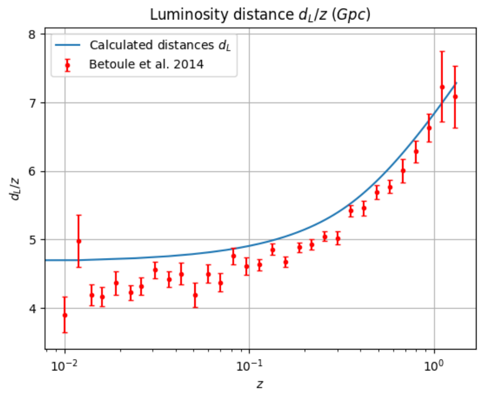

Fig. 2: Caption

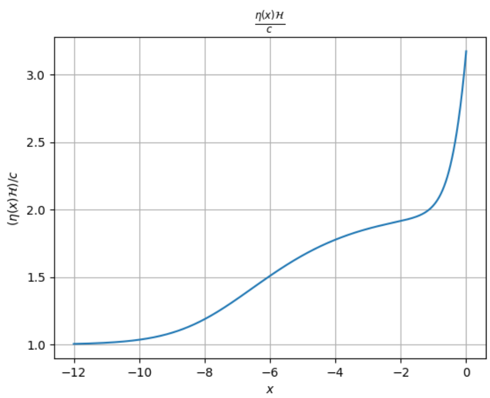

Fig. 3: Caption

3.1. Theory

The theory behind this milestone.

Start with eqs. 3 and 4.

$$\tau(\eta) = \int_{\eta}^{\eta_0} n_e \sigma_T a d\eta' \tag{3}$$

$$\boxed{\tau' = \frac{d\tau}{dx} = -\frac{c n_e \sigma_T}{H}.} \tag{4}$$

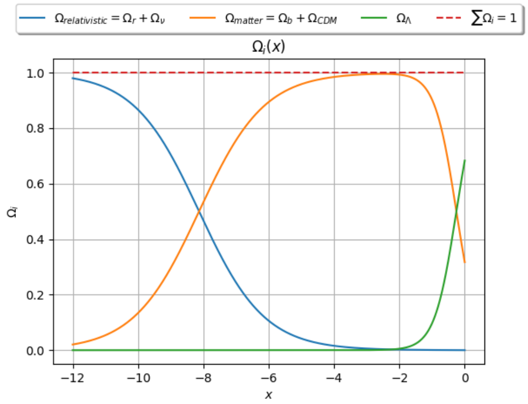

Fig. 4: Caption

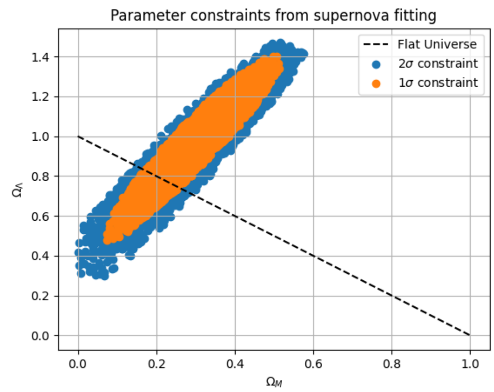

Fig. 5: Caption

We wish to compute the fractional electron density given by 5, where we assume all baryons are protons and there are no heavier elements. This approximation is acceptable for getting a simple reionization with clear falloff of electrons as they get absorbed into hydrogen atoms. By including ionization into Helium, our resulting ionization plot would have multiple bumps for ionization into different states of Hydrogen+Helium.

$$\boxed{X_e \equiv n_e/n_H}, \text{ with } n_H = n_b \approx \frac{\rho_b}{m_H} = \frac{\Omega_{b0}\rho_{c0}}{m_H a^3} \tag{5}$$

Saha approximation 6

$$\boxed{\frac{X_e^2}{1 - X_e} = \frac{1}{n_b}\left(\frac{m_e T_b}{2\pi}\right)^{3/2} e^{-\epsilon_0/T_b}} \tag{6}$$

Peebles equation 7, with the supporting definitions in eq. C.

$$\boxed{\frac{dX_e}{dx} = \frac{C_r(T_b)}{H}\left[\beta(T_b)(1 - X_e) - n_H \alpha^{(2)}(T_b) X_e^2\right],} \tag{7}$$

3.2. Implementation details

Something about the numerical work.

3.3. Results

Show and discuss the results.

4. Conclusions

Write a short summary and conclusion in the end.

1. Conclusive statement 1
2. Conclusive statement 2
3. Conclusive statement 3

Acknowledgements. Thank

References

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Collaboration, P., Aghanim, N., Akrami, Y., et al. 2020, A&A, 641, A6

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Hu, W., Seljak, U., White, M., & Zaldarriaga, M. 1998, Phys. Rev. D, 57, 3290

Winther, H. A. 2024, Cosmology II: Lecture Notes, https://cmb.wintherscoming.no/literature.php, (accessed 2025-03-17)

Appendix A: Code repository

All code used for this project is available at this Github repository. Description of code folders and places to look here.

Appendix B: Milestone I, extra plots

Evolution of some physical quantities in figs. B.1 and B.2.

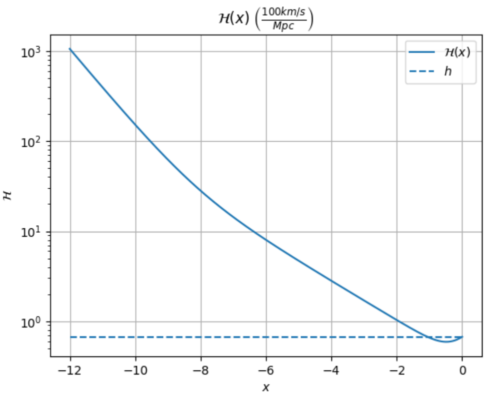


Fig. B.1: Caption

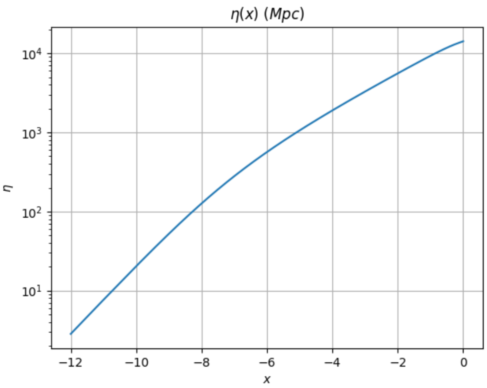
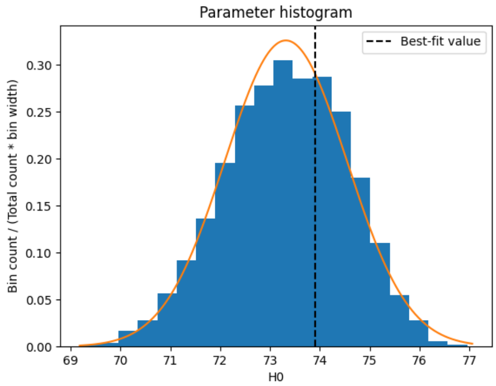
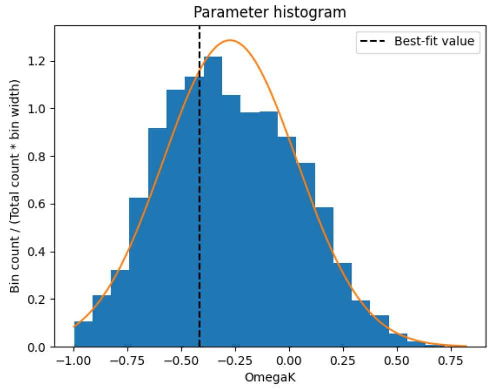


Fig. B.2: Caption

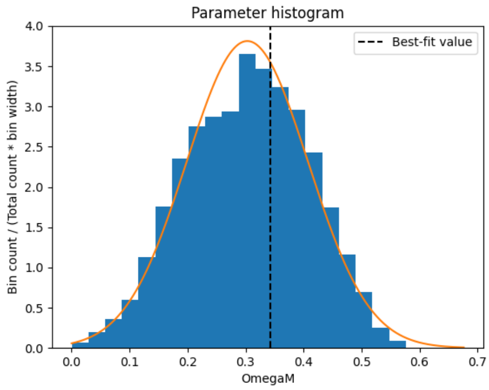
Histograms of parameter distribution in fig. B.3.



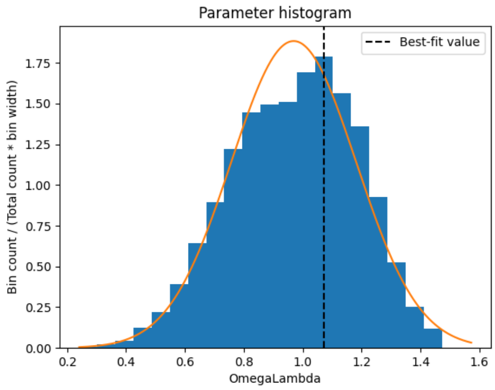
(a) Caption 1



(b) Caption 2



(c) Caption 3



(d) Caption 4

Fig. B.3: Whole figure caption

Appendix C: Milestone II, extra math

$$
\begin{aligned}
C_r(T_b) &= \frac{\Lambda_{2s\to 1s} + \Lambda_\alpha}{\Lambda_{2s\to 1s} + \Lambda_\alpha + \beta^{(2)}(T_b)}, \ \text{(dimensionless)}\\
H&, \ \text{(dimension 1/s)}\\
\Lambda_{2s\to 1s} &= 8.227\mathrm{s}^{-1}, \ \text{(dimension 1/s)}\\
\Lambda_\alpha &= H\frac{(3\epsilon_0)^3}{(8\pi)^2 n_{1s}}, \ \text{(dimension 1/s)}\\
n_{1s} &= (1-X_e)n_H, \ \text{(dimension 1/m}^3\text{)}\\
n_H &= (1-Y_p)\frac{3H_0^2\Omega_{b0}}{8\pi G m_H a^3}, \ \text{(dimension 1/m}^3\text{)}\\
\beta^{(2)}(T_b) &= \beta(T_b)e^{3\epsilon_0/4T_b}, \ \text{(dimension 1/s)}\\
\beta(T_b) &= \alpha^{(2)}(T_b)\left(\frac{m_e T_b}{2\pi}\right)^{3/2} e^{-\epsilon_0/T_b}, \ \text{(dimension 1/s)}\\
\alpha^{(2)}(T_b) &= \frac{64\pi}{\sqrt{27\pi}}\frac{\alpha^2}{m_e^2}\sqrt{\frac{\epsilon_0}{T_b}}\phi_2(T_b), \ \text{(dimension m}^3\text{/s)}\\
\phi_2(T_b) &= 0.448\ln(\epsilon_0/T_b), \ \text{(dimensionless)}\\
\alpha &\simeq \frac{1}{137.0359992}, \ \text{(dimensionless)}
\end{aligned}
$$