Introduction

Double Pendulum's Equation of Motion

總動能 (T)

$$T = \frac{1}{2}m_1 l_1^2 \dot{\theta}_1^{\ 2} + \frac{1}{2}m_2(l_1^2\dot{\theta}_1^{\ 2} + 2l_1l_2\cos(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2 + l_2^2\dot{\theta}_2^{\ 2})$$

總位能 (V)

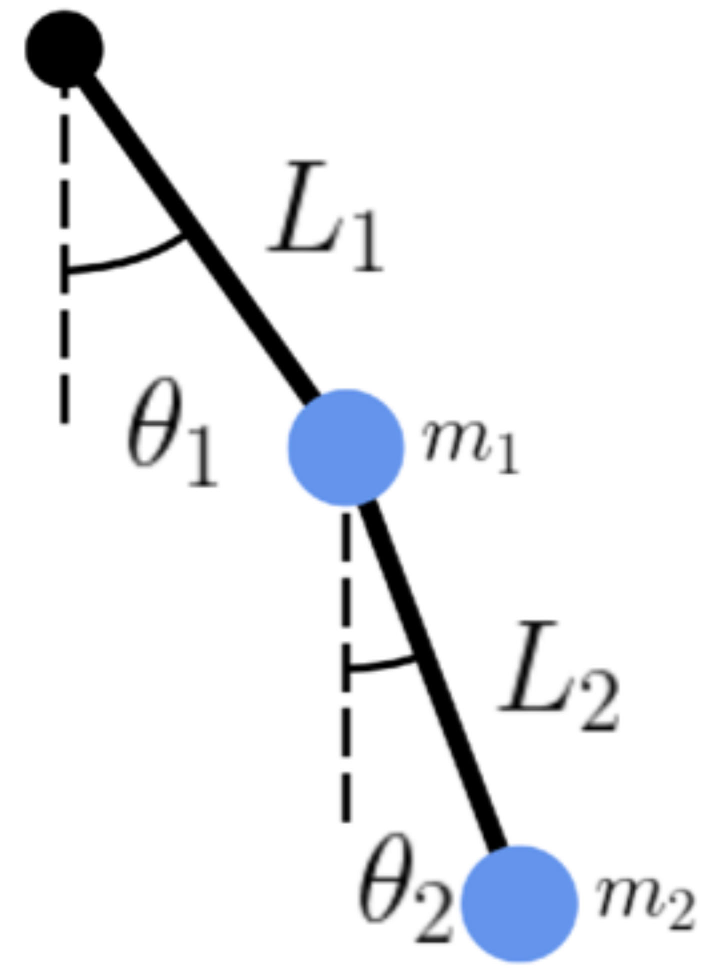
$$V = -m_1 g l_1 \cos(\theta_1) - m_2 g l_1 \cos(\theta_1) - m_2 g l_2 \cos(\theta_2)$$

拉格朗日值 ($\mathcal{L}$) 是總動能與總位能之差

$$\mathscr{L} = T - V$$

$$\mathscr{L} = \frac{1}{2}m_1 l_1^2\dot{\theta}_1^{\ 2} + \frac{1}{2}m_2 l_1^2\dot{\theta}_1^{\ 2} + m_2 l_1 l_2 \cos(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2 + \frac{1}{2}m_2 l_2^2\dot{\theta}_2^{\ 2}$$
$$+m_1 g l_1\cos(\theta_1) + m_2 g l_1 \cos(\theta_1) + m_2 g l_2 \cos(\theta_2)$$

Illustration of a double pendulum



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Double Pendulum’s Equation of Motion

拉格朗日力學指出：物理系統遵守歐拉-拉格朗日方程

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathscr{L}}{\partial \theta} = 0$$

列出偏微分聯立方程組：

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial \mathscr{L}}{\partial \dot{\theta}_1}\right) - \dfrac{\partial \mathscr{L}}{\partial \theta_1} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial \mathscr{L}}{\partial \dot{\theta}_2}\right) - \dfrac{\partial \mathscr{L}}{\partial \theta_2} = 0 \end{cases}$$

將雙擺物理參數帶入並做基本計算後，得雙擺角加速度方程：

$$\ddot{\theta}_{1_i} = \frac{m_2 g \sin\theta_2 \cos(\theta_1 - \theta_2) - m_2 \sin(\theta_1 - \theta_2)(l_1 z_1^2 \cos(\theta_1 - \theta_2) + l_2 z_2^2) - (m_1 + m_2) g \sin\theta_1}{l_1(m_1 + m_2 \sin^2(\theta_1 - \theta_2))}$$

$$\ddot{\theta}_{2_i} = \frac{(m_1 + m_2)[l_1 z_1^2 \sin(\theta_1 - \theta_2) - g \sin\theta_2 + g \sin\theta_1 \cos(\theta_1 - \theta_2)] + m_2 l_2 z_2^2 \sin(\theta_1 - \theta_2)\cos(\theta_1 - \theta_2)}{l_2[m_1 + m_2 \sin^2(\theta_1 - \theta_2)]}$$

Illustration of a double pendulum