Introduction

Double Pendulum's Equation of Motion

拉格朗日力學指出：物理系統遵守歐拉-拉格朗日方程

$$\frac{d}{dt}(\frac{\partial \mathscr{L}}{\partial \dot{\theta}}) - \frac{\partial \mathscr{L}}{\partial \theta} = 0$$

列出偏微分聯立方程組：

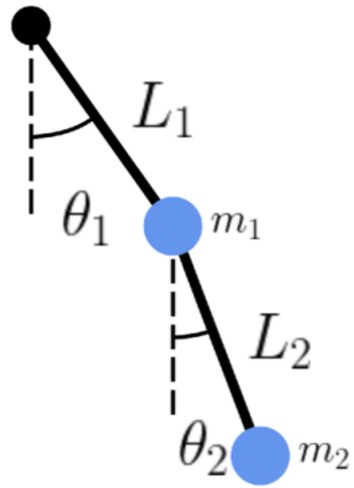
$$\begin{cases} \frac{d}{dt}(\frac{\partial \mathscr{L}}{\partial \dot{\theta}_1}) - \frac{\partial \mathscr{L}}{\partial \theta_1} = 0 \\ \frac{d}{dt}(\frac{\partial \mathscr{L}}{\partial \dot{\theta}_2}) - \frac{\partial \mathscr{L}}{\partial \theta_2} = 0 \end{cases}$$

將雙擺物理參數帶入並做基本計算後，得雙擺角加速度方程：

$$\ddot{\theta}_{1_i} = \frac{m_2 g \sin\theta_2 \cos(\theta_1 - \theta_2) - m_2 \sin(\theta_1 - \theta_2)(l_1 z_1^2 \cos(\theta_1 - \theta_2) + l_2 z_2^2) - (m_1 + m_2) g \sin\theta_1}{l_1(m_1 + m_2 \sin^2(\theta_1 - \theta_2))}$$

$$\ddot{\theta}_{2_i} = \frac{(m_1 + m_2)[l_1 z_1^2 \sin(\theta_1 - \theta_2) - g \sin\theta_2 + g \sin\theta_1 \cos(\theta_1 - \theta_2)] + m_2 l_2 z_2^2 \sin(\theta_1 - \theta_2)\cos(\theta_1 - \theta_2)}{l_2[m_1 + m_2 \sin^2(\theta_1 - \theta_2)]}$$

Illustration of a double pendulum



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用於數值積分之雙擺動力方程：

$$\theta_{1_i} = \theta_{1_{i-1}} + \dot{\theta}_{1_i}\Delta t$$

$$\theta_{2_i} = \theta_{2_{i-1}} + \dot{\theta}_{2_i}\Delta t$$

$$\dot{\theta}_{1_i} = \dot{\theta}_{1_{i-1}} + \frac{m_2 g \sin\theta_{2_i}\cos(\theta_{1_i}-\theta_{2_i}) - m_2\sin(\theta_{1_i}-\theta_{2_i})(l_1 z_1^2\cos(\theta_{1_i}-\theta_{2_i}) + l_2 z_2^2) - (m_1+m_2)g\sin\theta_{1_i}}{l_1(m_1 + m_2\sin^2(\theta_{1_i}-\theta_{2_i}))}\Delta t$$

$$\dot{\theta}_{2_i} = \dot{\theta}_{2_{i-1}} + \frac{(m_1+m_2)[l_1 z_1^2\sin(\theta_{1_i}-\theta_{2_i}) - g\sin\theta_{2_i} + g\sin\theta_{1_i}\cos(\theta_{1_i}-\theta_{2_i})] + m_2 l_2 z_2^2\sin(\theta_{1_i}-\theta_{2_i})\cos(\theta_{1_i}-\theta_{2_i})}{l_2[m_1 + m_2\sin^2(\theta_{1_i}-\theta_{2_i})]}\Delta t$$

模擬變數：

θ_1：擺角 1 的角度

θ_2：擺角 2 的角度

$\dot{\theta}_1$：擺角 1 的角速度

$\dot{\theta}_2$：擺角 2 的角速度

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