Economics 136

Homework video instructions:

Estimating the probability that a stock price will be above an option strike price at expiration.

Advice:

Look at the slides without sound first.

When using the video, pause and look at the slide referred to in the narration.

Give any feedback to Prof E.

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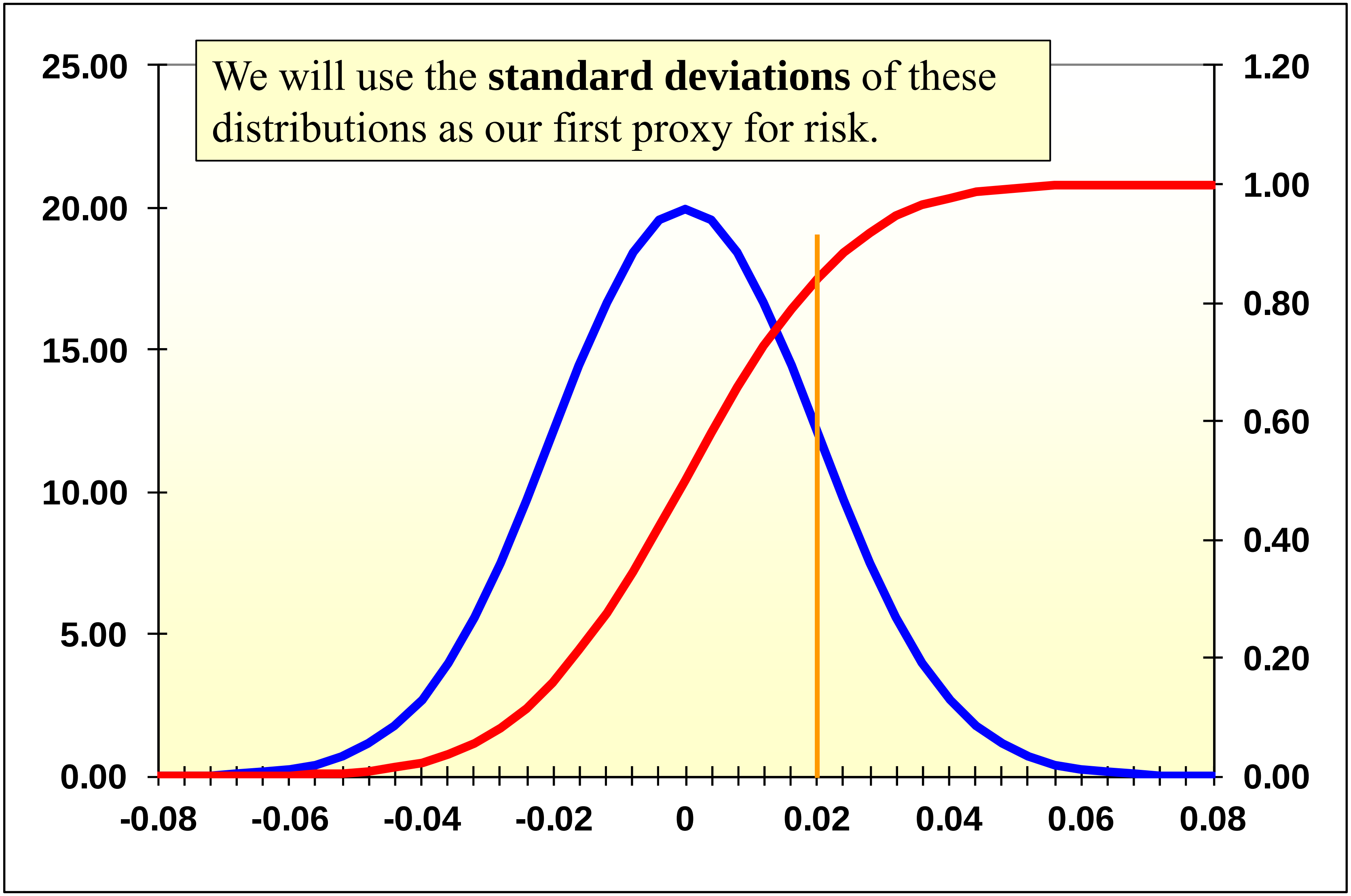
Tools and objectives ...

Files:

1. **NormBase:** An Excel workbook that shows how to map normal distributions and their log transformations. There is no HW associated with this, but it is useful to peruse and the structure and commands might come in handy.
2. **SPPC (Strike Price Probability Calculator):** Using the volatility estimates from HW1, this Excel workbook is used in this HW to calculate the probability of a stock price settling on the other side of an option strike price upon maturity. This is an empty interface. You must used the lectures and possibly material from NormBase to figure out how to do this.

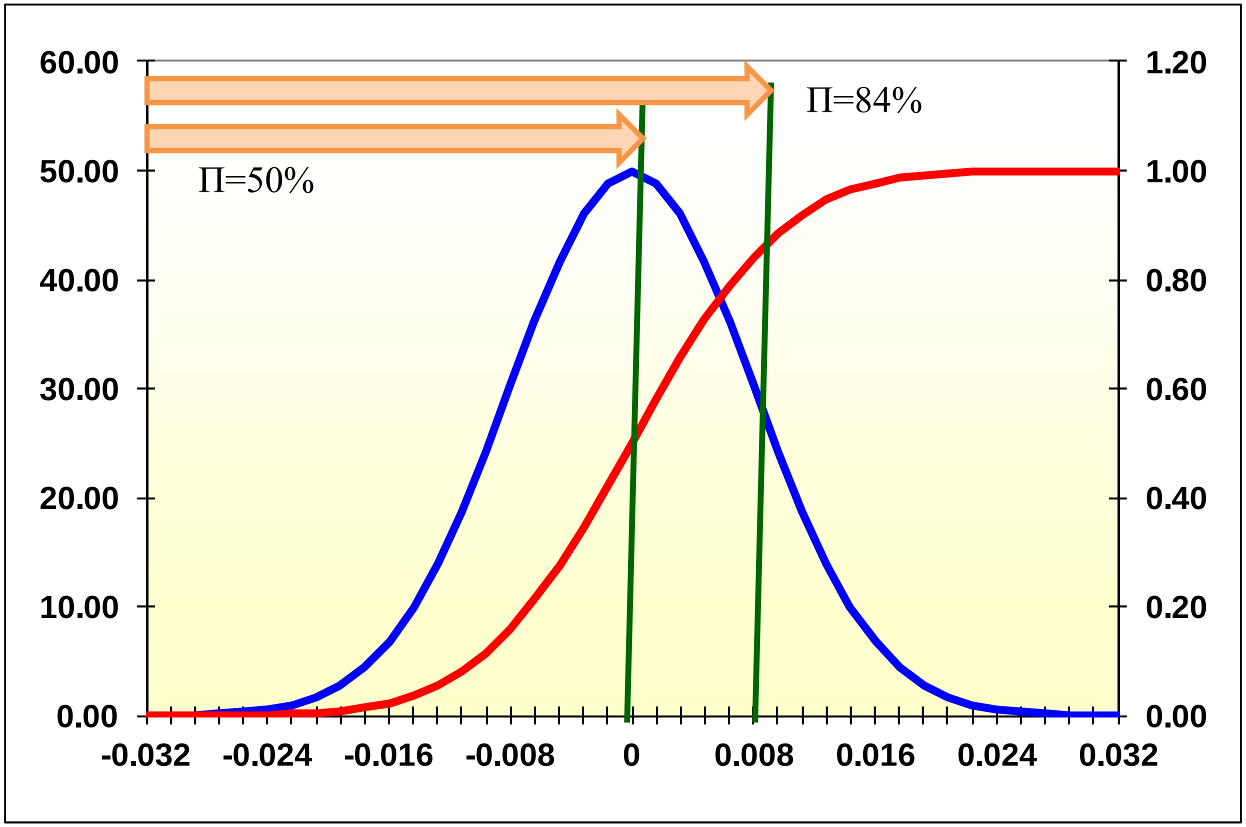
   If Python is being used in this class, see if you can set up a Python object that will make this calculation.
3. **HVHW:** You already have this. This file has your answer to Homework 1, where we calculated historical CGRs and volatility measures for SPY. You will need that data and those results to this homework. So go find it.

The assumed probability distribution of FA continuous growth rates (or similar): Gaussian (normal)



Think about this ...

If we discover that our stock more or less fits a Gaussian distribution and has a standard deviation of 0.008, and we assume for the moment that the drift (alpha) is zero (an assumption that we can later over-ride), can we calculate the probability that an OOM strike price will be hit or exceeded in one day? If the stock is **225** today and we have a strike price of **227**, what is the probability?



Hint:

1. What will

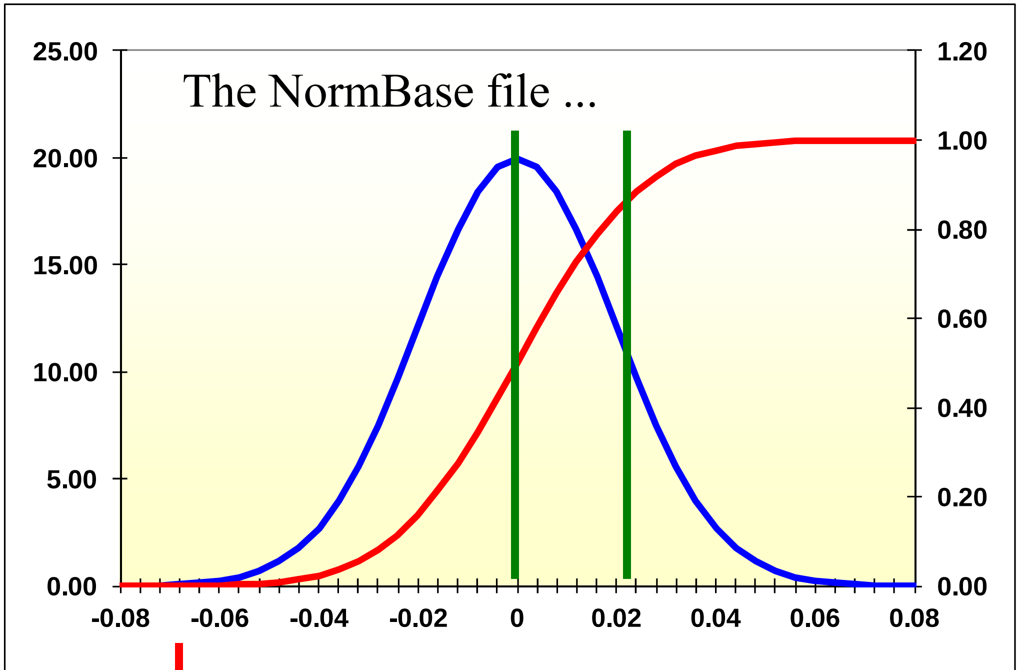
$$\ln\left(\frac{227}{225}\right)$$

tell us?

2. What will the answer to **1** divided by **SD**

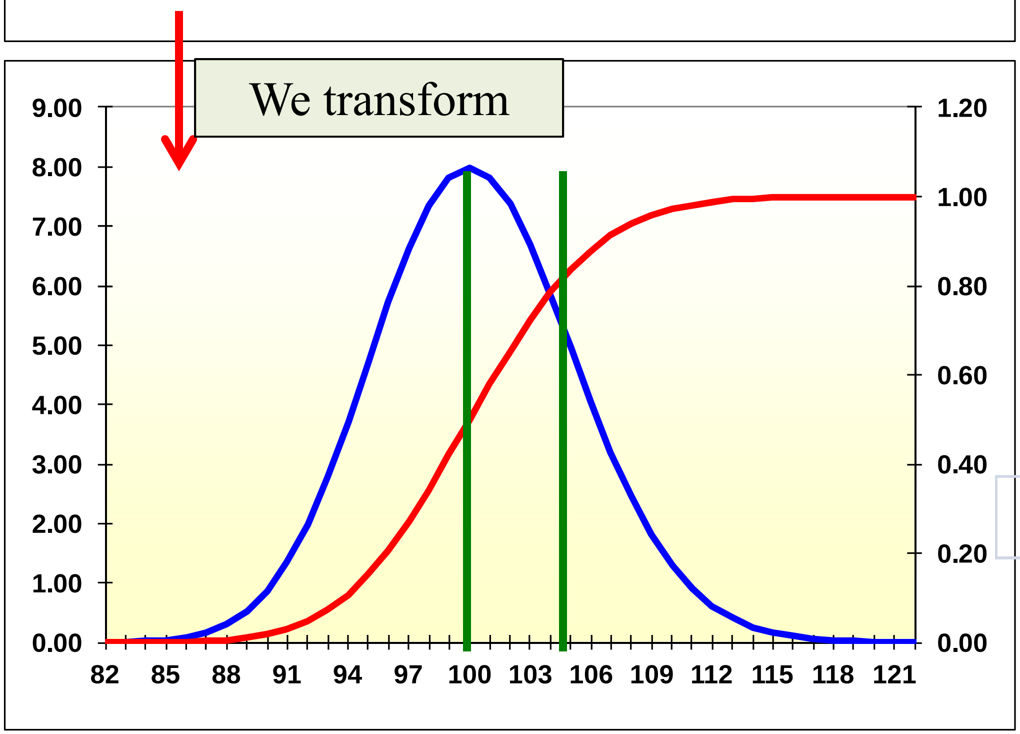
$$\ln\left(\frac{227}{225}\right)\Big/\sigma$$

tell us?



When you ask the question, "What is the prob that price will go from 100 to 105?", you are also asking "How many standard deviations is 105 away from 100, and what is the probability of that?"

Ranges		Mean to	Value	Cumulative Probs	
+/- 3	0.9973	0.4987	-0.0400	-2	0.0228
+/- 2	0.9545	0.4772	-0.0200	-1	0.1587
+/-1.5	0.8664	0.4332	0.0000	0	0.5000
+/-1	0.6827	0.3413	0.0200	1	0.8413
+/-0.5	0.3829	0.1915	0.0600	2	0.9772



Value	Cumulative Probs	
90.4837	-2	0.022750
95.1229	-1	0.158655
100.0000	0	0.500000
105.1271	1	0.841345
110.5171	2	0.977250

Probability of X greater than:	105	is	0.164581

We use daily volatility, but if we need to do this for more than one day, and we usually will, then adjust for the number of days using this time volatility formula that we have seen before ...

$$\sigma_t = \sqrt{t}\,\sigma_d$$

And what if we wanted to add an alpha drift? Just keep in mind that your expected value will shift daily by approximately this formula ...

$$P_t = P_0 e^{rt}$$

... so maybe calculate this first, then calculate the spread between this new value and the strike price.

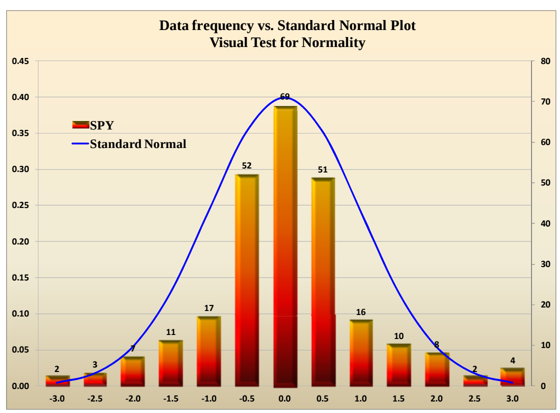
One Year:	
Average DGR:	0.00084
Standard Deviation:	0.00762
Average ABS DCGR:	0.00538
Min:	-0.03657
MaX:	0.02408
Min Norm DCGR:	-4.90659
Max Norm DCGR:	3.04865
<F9> to calculate	
60 day:	
Average DGR:	0.00111
Standard Deviation:	0.00518
Average ABS DCGR:	0.00390
Min:	-0.00830
MaX:	0.02182
Min Norm DCGR:	-1.81595
Max Norm DCGR:	3.99837
30 Day:	
Average DGR:	0.00119
Standard Deviation:	0.00463
Average ABS DCGR:	0.00375
Min:	-0.00830
MaX:	0.01300
Min Norm DCGR:	-2.04984
Max Norm DCGR:	2.55050
Stock Symbol:	SPY

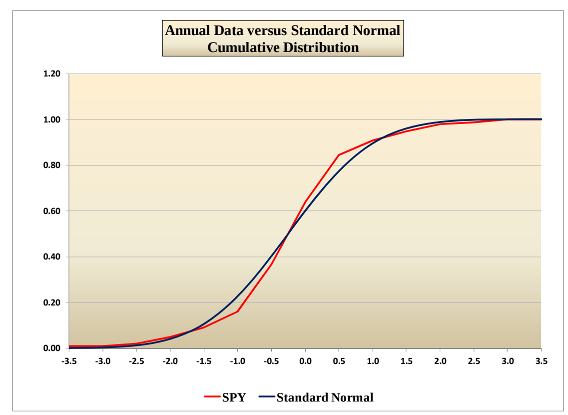
You calculated daily volatility (SD) for one year, 60 days and 30 days.

The results shown here are for 2016. Your assignment may differ.

You will use the 252-day standard deviation as your default volatility measure (0.00762 in this example).

You will use the Average one-year DCGR as your *alpha* in your homework (the true *alpha* equals *Mu* minus one-half variance), equal to 0.00084 in this example.





The Homework assignment ...

SPY 229.05 +1.45 (+0.64%) U1395405

Call Last	Call Change	Call Bid	Call Ask	Call Volume	Call OI	Description	Put Last	Put Change	Put Bid	Put Ask	Put Volume	Put OI
						FEB 03 '17						
4.38	+1.24	4.34	4.39	381	6.28K	225	0.26	-0.27	0.27	0.28	4.36K	26.3K
3.72	+0.99	3.88	3.94	31	2.54K	225.5	0.33	-0.29	0.31	0.32	769	4.55K
3.53	+1.19	3.44	3.49	517	6.48K	226	0.37	-0.36	0.37	0.38	2.79K	8.21K
3.03	+1.06	3.02	3.06	384	6.44K	226.5	0.44	-0.42	0.44	0.45	2.99K	10.8K
2.61	+0.99	2.62	2.65	1.07K	9.41K	227	0.52	-0.49	0.53	0.54	2.19K	10.6K
2.17	+0.86	2.23	2.26	393	11.3K	227.5	0.63	-0.57	0.63	0.65	1.71K	4.66K
1.95	+0.92	1.87	1.89	8.85K	21.3K	228	0.76	-0.66	0.77	0.78	6.67K	3.38K
1.59	+0.78	1.54	1.56	5.28K	22.4K	228.5	0.95	-0.75	0.94	0.95	3.12K	1.30K
1.24	+0.63	1.24	1.26	8.25K	22.8K	229	1.13	-0.87	1.14	1.16	13.2K	2.03K
1.03	+0.57	0.99	1.01	14.3K	6.14K	229.5	1.39	-0.96	1.39	1.41	2.79K	975
0.81	+0.47	0.78	0.79	42.4K	35.7	230	1.67	-1.05	1.67	1.70	2.29K	1.32K
0.63	+0.38	0.61	0.62	63.7K	6.07K	230.5	1.95	-1.19	2.00	2.03	176	204
0.47	+0.29	0.47	0.49	1.49K	3.33K	231	2.35	-1.23	2.36	2.39	289	740
0.37	+0.23	0.37	0.38	1.33K	4.00K	231.5	2.75	-1.28	2.74	2.79	480	70
0.35	+0.25	0.29	0.30	2.28K	2.86K	232	3.14	-1.37	3.19	3.22	131	175

Generally, what is the probability that the call option shown above will be above the strike price at expiration. This was taken on January 25, 2017 at 12:45PM (NYT), so expiration is **7** trading days away.

Let us assume that we bought this at Besk Ask (**0.79**) when the stock was trading for **229.05** (circled in red at the top).

The Homework assignment (detail) ...

In this video example for 2017 homework assignment, we are assuming that on Wednesday, February 25, 2017 at 12:45 PM we have bought a single contract (not relevant) of an **SPY** call at the **230** strike price for the Friday, February 3rd, expiration, which we will treat as **7** days away. At the moment we made this purchase, **SPY** had just traded for **229.05** (in a different context, we might use **SPY** Best Bid or Best Ask but for this homework use the most recent trade). We bought the call option at Best Ask, **0.79** (not relevant to this homework problem but relevant to later problems that build on this).

Homework Part 1: What is the probability of the stock price being above the strike price by the expiration date if I assume that the SPY alpha is zero?

Homework Part 2: What is the probability of the stock price being above the strike price by the expiration date if I assume that the SPY alpha is based upon my estimate of the mean adjusted CGR?

Note: We are **not** estimating the probability that the stock price will rise above the strike price at least once during the 7-day duration. To estimate that we would have to use **Baye's Theorem**, the **Conditional Probability Hypothesis**.

Calculating the probability of being above the strike price at expiration.

STRIKE PRICE PROBABILITY CALCULATOR

Analyst's Name:	
Date Today:	1/25/17
Stock	
Stock Symbol:	SPY
Stock Price:	229.050
Option	
Option Strike Price:	230
Option Expiration Date:	2/3/17
Option Price (Best Ask):	1.690
Days to expiration:	7
Historical Data	
Log Growth Rate Price to Strike Price:	
Sample Mean Daily Growth Rate:	
Sample Daily Standard Deviation:	
Time-adjusted Standard Deviation:	
Calculation	
Probability Stock Price > Strike Price at Expiration, 0 alpha:	
Probability Stock Price > Strike Price at Expiration, estimated alpha:	

We will start by doing this in Excel (for the GUI), although we want to transfer the project to Python. For the data use your results from **HW1** and use the data for **252** observations.

For HW part 1, assume the alpha is zero, which is loaded as a default.

For HW part 2, use the historical alpha (252 observations) given your results from HW1.

The daily volatility is adjusted for duration volatility by multiplying SD by the square root of the number of days to expiration.

This interface is designed as a learning interface – taking you through a sequence of steps.

A Python portion of this problem

```
# Calculate a standard normal (alpha zero, sigma 1) at value 1 SD:
# CSND integrates a standard normal distribution up to some sigma.
#
def csnd(point):
    return (1.0 + math.erf(point/math.sqrt(2.0)))/2.0
#
sigma = 0.5
x = csnd(sigma)
print
print "In the standard normal distribution, the cumulative"
print "probability at ",sigma, "standard deviations equals", "%.5f" % x, "."
print
print
#
# Calculate a normal at some alpha and sigma.
# CND integrates a Gaussian distribution up to some value.
#
def cnd(point):
    return (1.0 + math.erf((point - alpha)/(sigma*math.sqrt(2.0))))/2.0
#
alpha = 40
sigma = 1.5
val = 42
x = cnd(val)
print "When the mean is", "%.3f" % alpha, "and the sigma is", "%.3f" % sigma, "and when"
print "our point is", "%.3f" % val, "the cumulative probability is", "%.5f" % x, "."
```