

First, define  $q = [q_p^T, q_a^T]^T$ , where  $q_p$  are the passive (floating base) joints, and  $q_a$  are the actuated joints. If I assume that the ground contacts on the feet will stay stationary, then I have

$$\begin{aligned}\dot{x}_{gc} &= [J_{gc}^p, J_{gc}^a] \begin{bmatrix} \dot{q}_p \\ \dot{q}_a \end{bmatrix} = 0, \\ \Rightarrow J_{gc}^p \dot{q}_p &= -J_{gc}^a \dot{q}_a, \\ \Rightarrow \dot{q}_p &= -J_{gc}^{p+} J_{gc}^a \dot{q}_a, \\ \Rightarrow \dot{q} &= \begin{bmatrix} -J_{gc}^{p+} J_{gc}^a \\ I \end{bmatrix} \dot{q}_a = A \dot{q}_a,\end{aligned}$$

where  $J_{gc}$  is the kinematic Jacobian of the ground contacts, the superscripts  $a$  and  $p$  denote the subsets of this matrix related to  $q_a$  and  $q_p$ , and the superscript  $+$  indicates a pseudo-inverse. In the planar case,  $J_{gc}^p$  will likely have full column rank if there are two contact points (even on a single foot); in the 3D case you'd like to have three contact points (presumably on at least two different bodies).

For control, I want to regulate the  $x$  and  $y$  positions of the center of mass (first priority), then regulate hand position (second priority), then stay near a comfortable posture (last priority). For a good summary of kinematic Jacobian control, see [1]. We will use

$$\begin{aligned}\dot{x}_{com,xy} &= [I \quad 0] J_{com} A \dot{q}_a = J_1 \dot{q}_a, \quad x_{com,xy}^d = x_{csp}, \text{ and} \\ \dot{x}_{rhand} &= J_{rhand} A \dot{q}_a = J_2 \dot{q}_a,\end{aligned}$$

where COM stands for “center of mass” and CSP stands for “center of support polygon”. Set

$$\begin{aligned}\dot{x}_1 &= \eta_1 (x_{com,xy}^d - x_{com,xy}) \\ \dot{x}_2 &= \eta_2 (x_{rhand}^d - x_{hand}) \\ \dot{q}_0 &= \eta_3 (q_0^d - q_0)\end{aligned}$$

Finally, the control law is given by ([1] eq 15):

$$q_d = q + J_1^+ \dot{x}_1 + (I - J_1^+ J_1) J_2^+ (\dot{x}_2 - J_2 J_1^+ \dot{x}_1 + \dot{x}) + (I - J_1^+ J_1) (I - J_2^+ J_2) \dot{q}_0.$$

## References

- [1] Bruno Siciliano. Kinematic control of redundant robot manipulators: A tutorial. *Journal of Intelligent and Robotic Systems*, 3:201–212, 1990.