

The Space of Collider Events

BOOST 2019

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Massachusetts Institute of Technology

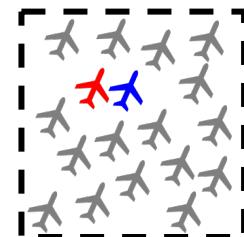
Joint work with Patrick Komiske, Radha Mastandrea, Preksha Naik, and Jesse Thaler

[\[1902.02346\]](https://arxiv.org/abs/1902.02346), to appear in PRL

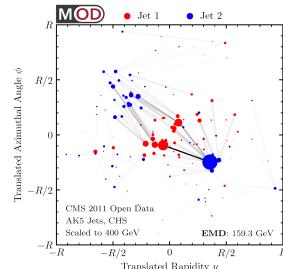
[19xx.xxxxx]

July 22, 2019

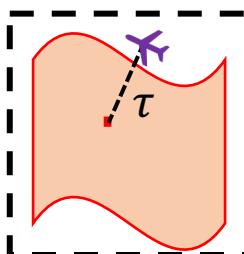
Outline



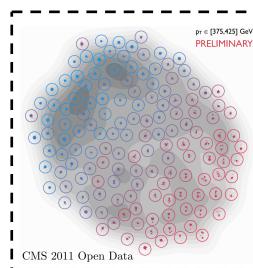
When are two jets similar?



Energy Mover's Distance

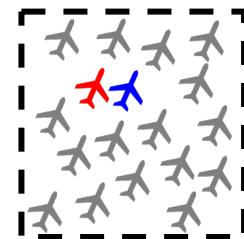


Quantifying Jet Similarity

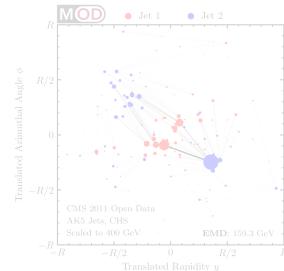


Exploring the Space of Jets

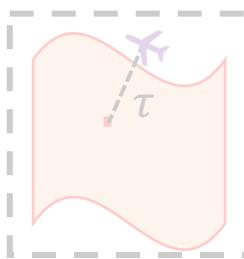
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Energy Mover's Distance



Quantifying Jet Similarity

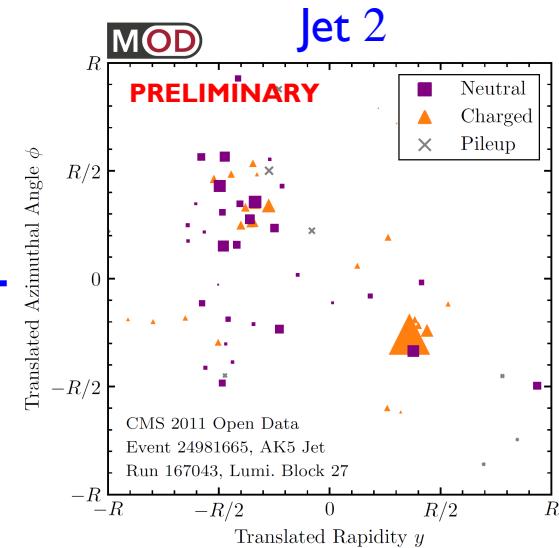
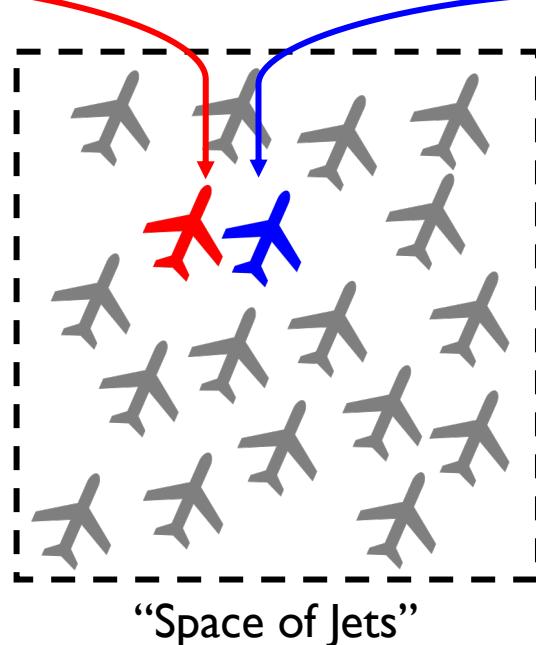
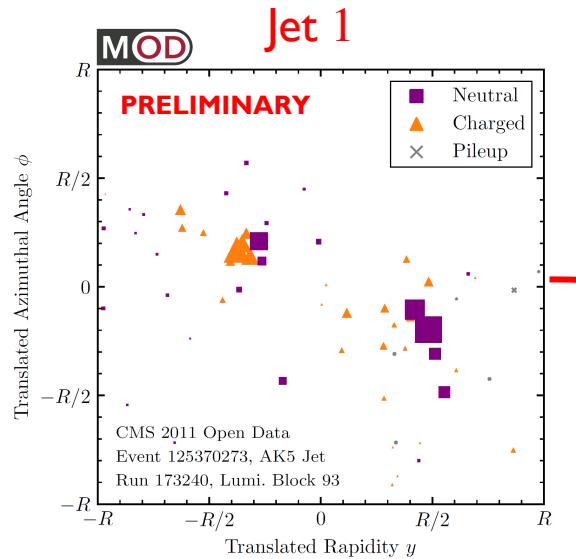


Exploring the Space of Jets

When are two jets similar?

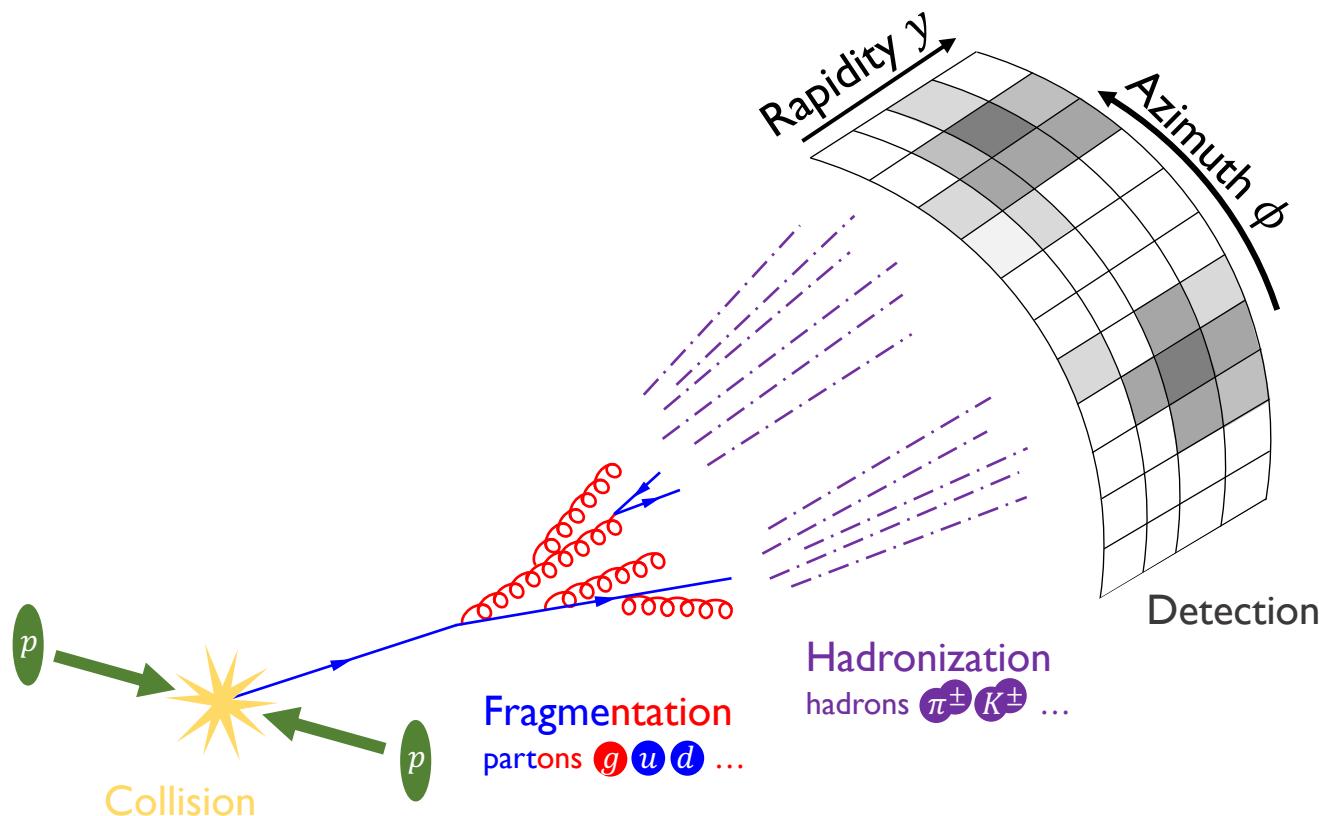
These two jets “look” similar, but have different numbers of particles, flavors, and locations.

How do we quantify this?



400 GeV AK5 Jets from CMS Open Data
See [Radha's talk](#) on Thursday for more!

When are two jets similar?

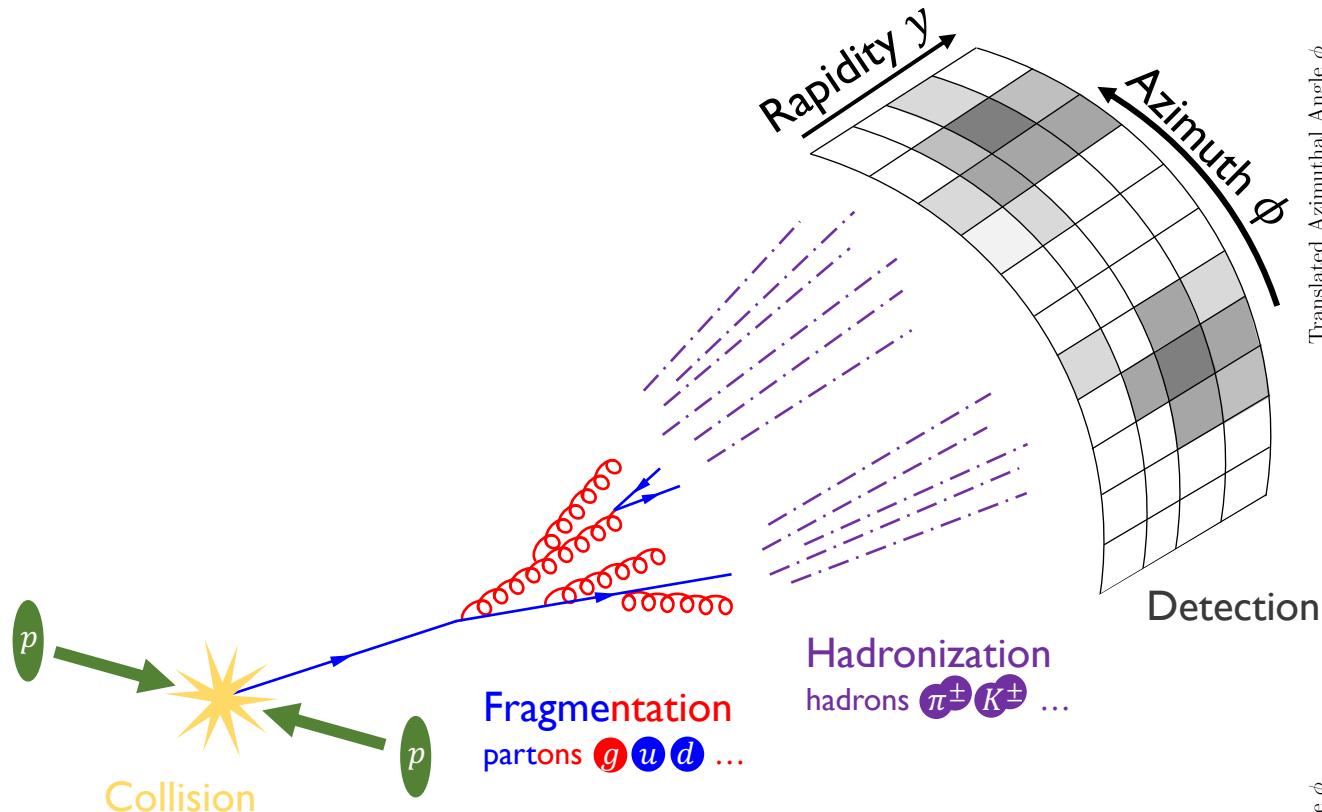


The energy flow (distribution of energy) is the information that is robust to:
fragmentation, hadronization, detector effects, ...

[\[N.A. Sveshnikov, F.V. Tkachov, 9512370\]](#)
[\[F.V. Tkachov, 9601308\]](#)
[\[P.S. Cherzor, N.A. Sveshnikov, 9710349\]](#)

Energy flow \Leftrightarrow Infrared and Collinear (IRC) Safe information

When are two jets similar?

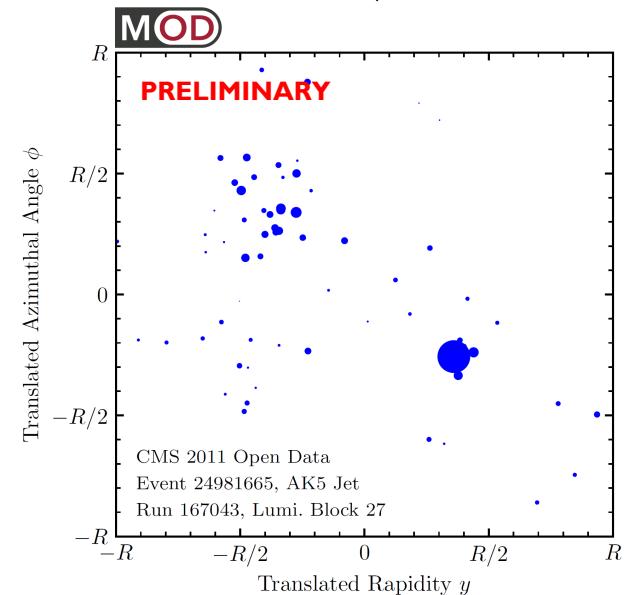
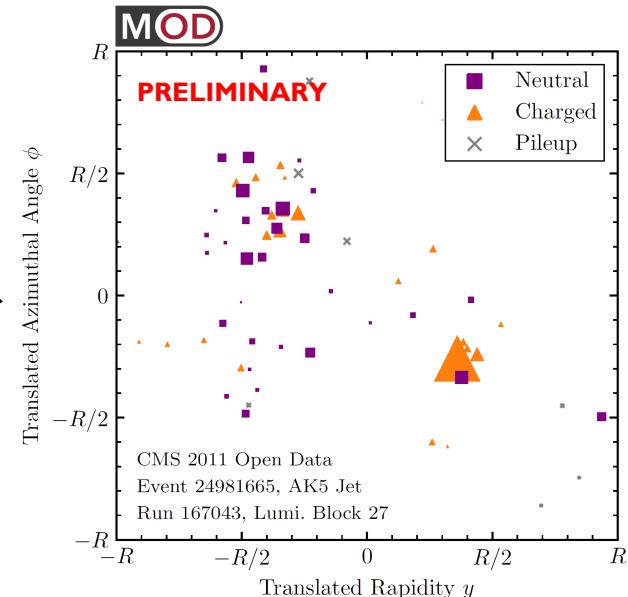


Treat jets as distributions of energy:

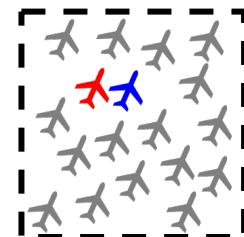
$$\mathcal{E}(\hat{n}) = \sum_{i=1}^M E_i \delta(\hat{n} - \hat{n}_i)$$

↑
energy ↑
direction

Ignoring particle flavor, charge...

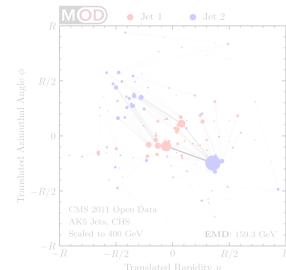


Outline

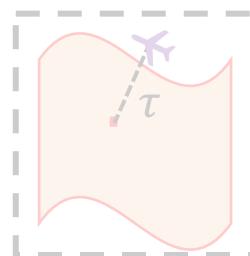


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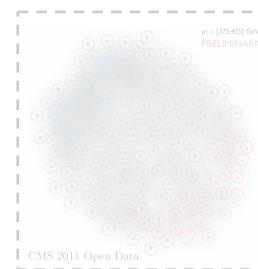
When they have similar distributions of energy



Energy Mover's Distance

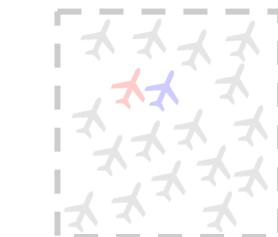


Quantifying Jet Similarity



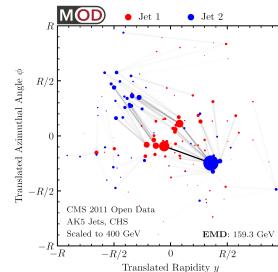
Exploring the Space of Jets

Outline

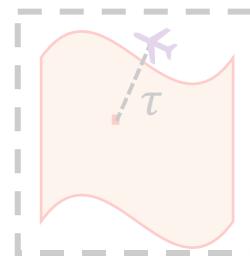


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When they have similar distributions of energy



Energy Mover's Distance



Quantifying Jet Similarity



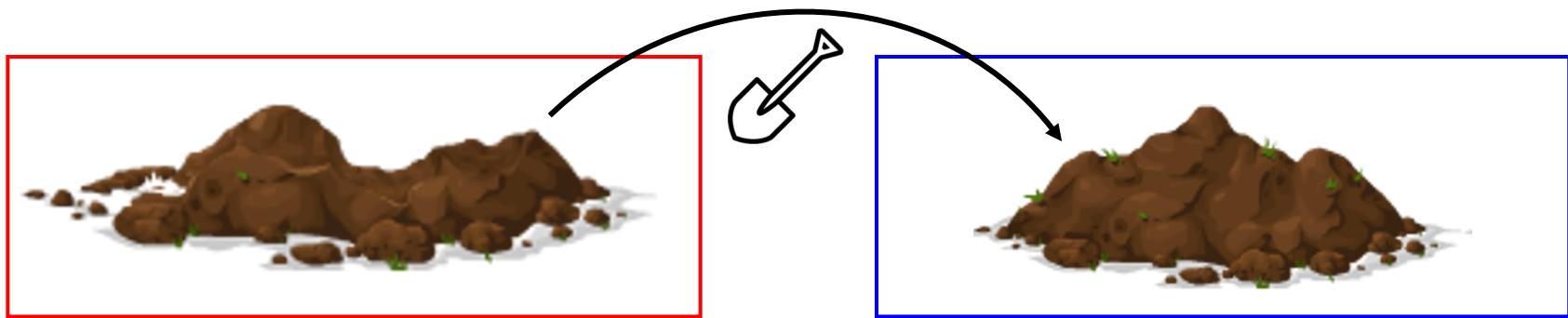
Exploring the Space of Jets

The Energy Mover's Distance

Review: *The Earth Mover's Distance*

Earth Mover's Distance: the minimum “work” ($\text{stuff} \times \text{distance}$) to rearrange one pile of dirt into another

[[Peleg, Werman, Rom](#)]
[[Rubner, Tomasi, Guibas](#)]



Metric on the space of (normalized) distributions: *symmetric, non-negative, triangle inequality*

Distributions are close in EMD \Leftrightarrow their expectation values are close.

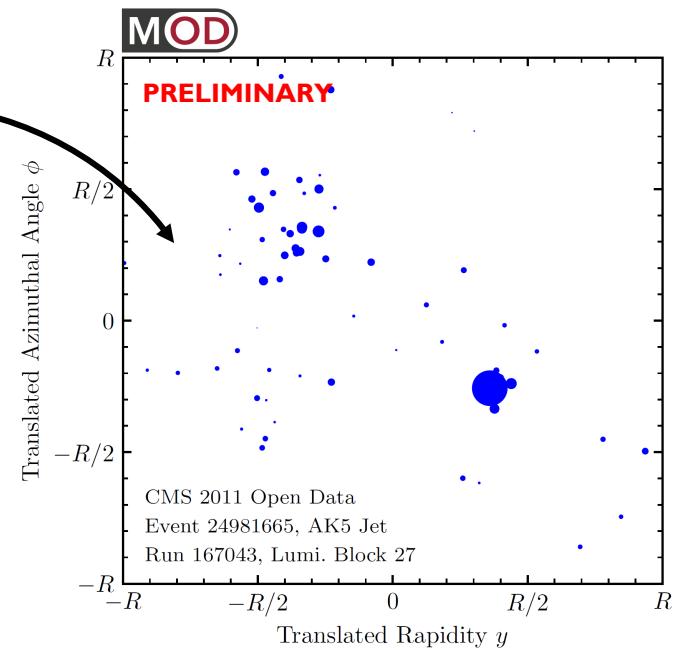
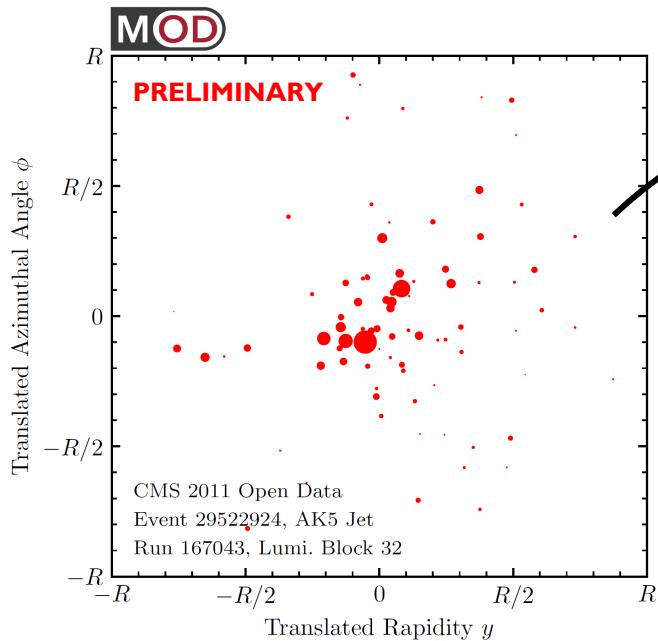
Also known as the 1-Wasserstein metric.

The Energy Mover's Distance

From Earth to Energy

Energy Mover's Distance: the minimum “work” (**energy** \times angle) to rearrange one jet (pile of energy) into another

[Komiske, EMM, Thaler, 1902.02346]



$$\text{EMD}(\mathcal{E}, \mathcal{E}') = \min_{\{f\}} \sum_{i=1}^M \sum_{j=1}^{M'} f_{ij} \frac{\theta_{ij}}{R} + \left| \sum_{i=1}^M E_i - \sum_{j=1}^{M'} E'_j \right|$$

Difference in radiation pattern Difference in total energy

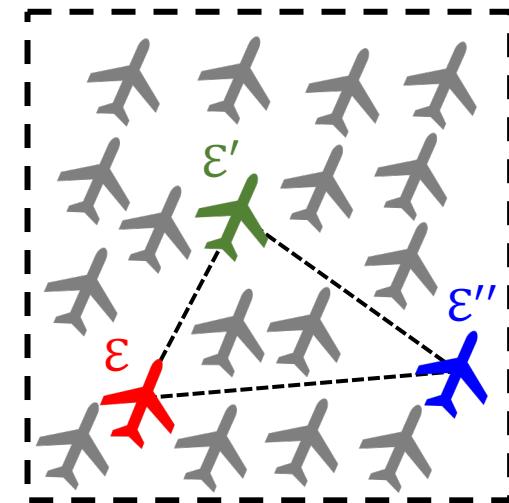
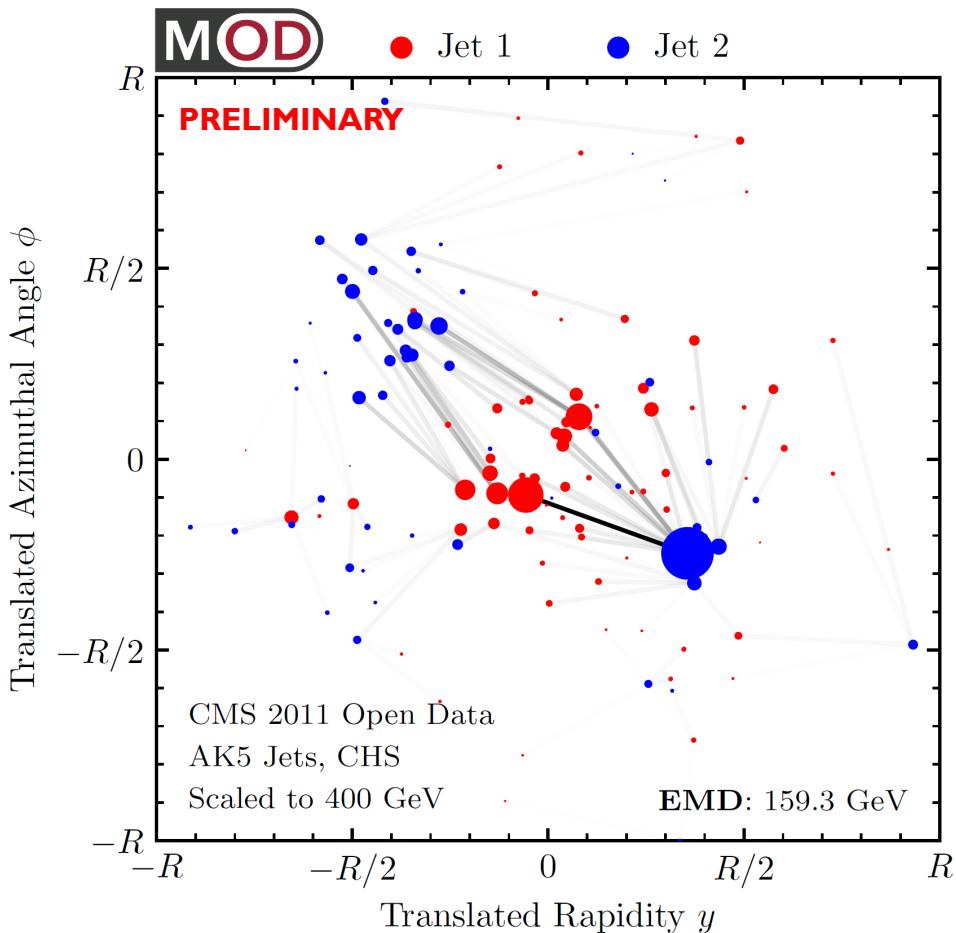
Diagram illustrating the Energy Mover's Distance (EMD) calculation. Two jets are shown: a red jet with energy E_i and a blue jet with energy E'_j . The angle between them is θ_{ij} . The fraction f_{ij} represents the fraction of energy moved from the red jet to the blue jet.

The Energy Mover's Distance

From Earth to Energy

Energy Mover's Distance: the minimum “work” (**energy** \times angle) to rearrange one event (pile of energy) into another

[\[Komiske, EMM, Thaler, 1902.02346\]](#)



EMD has dimensions of energy

True metric as long as $R \geq \frac{1}{2}\theta_{\max}$
 $R \geq$ the jet radius, for conical jets

Solvable via Optimal Transport problem.

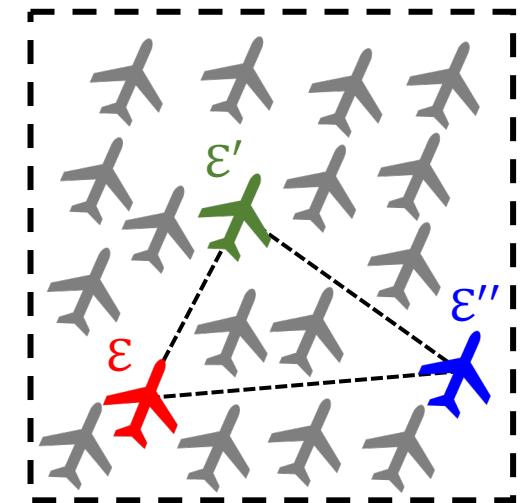
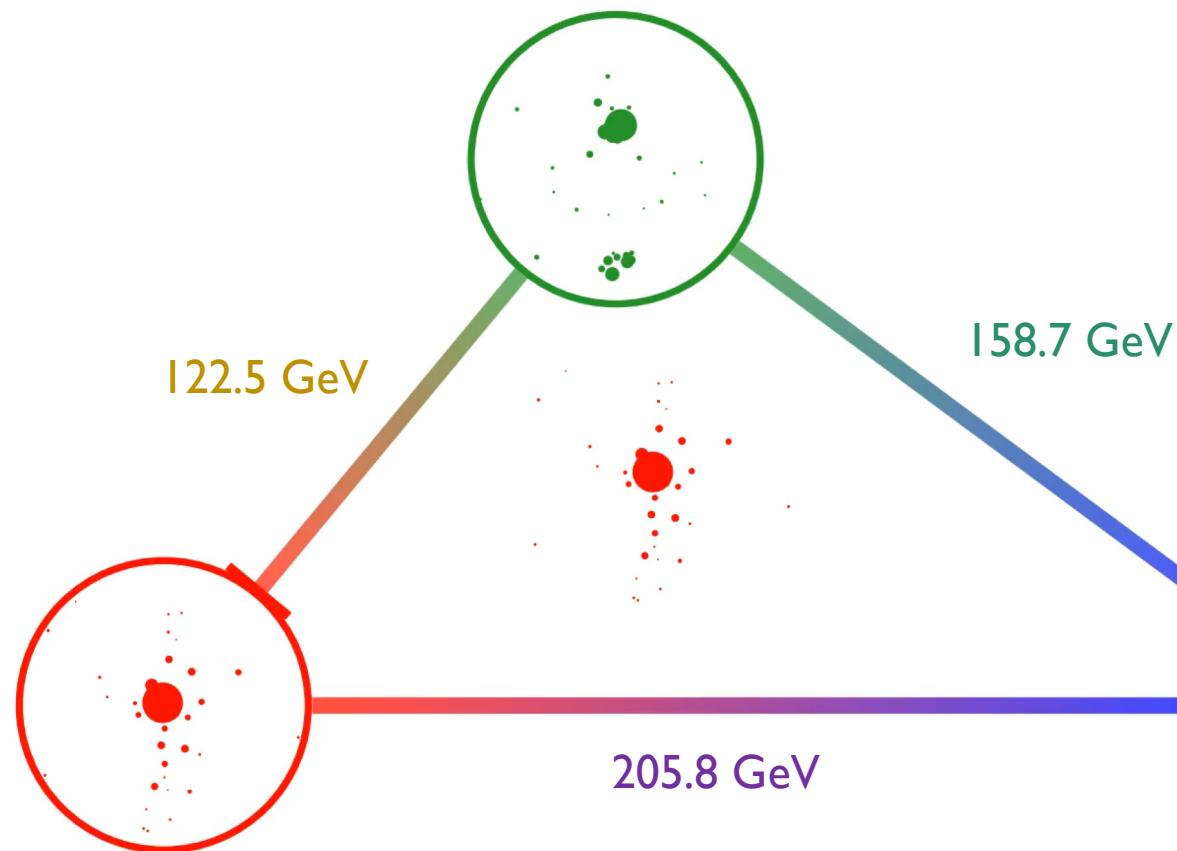
~1ms to compute EMD for two jets with 100 particles.

The Energy Mover's Distance

From Earth to Energy

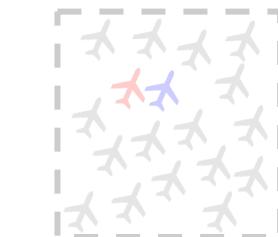
Energy Mover's Distance: the minimum “work” (**energy** × angle) to rearrange one event (pile of energy) into another

[Komiske, EMM, Thaler, 1902.02346]



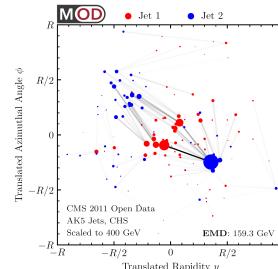
<https://energyflow.network>

Outline



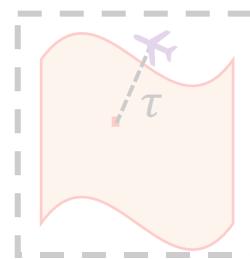
When are two jets similar?

When they have similar distributions of energy



Energy Mover's Distance

The “work” to rearrange one jet into another

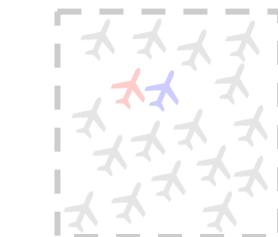


Quantifying Jet Similarity



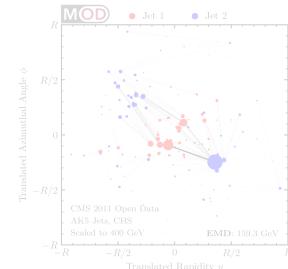
Exploring the Space of Jets

Outline



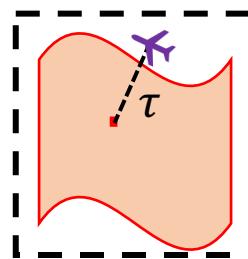
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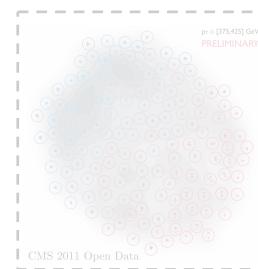


Energy Mover's Distance

The “work” to rearrange one jet into another



Quantifying Jet Similarity



Exploring the Space of Jets

Energy Moving and IRC Safety

Events close in EMD are close in any infrared and collinear safe observable!

Additive IRC-safe observables:

Energy Mover's
Distance

$$\text{EMD}(\mathcal{E}, \mathcal{E}') \geq \frac{1}{RL} |\mathcal{O}(\mathcal{E}) - \mathcal{O}(\mathcal{E}')|$$

Difference in
observable values

“Lipschitz constant” of Φ
i.e. bound on its derivative

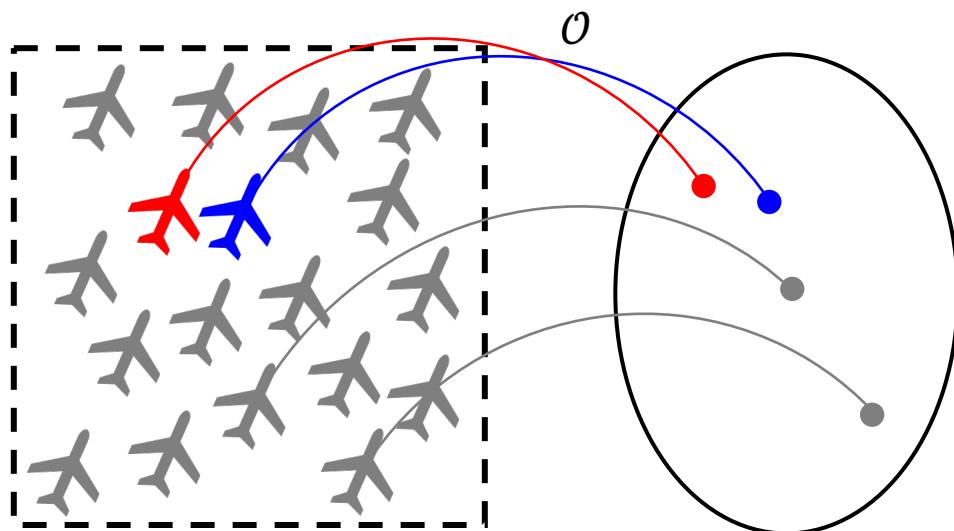
$$\mathcal{O}(\mathcal{E}) = \sum_{i=1}^M E_i \Phi(\hat{n}_i)$$

e.g. $\beta \geq 1$ jet angularities:

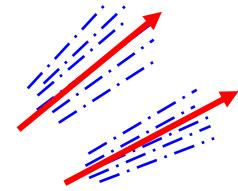
[\[Berger, Kucs, Sterman, 0303051\]](#)

[\[Larkoski, Thaler, Waalewijn, 1408.3122\]](#)

$$|\lambda^{(\beta)}(\mathcal{E}) - \lambda^{(\beta)}(\mathcal{E}')| \leq \beta \text{EMD}(\mathcal{E}, \mathcal{E}')$$



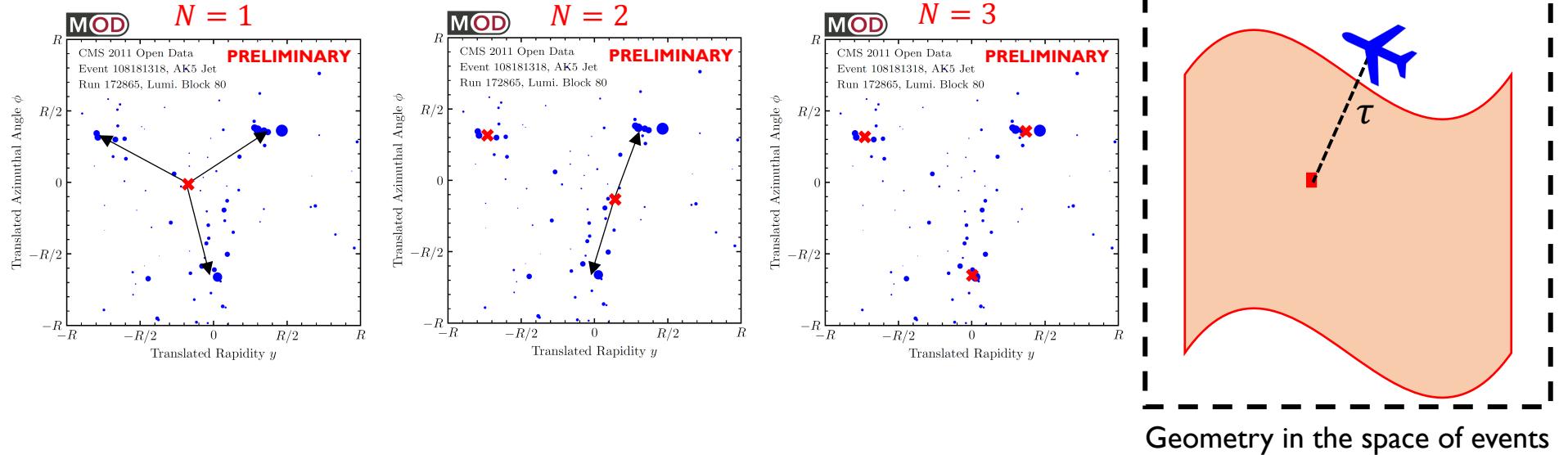
Old Observables in a New Language



N -subjettiness is the EMD between the event and the closest N -particle event.

$$\tau_N^{(\beta)}(\mathcal{E}) = \min_{N \text{ axes}} \sum_{i=1}^M E_i \min\{\theta_{1,i}^\beta, \theta_{2,i}^\beta, \dots, \theta_{N,i}^\beta\} \longrightarrow \tau_N(\mathcal{E}) = \min_{|\mathcal{E}'|=N} \text{EMD}(\mathcal{E}, \mathcal{E}').$$

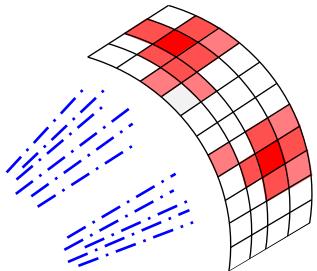
$\beta \geq 1$ is p -Wasserstein distance with $p = \beta$.



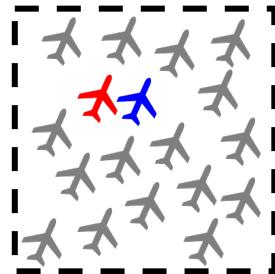
Thrust is the EMD between the event and two back-to-back particles.

$$t(\mathcal{E}) = E - \max_{\hat{n}} \sum_i |\vec{p}_i \cdot \hat{n}| \longrightarrow t(\mathcal{E}) = \min_{|\mathcal{E}'|=2} \text{EMD}(\mathcal{E}, \mathcal{E}') \quad \text{with } \theta_{ij} = \hat{n}_i \cdot \hat{n}_j, \hat{n} = \vec{p}/E$$

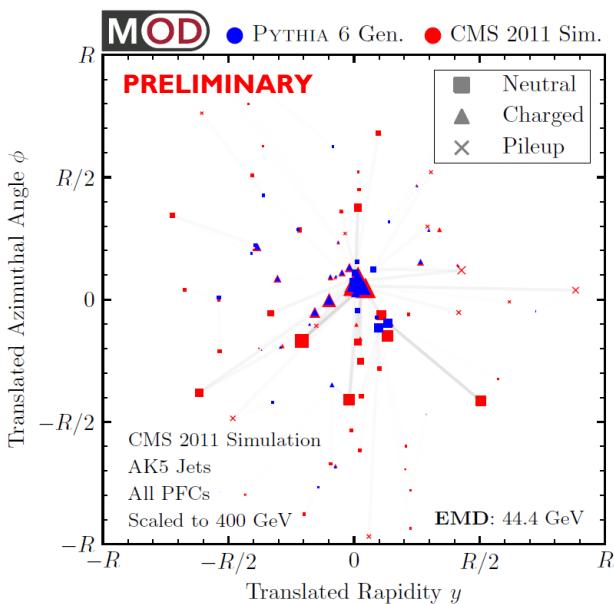
Quantifying Pileup and Detector Effects with EMD



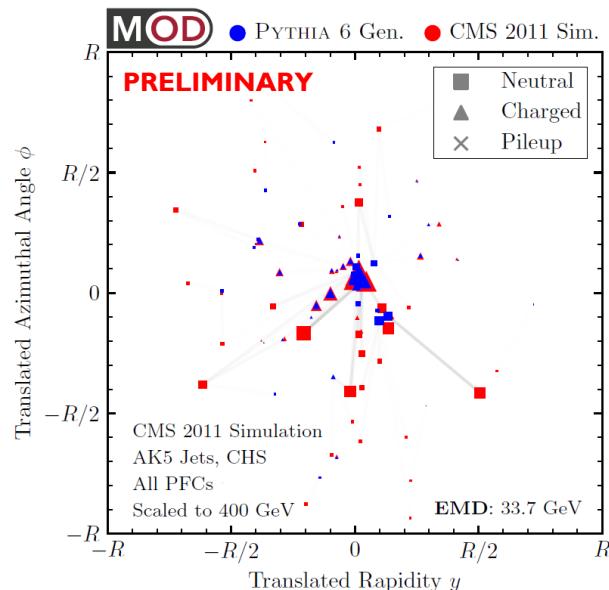
EMD universally quantifies pileup and detector effects.



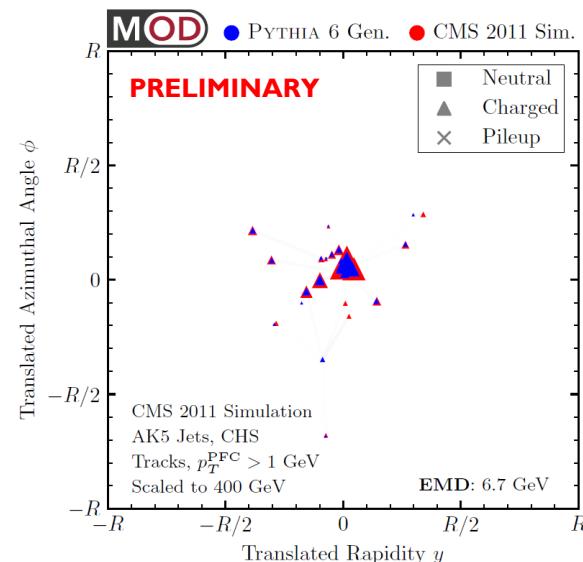
Gen./Sim. EMD: 44.4 GeV



Gen./Sim. EMD: 33.7 GeV



Gen./Sim. EMD: 6.7 GeV

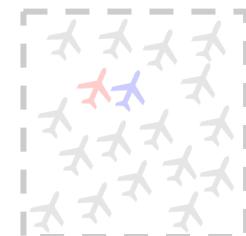


+ charged hadron subtraction

+ Tracks only, $p_T^{\text{PFC}} > 1 \text{ GeV}$ cut

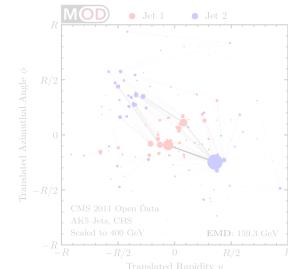
See extra slides for histograms. Can also quantify hadronization effects this way.

Outline



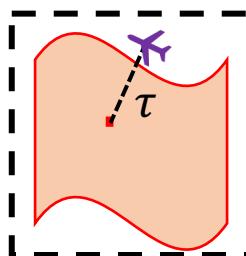
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When they have similar distributions of energy



Energy Mover's Distance

The “work” to rearrange one jet into another



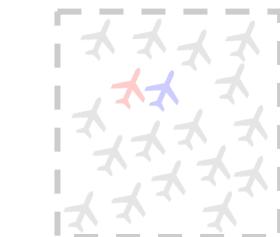
Quantifying Jet Similarity

Geometry of the space of jets. Bounds for pileup, detector effects



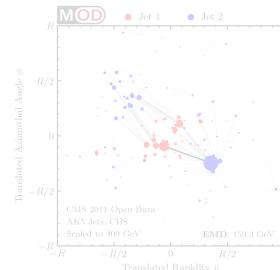
Exploring the Space of Jets

Outline



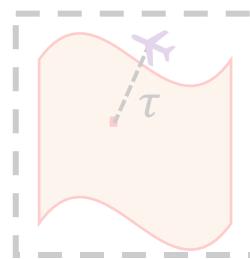
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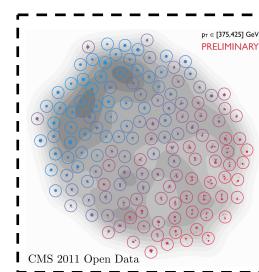
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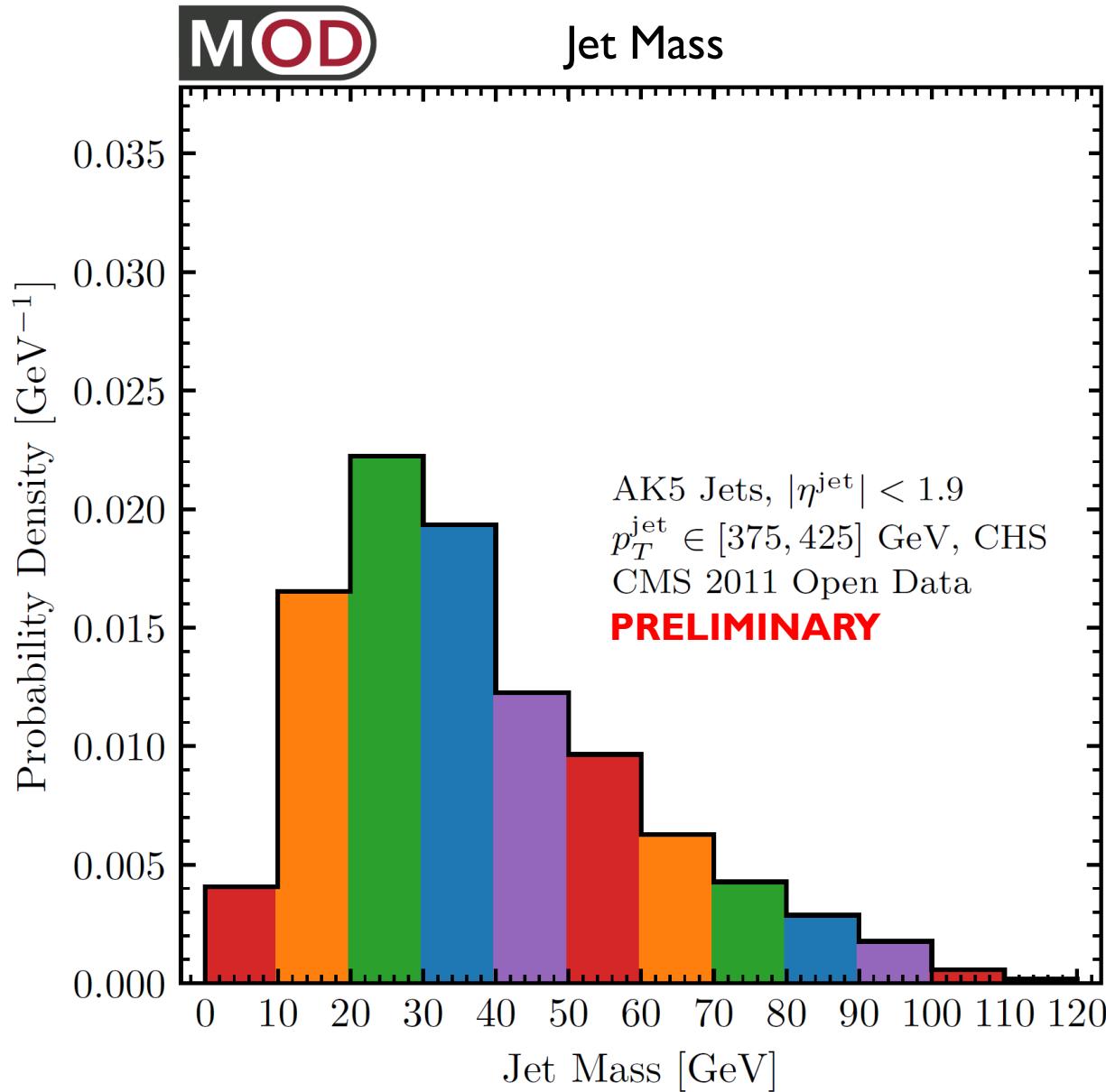
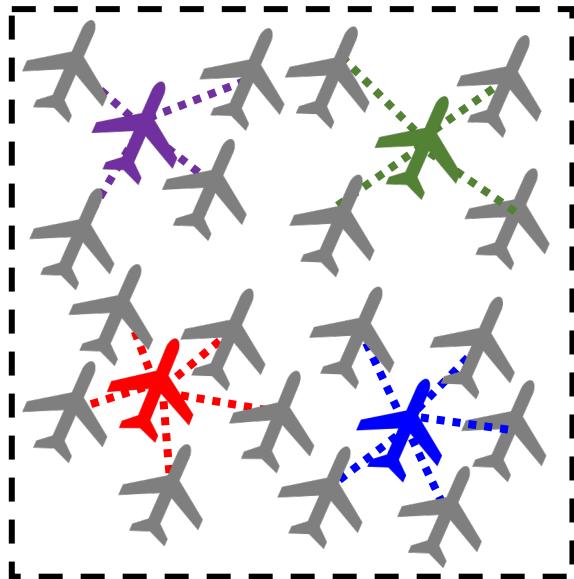
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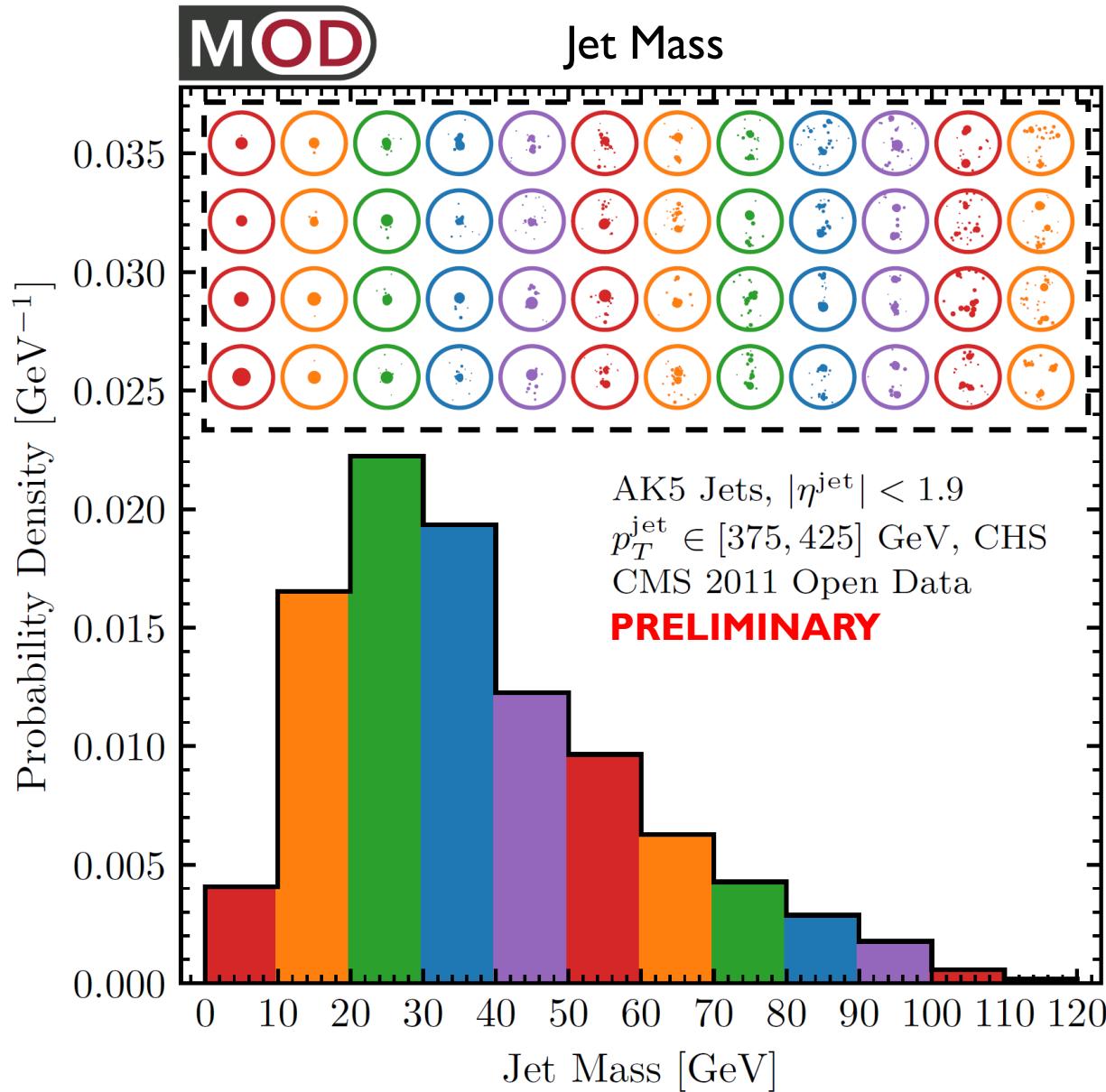
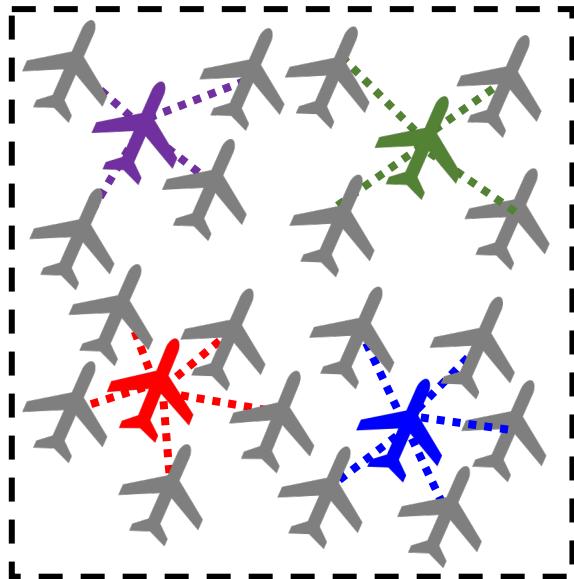


Exploring the Space of Jets

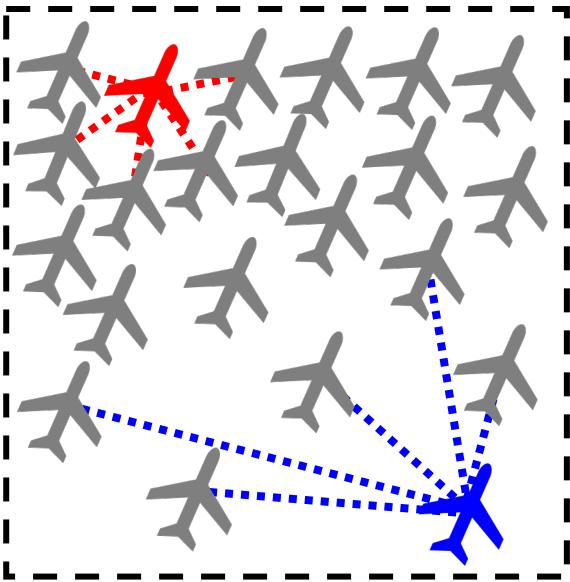
Most Representative Jets: K-medoids



Most Representative Jets: K-medoids



Towards Anomaly Detection



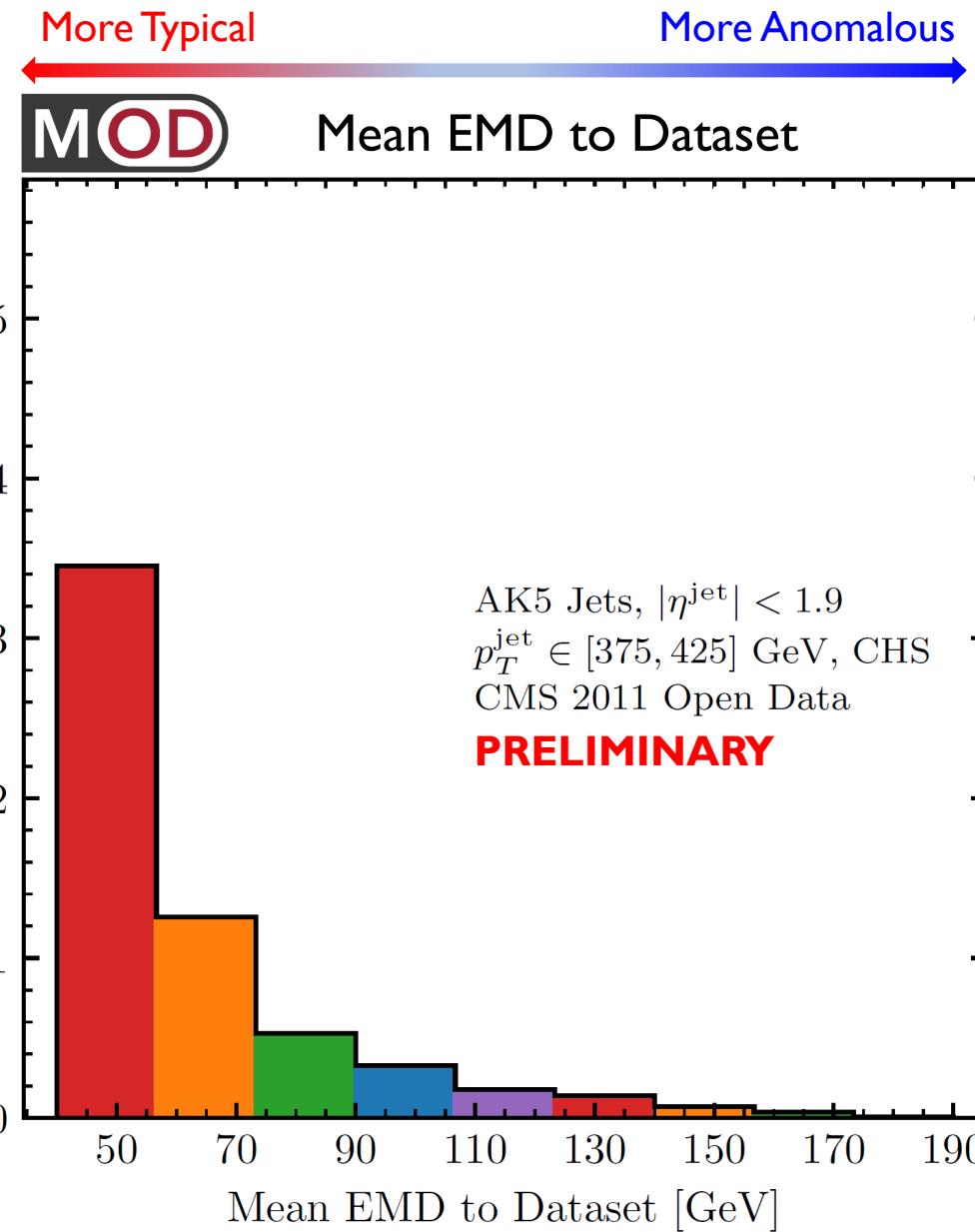
Complements recent developments in anomaly detection for collider physics.

[\[Collins, Howe, Nachman, 1805.02664\]](#)

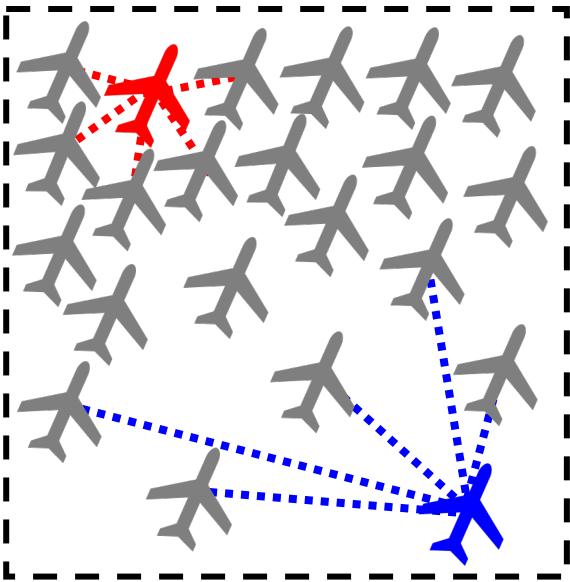
[\[Heimel, Kasieczka, Plehn, Thompson, 1808.08979\]](#)

[\[Farina, Nakai, Shih, 1808.08992\]](#)

[\[Cerri, Nguyen, Pierini, Spiropulu, Vlimant, 1811.10276\]](#)



Towards Anomaly Detection



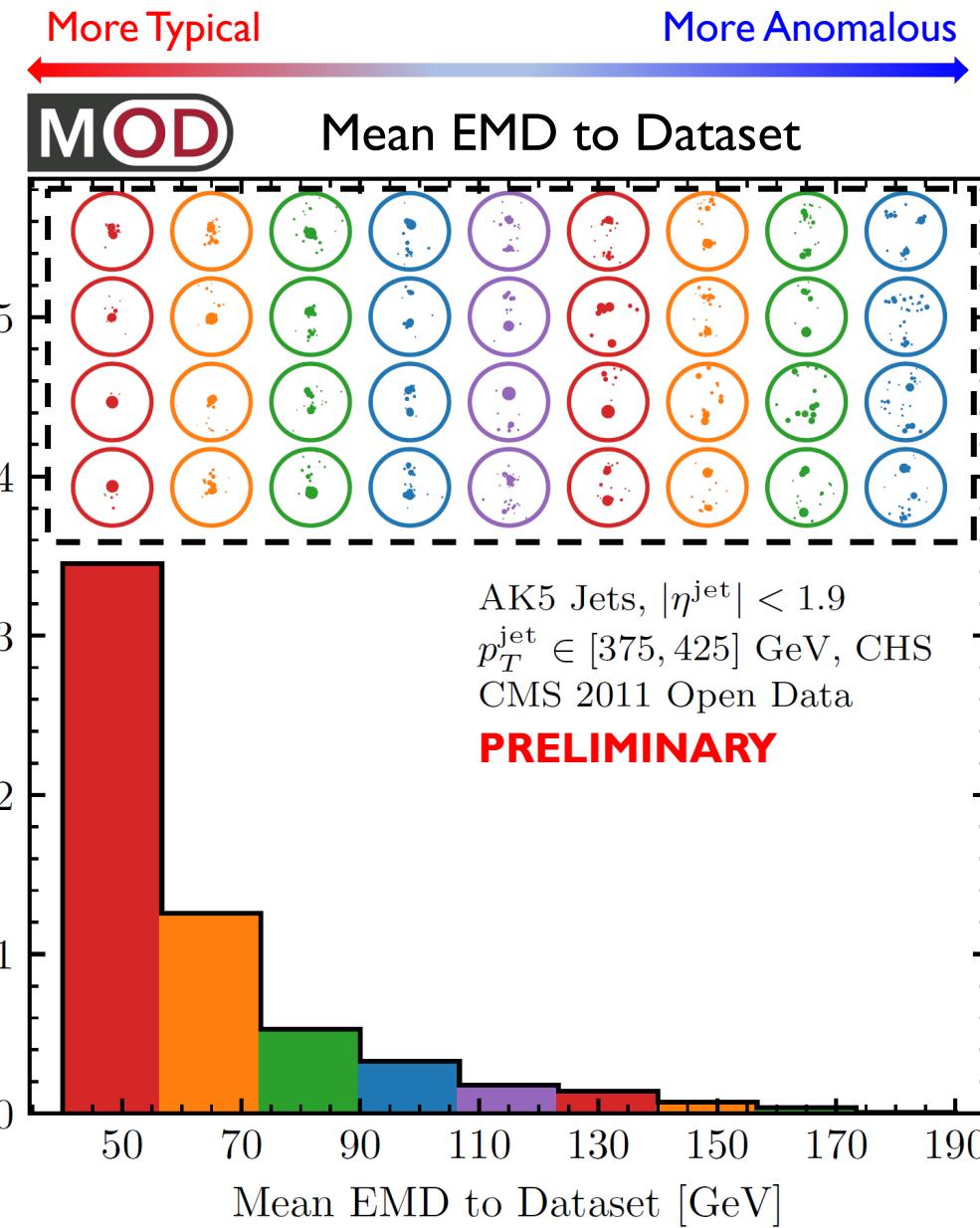
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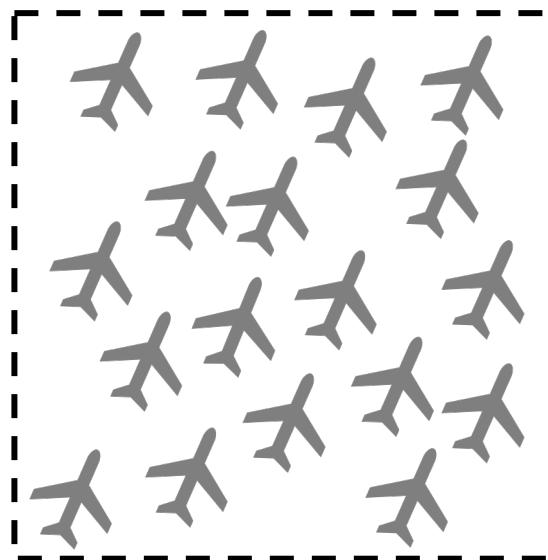


Exploring the Space of Jets: Visualizing the Manifold

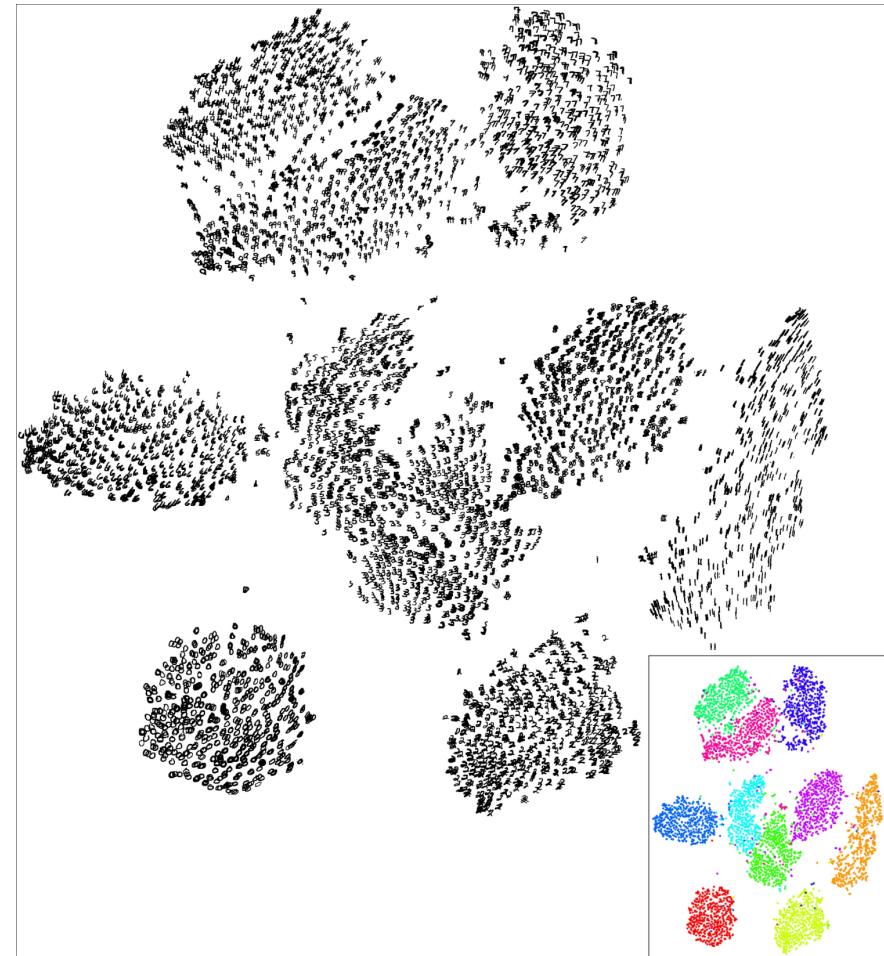
Visualize the space of events with t-Distributed Stochastic Neighbor Embedding (t-SNE).

[\[L. van der Maaten, G. Hinton\]](#)

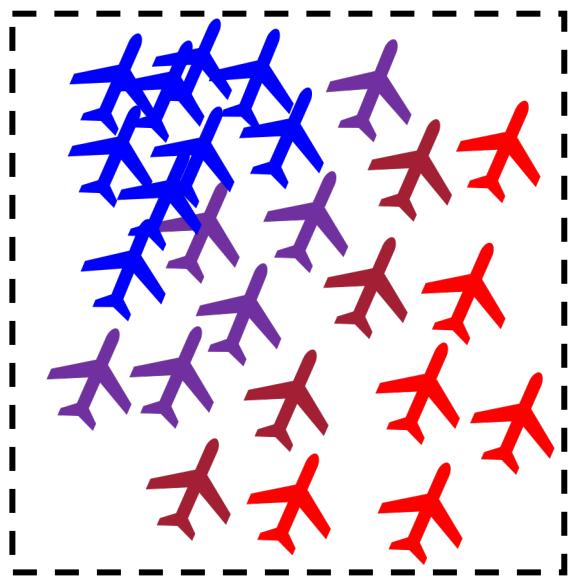
Finds an embedding into a low-dimensional manifold that respects distances.



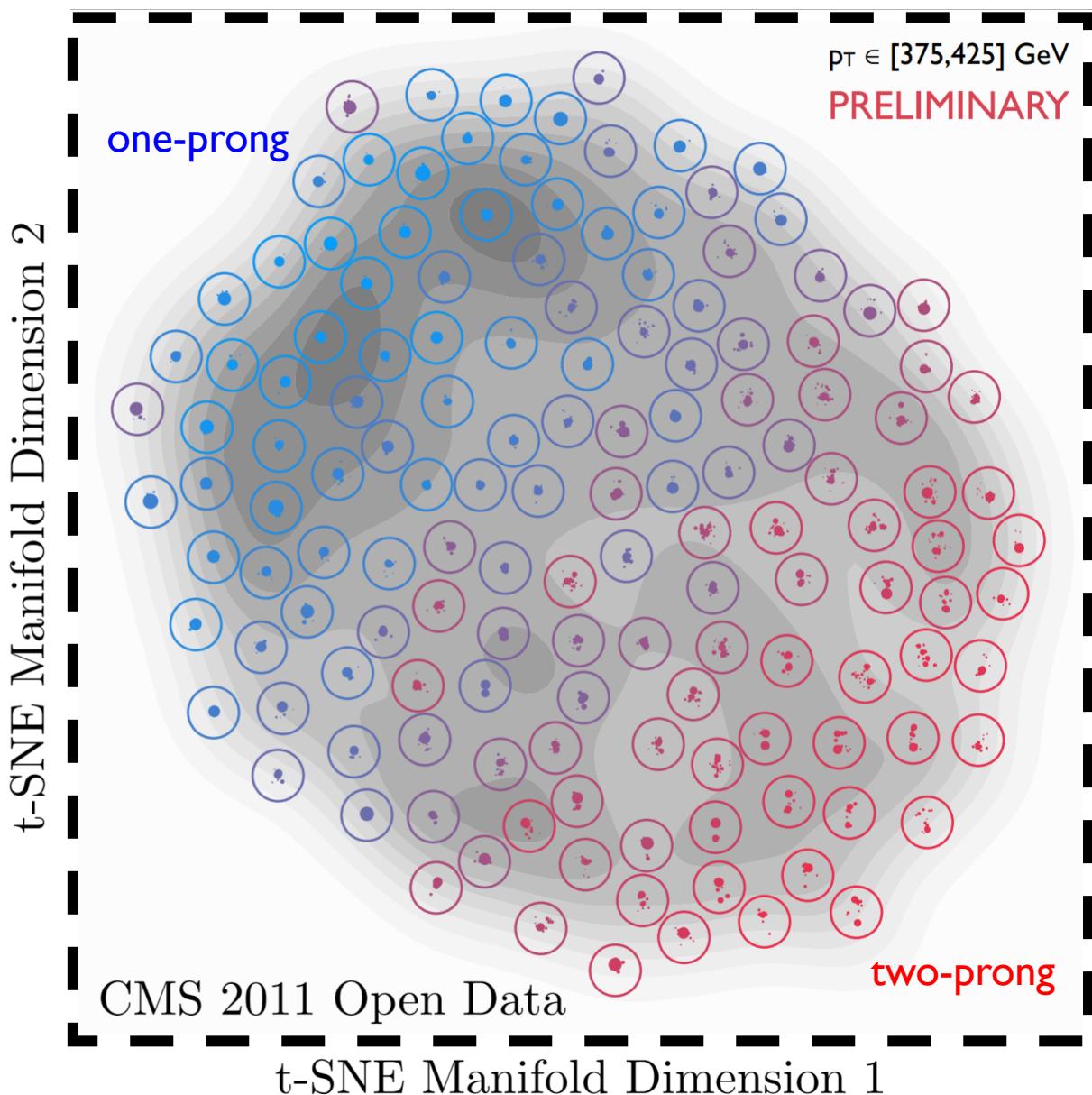
What does the space
of jets look like?



Exploring the Space of Jets: Visualizing the Manifold



What does the space
of jets look like?



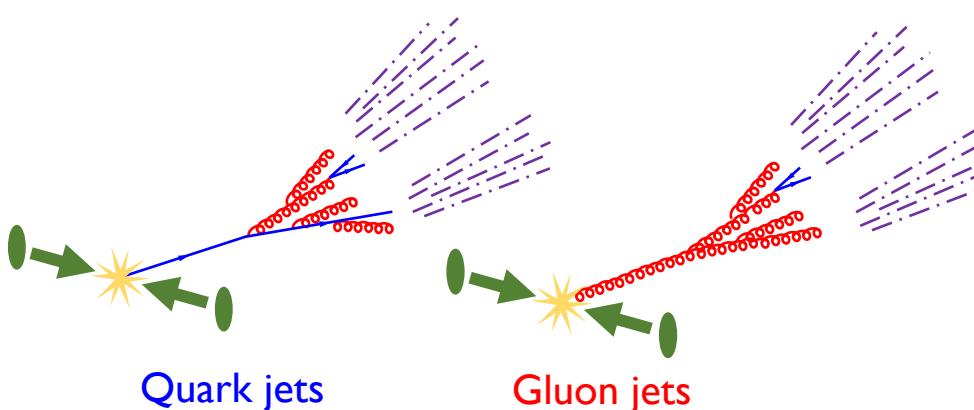
Exploring the Space of Jets: Correlation Dimension

Correlation dimension:

$$\dim(Q) = Q \frac{\partial}{\partial Q} \ln \sum_{i=1}^N \sum_{j=1}^N \Theta[\text{EMD}(\varepsilon_i, \varepsilon_j) < Q]$$

Energy scale Q
dependence

Count neighbors in
ball of radius Q

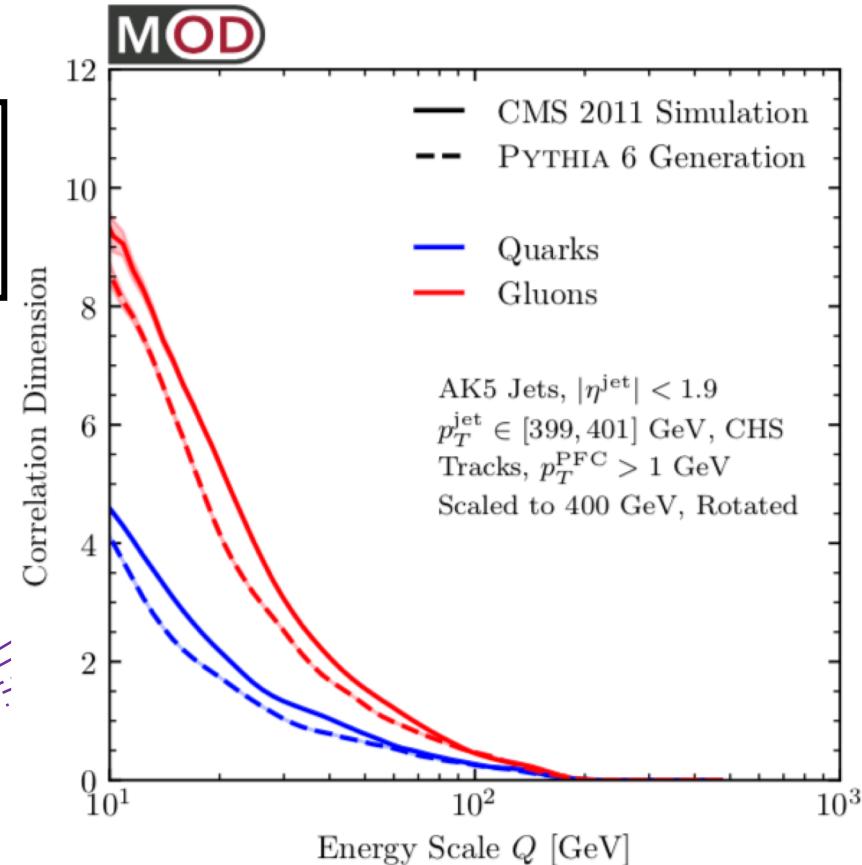


$$\text{At LL: } \dim_{q/g}(Q) = -\frac{8\alpha_s C_{q/g}}{\pi} \ln \frac{Q}{p_T/2}$$

MOD

- CMS 2011 Simulation
- PYTHIA 6 Generation
- Quarks
- Gluons

AK5 Jets, $|\eta^{\text{jet}}| < 1.9$
 $p_T^{\text{jet}} \in [399, 401] \text{ GeV, CHS}$
 Tracks, $p_T^{\text{PFC}} > 1 \text{ GeV}$
 Scaled to 400 GeV, Rotated

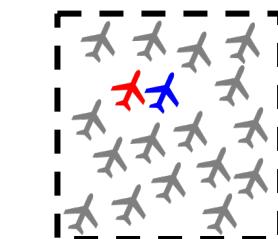


Energy Scale Q [GeV]

Dimension blows up at
low energies.

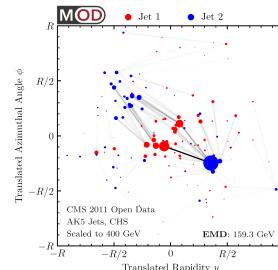
Jets are “more than fractal”

Outline



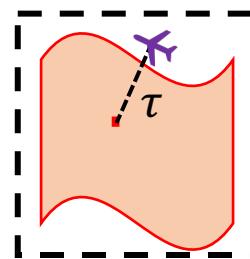
When are two jets similar?

When they have similar distributions of energy



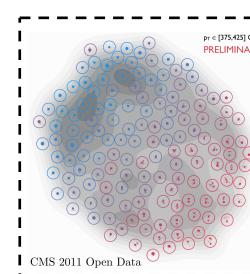
Energy Mover's Distance

The “work” to rearrange one jet into another



Quantifying Jet Similarity

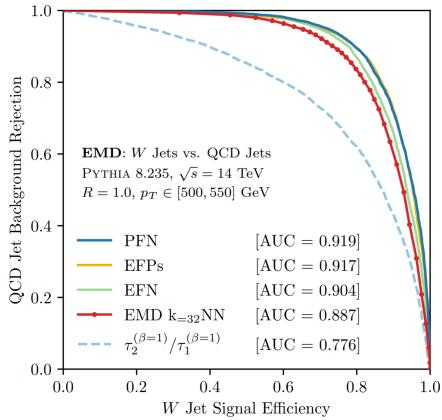
Geometry of the space of jets. Bounds for pileup, detector effects



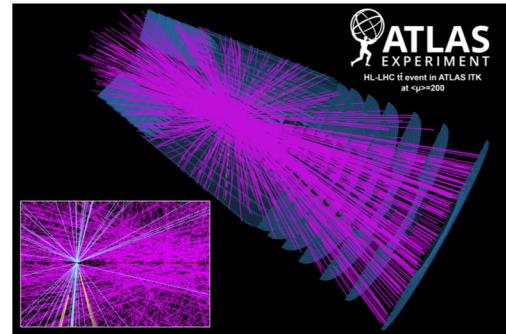
Exploring the Space of Jets

New ways to visualize and probe jet data

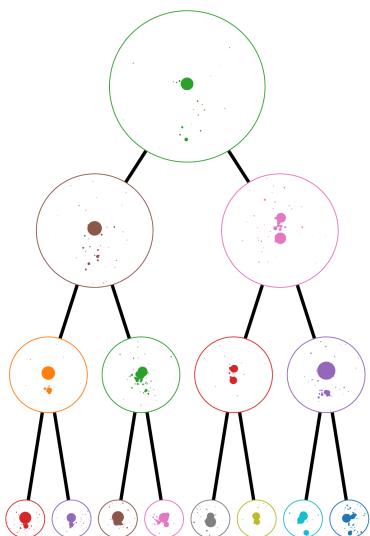
Going Beyond



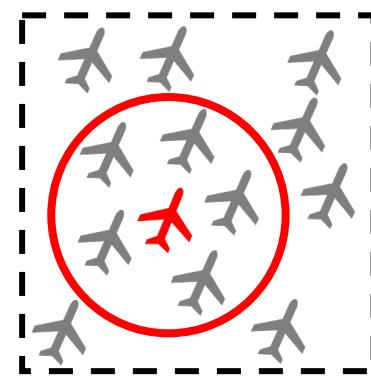
Classification with EMD



Clustering sets of events

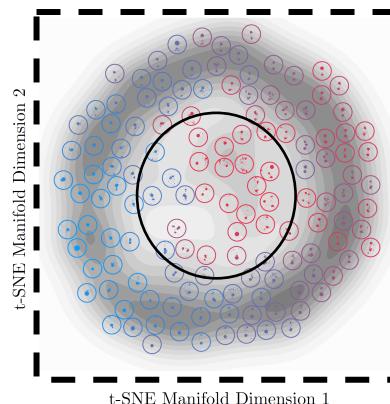


New observables through EMD geometry?



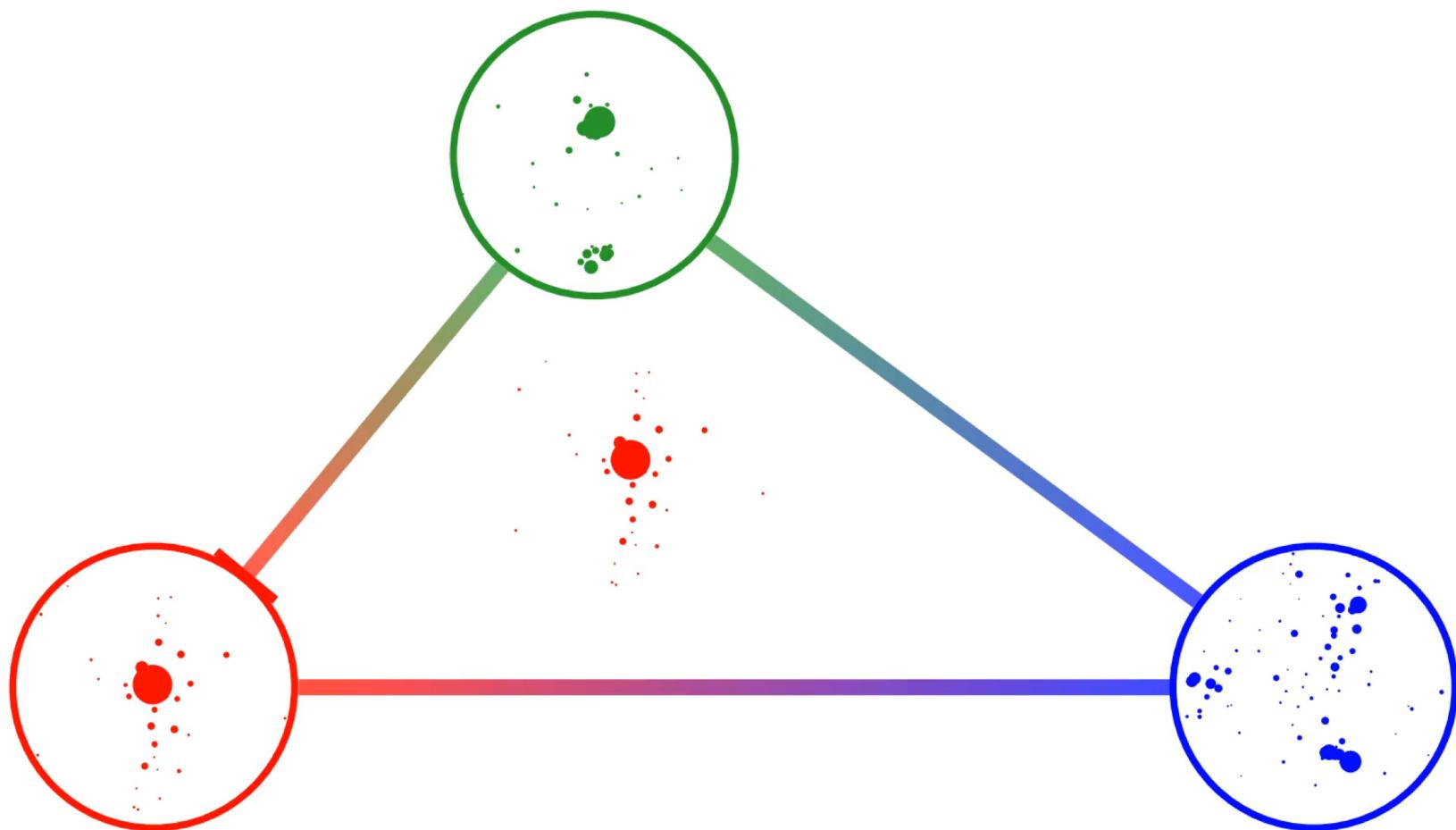
“Event” mover’s distance between ensembles?

Include flavor information?

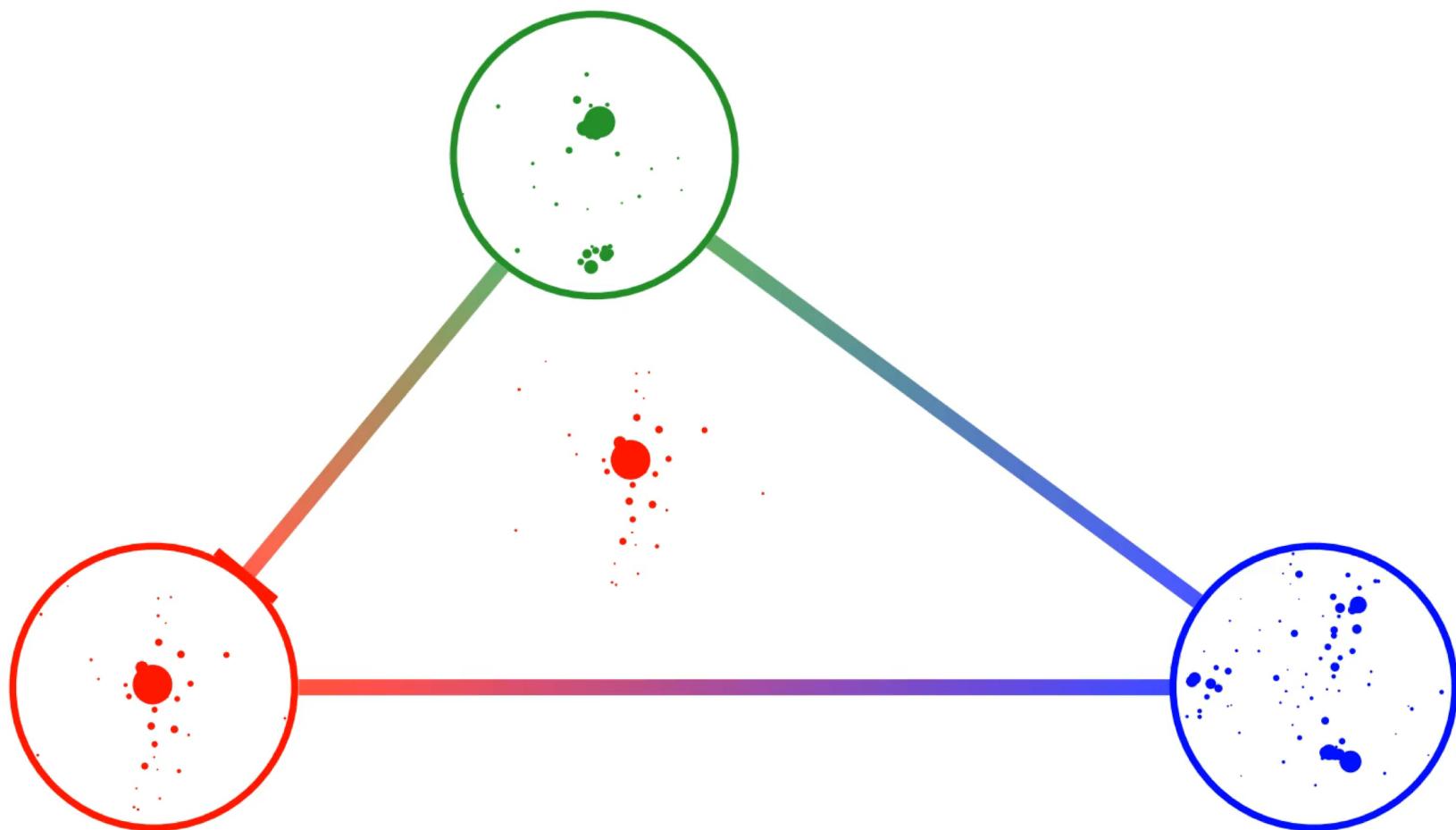


The End

Thank you!



Extra Slides



EnergyFlow

<https://energyflow.network>

pip install energyflow

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Welcome to EnergyFlow

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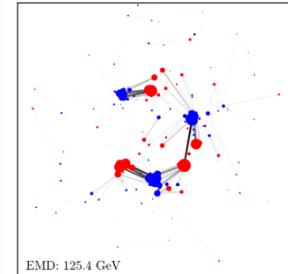
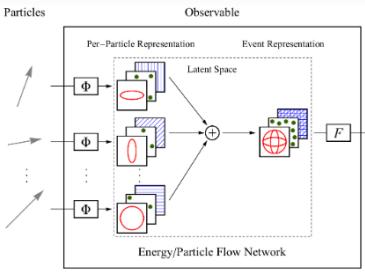
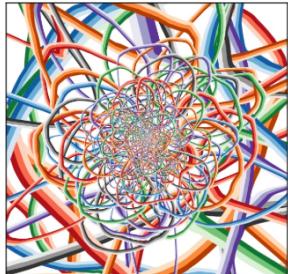
Getting Started

- Installation
- Demos
- Examples
- FAQs
- Release Notes

Documentation

- Energy Flow Polynomials
- Architectures
- EMD
- Measures
- Multigraph Generation
- Utils
- Datasets

Welcome to EnergyFlow



EnergyFlow is a Python package containing a suite of particle physics tools. Originally designed to compute Energy Flow Polynomials (EFPs), as of version 0.10.0 the package expanded to include implementations of Energy Flow Networks (EFNs) and Particle Flow Networks (PFNs). As of version 0.11.0, functions for facilitating the computation of the Energy Mover's Distance (EMD) on particle physics events are included. To summarize the main features:

- **Energy Flow Polynomials:** EFPs are a collection of jet substructure observables which form a complete linear basis of IRC-safe observables. EnergyFlow provides tools to compute EFPs on events for several energy and angular measures as well as custom measures.
- **Energy Flow Networks:** EFNs are infrared- and collinear-safe models designed for learning from collider events as unordered, variable-length sets of particles. EnergyFlow contains customizable Keras implementations of EFNs.
- **Particle Flow Networks:** PFNs are general models designed for learning from collider events as

CMS Open Data

An amazing resource for physics exploration and proof-of-principle studies.

opendata.cern.ch



opendata
CERN

About ▾

Explore more than **two petabytes**
of open data from particle physics!

jet primary dataset

search examples: [collision datasets](#), [keywords:education](#), [energy:7TeV](#)

Explore

[datasets](#)
[software](#)
[environments](#)
[documentation](#)

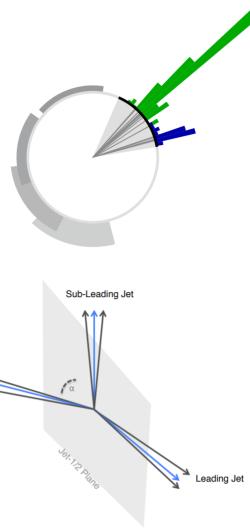
Focus on

[ATLAS](#)
[ALICE](#)
[CMS](#)
[LHCb](#)
[OPERA](#)
[Data Science](#)

▼ Get started ▼

CMS Open Data

Many exciting physics applications with the CMS Open Data already.



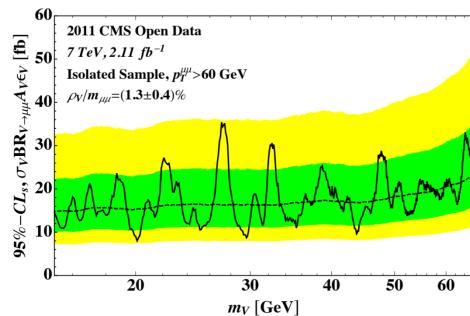
Exposing the QCD splitting function

[\[Tripathee, Xue, Larkoski, Marzani, Thaler, 1704.05842\]](#)

[\[Larkoski, Marzani, Thaler, Tripathee, Xue, 1704.05066\]](#)

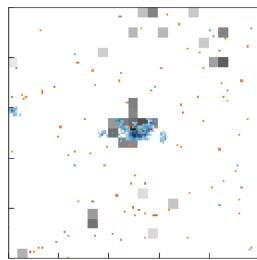
Looking for parity violation in jets

[\[Lester, Schott, 1904.11195\]](#)



Searching for dimuon resonances

[\[Cesarotti, Soreq, Strassler, Thaler, Xue, 1902.04222\]](#)



Analyzing collision data with deep learning techniques

[\[Madrazo, Cacha, Iglesias, de Lucas, 1708.07034\]](#)

[\[Andrews, Paulini, Gleyzer, Poczos, 1807.11916\]](#)

[\[Andrews, et al., 1902.08276\]](#)

Exploring the Space of Jets: Correlation Dimension

VOLUME 50, NUMBER 5

PHYSICAL REVIEW LETTERS

31 JANUARY 1983

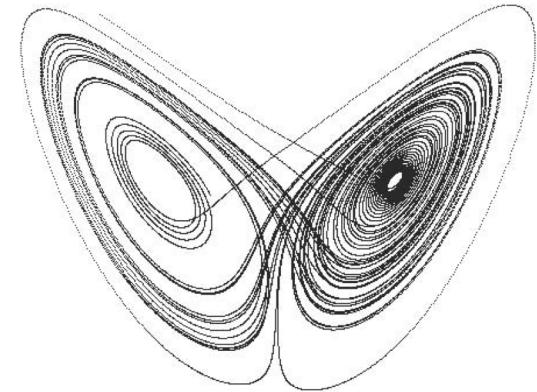
Characterization of Strange Attractors

Peter Grassberger^(a) and Itamar Procaccia

Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel

(Received 7 September 1982)

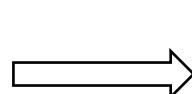
A new measure of strange attractors is introduced which offers a practical algorithm to determine their character from the time series of a single observable. The relation of this new measure to fractal dimension and information-theoretic entropy is discussed.



Intuition:

$$N_{\text{neighboring}}(r) \propto r^{\dim}$$

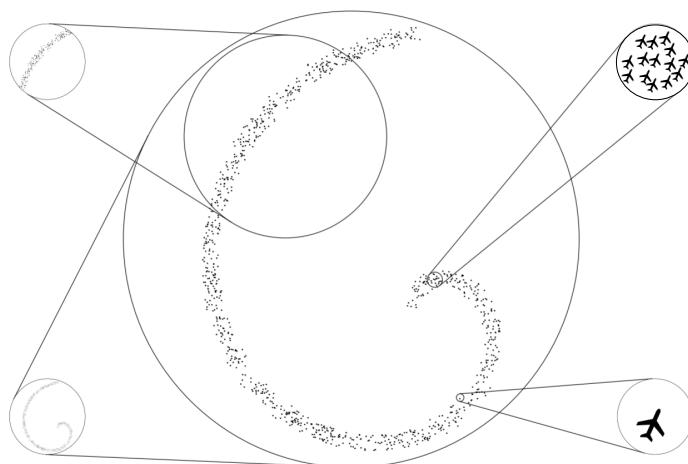
points



$$\dim(r) = r \frac{\partial}{\partial r} \ln N_{\text{neighbors}}(r)$$

$\dim \approx 1$

$\dim \approx 2$



$\dim \approx 2$

(eventually 0)

$\dim \approx 0$

Correlation dimension:

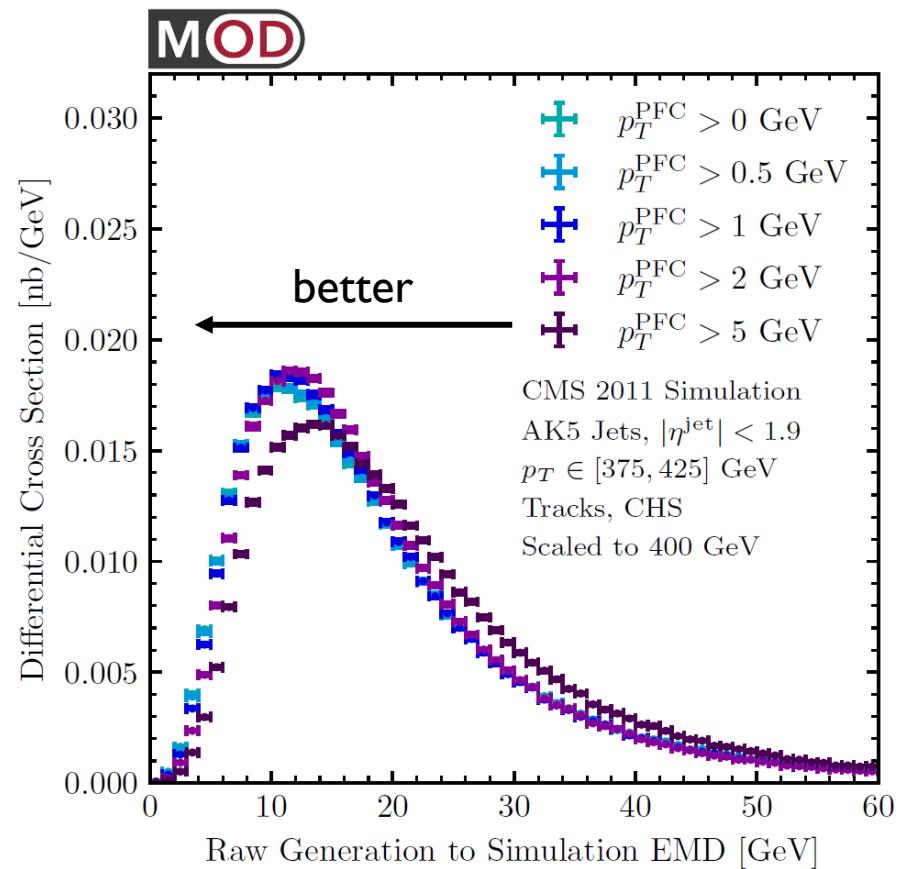
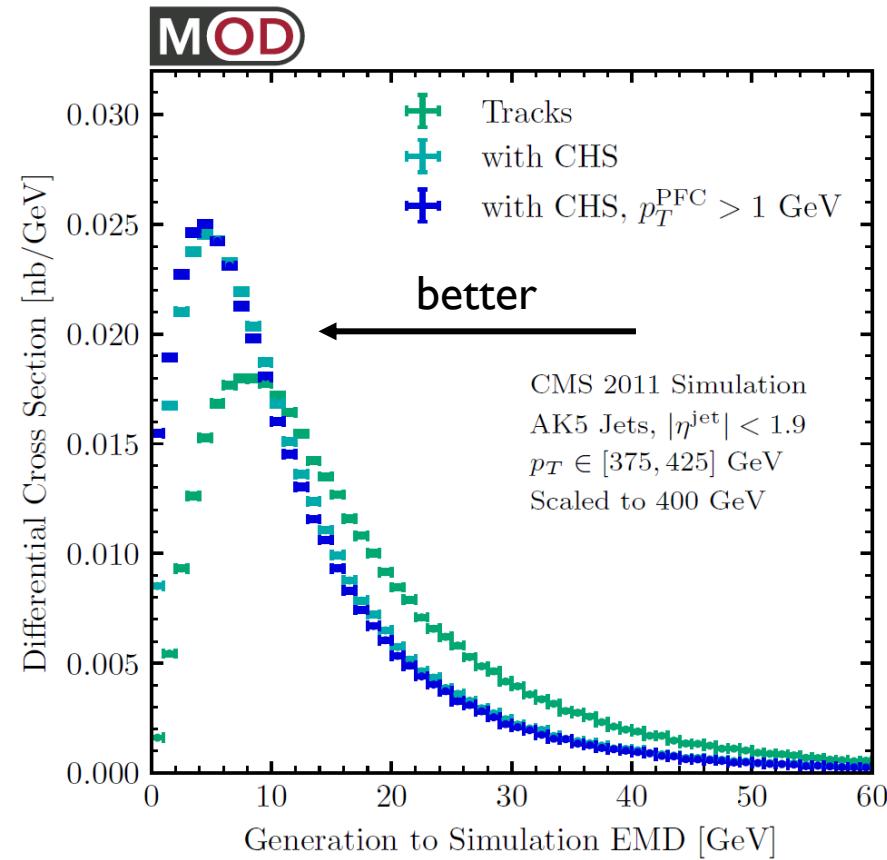
$$\dim(Q) = Q \frac{\partial}{\partial Q} \ln \sum_{i=1}^N \sum_{j=1}^N \Theta[\text{EMD}(\varepsilon_i, \varepsilon_j) < Q]$$

Energy scale Q
dependence

Count neighbors in
ball of radius Q

Quantifying Pileup and Detector Effects with EMD

Gen./Sim. EMD universally quantifies pileup mitigation and detector effects.

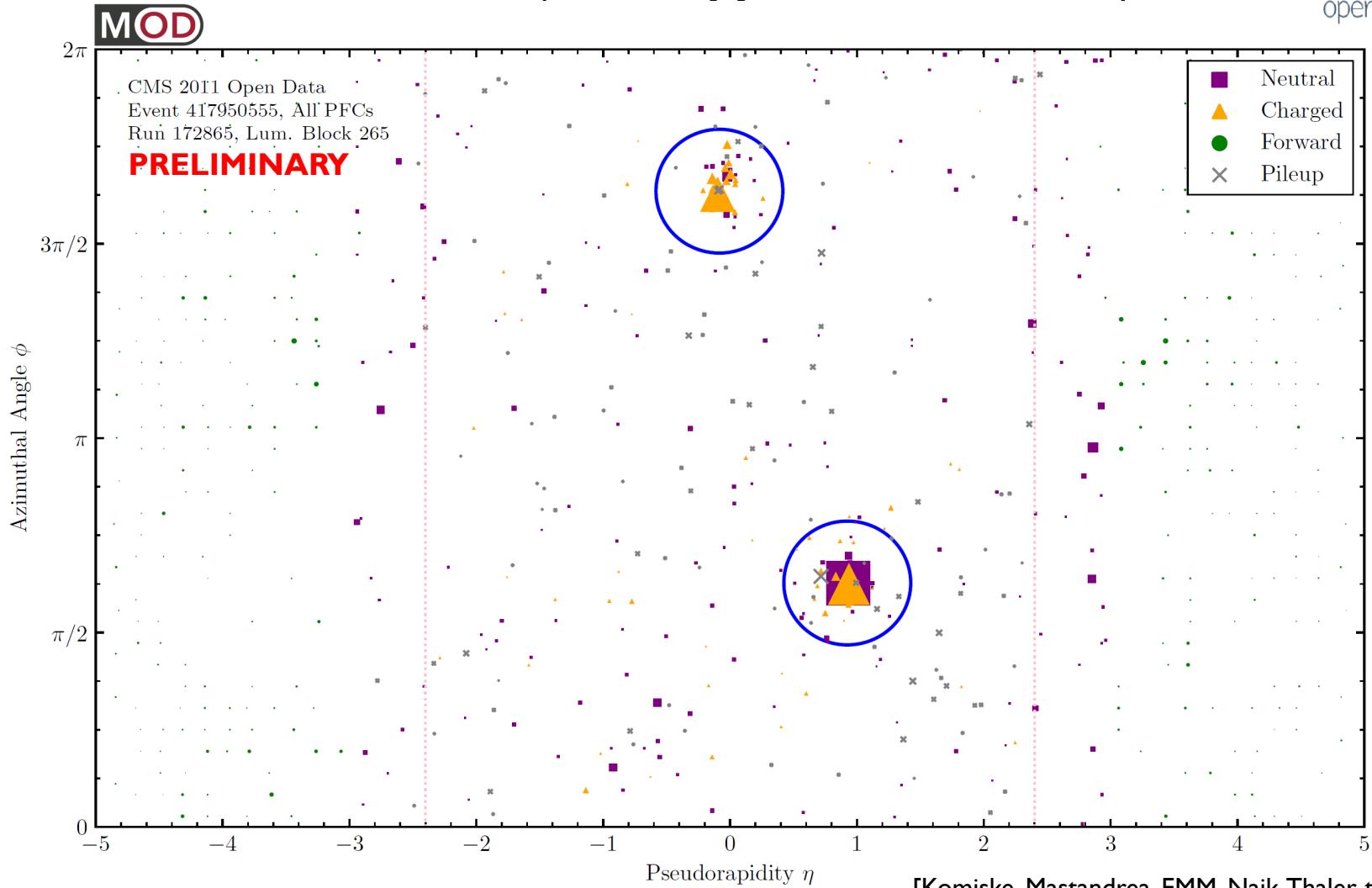




CMS 2011 A Jet Primary Dataset (+ Simulation)

2.3 fb^{-1} of 7 TeV proton-proton collision data. [\[link\]](#)

~1 million $R = 0.5$ jets with $p_T \in [375, 425]\text{GeV}$, $|\eta| < 1.9$



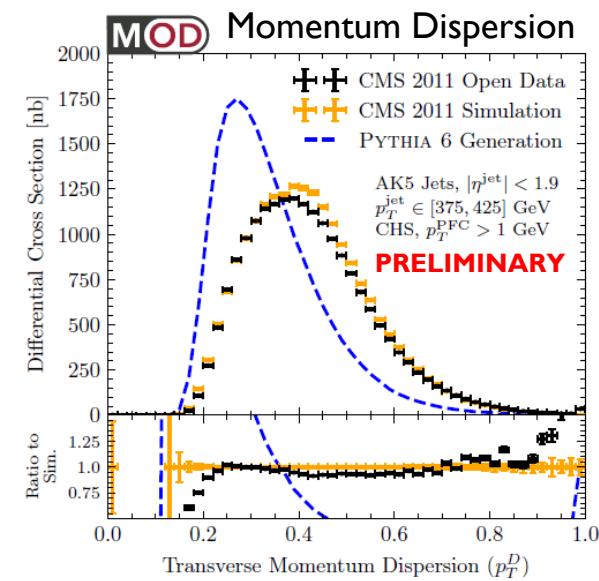
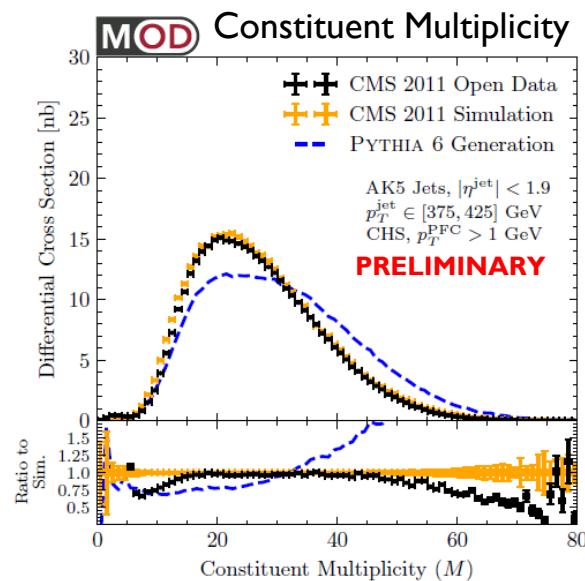
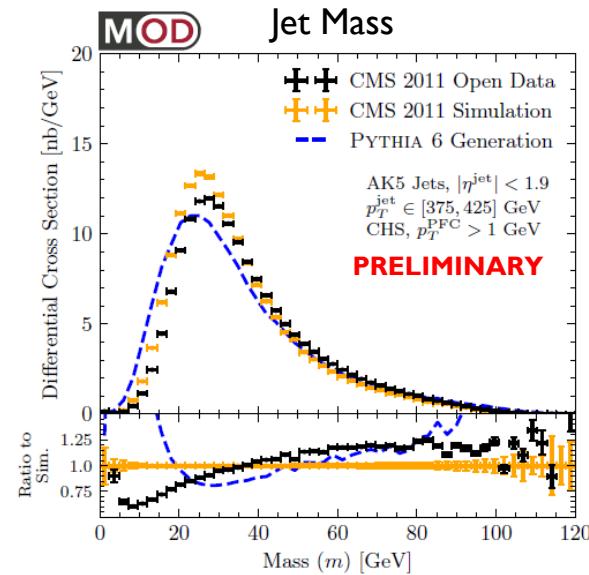
Jet Substructure Observables

Study jet substructure at truth and detector level.

$$m^2 = \left(\sum_{i \in \text{Jet}} p_i^\mu \right)^2$$

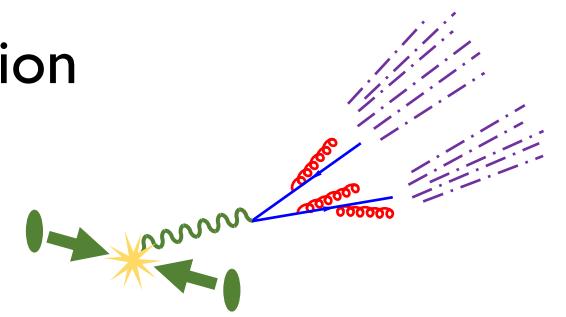
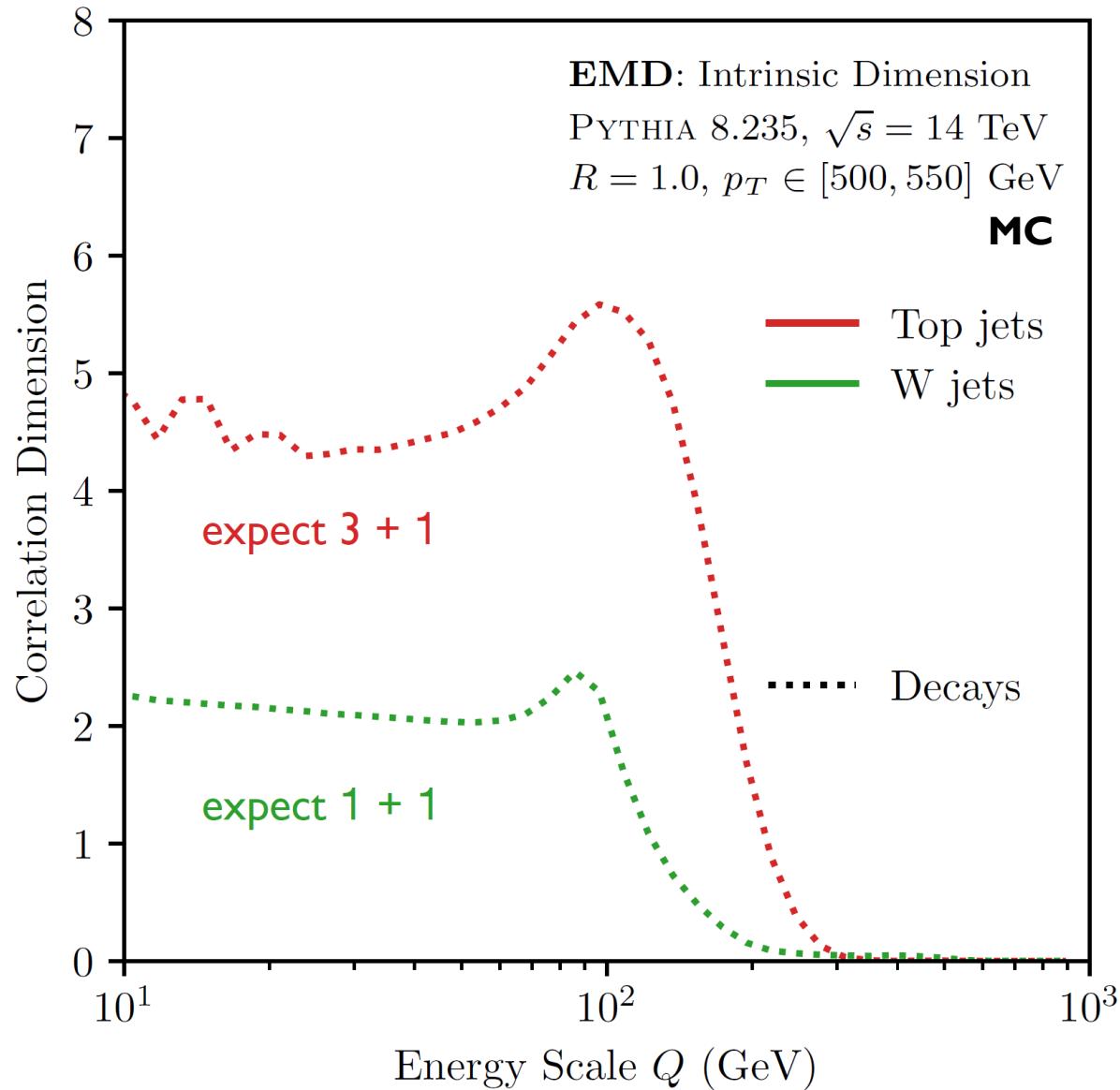
$$M = \sum_{i \in \text{Jet}} 1$$

$$p_T^D = \frac{\sum_{i \in \text{Jet}} p_{T,i}^2}{\left(\sum_{i \in \text{Jet}} p_{T,i} \right)^2}$$



Similar to: [\[Larkoski, Marzani, Thaler, Tripathee, Xue, 1704.05066\]](#)

Exploring the Space of Jets: Correlation Dimension



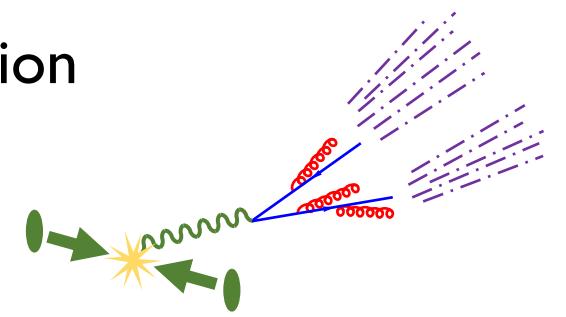
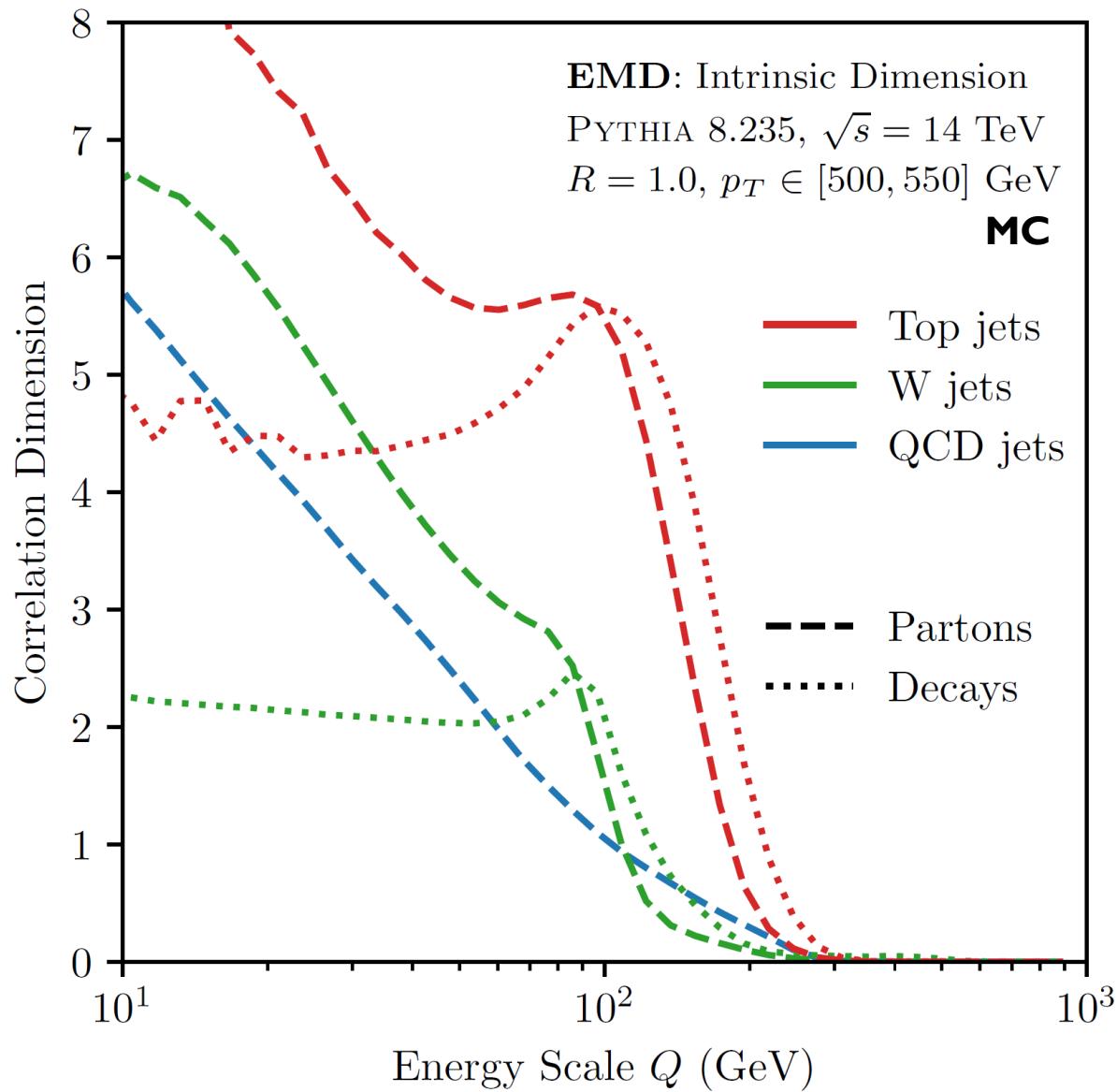
QCD jets are simplest.

W jets are more complicated.

Top jets are most complex.

“Decays” have \sim constant dimension.

Exploring the Space of Jets: Correlation Dimension



QCD jets are simplest.

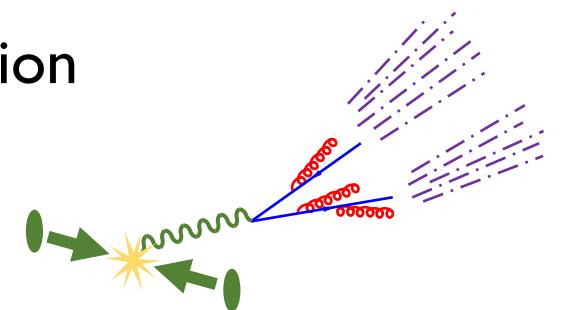
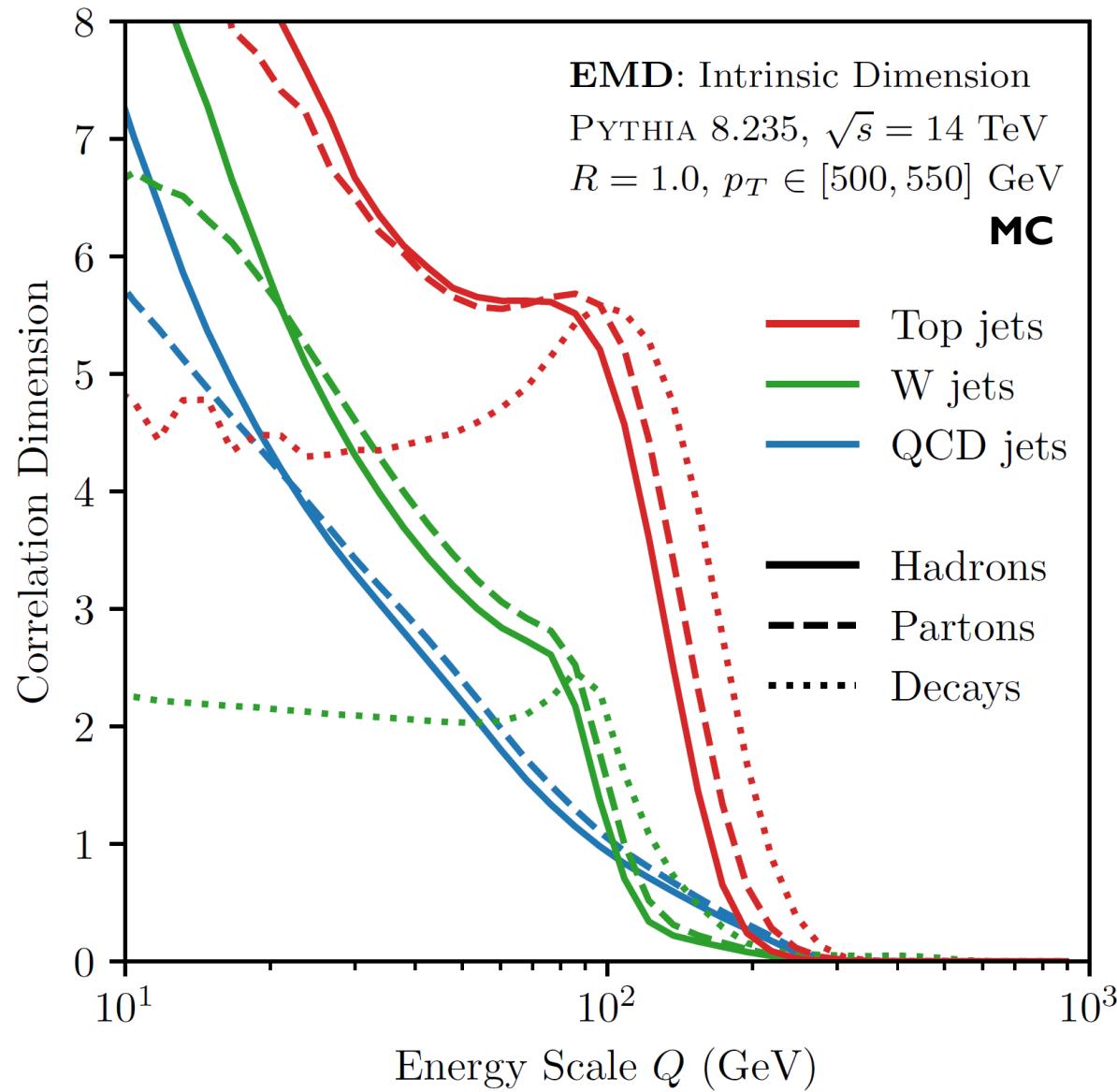
W jets are more complicated.

Top jets are most complex.

“Decays” have ~constant dimension.

Fragmentation becomes more complex at lower energy scales.

Exploring the Space of Jets: Correlation Dimension



QCD jets are simplest.

W jets are more complicated.

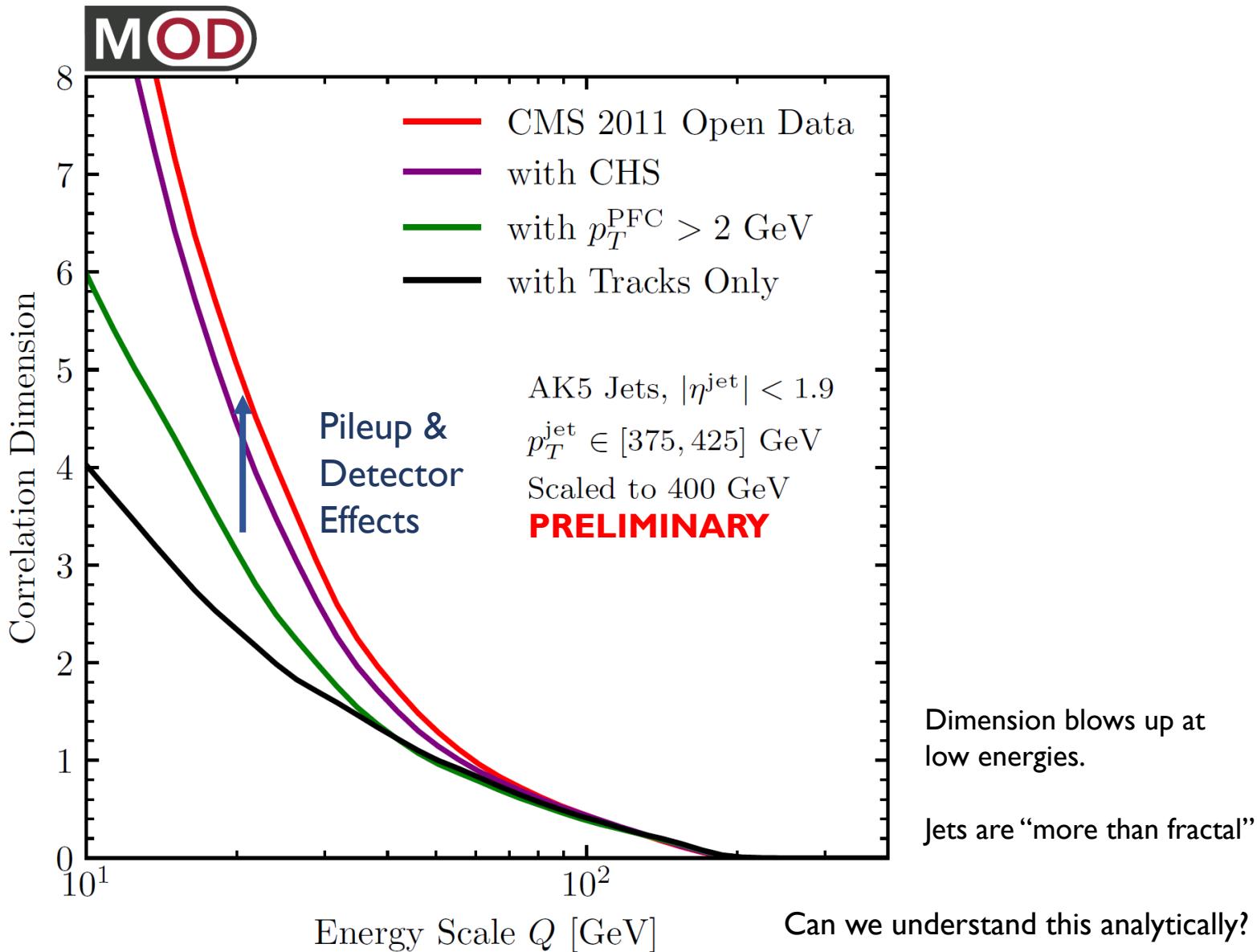
Top jets are most complex.

“Decays” have ~constant dimension.

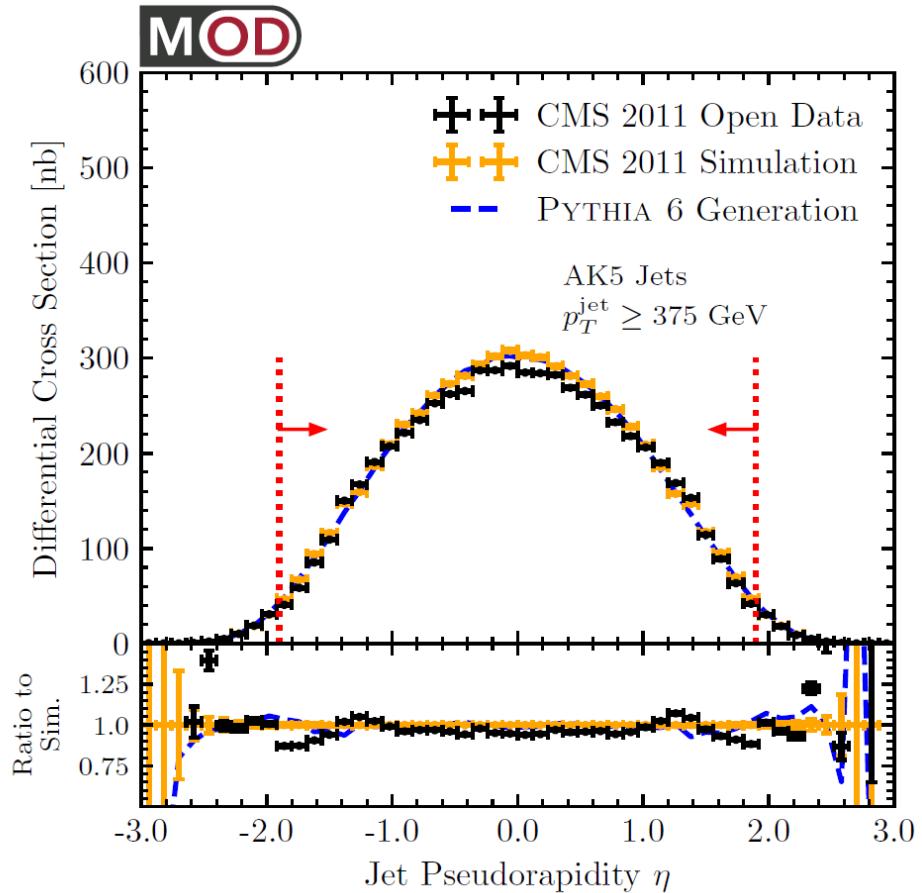
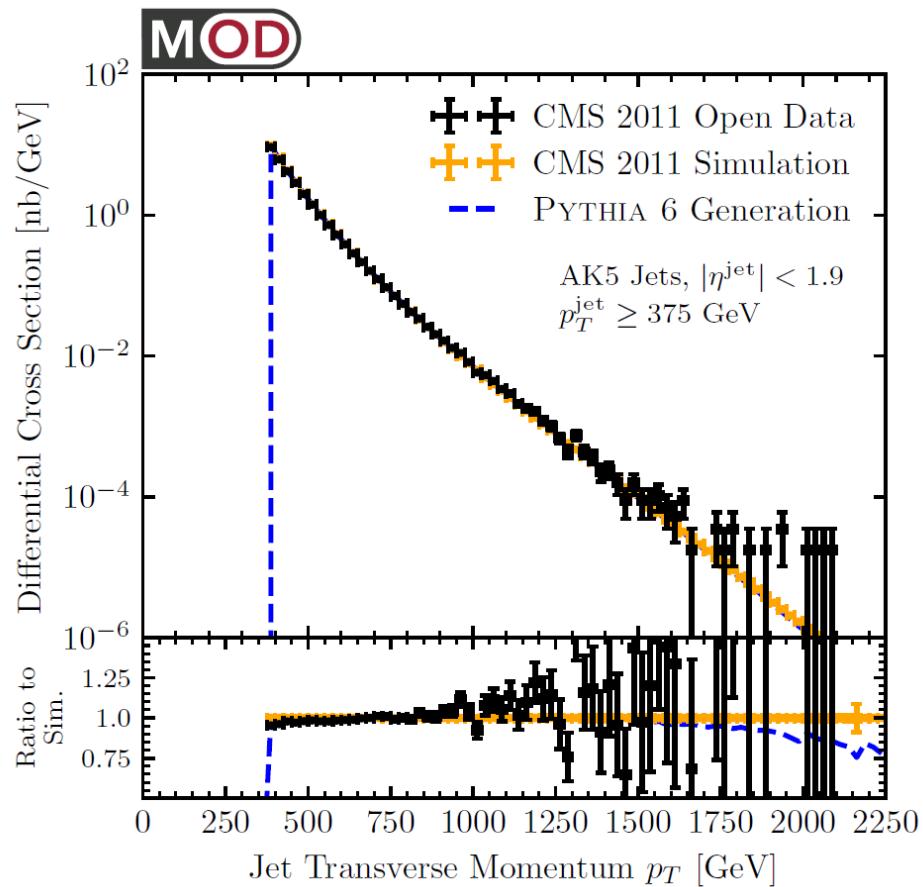
Fragmentation becomes more complex at lower energy scales.

Hadronization becomes relevant at scales around 20 GeV.

Exploring the Space of Jets: Correlation Dimension

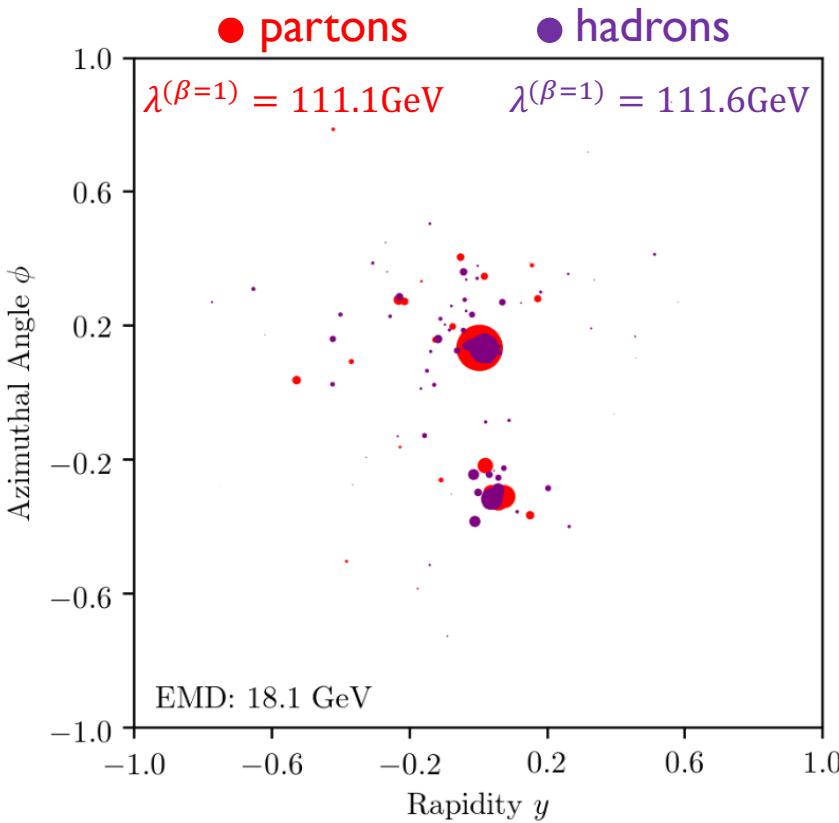


Jet Kinematic Distributions



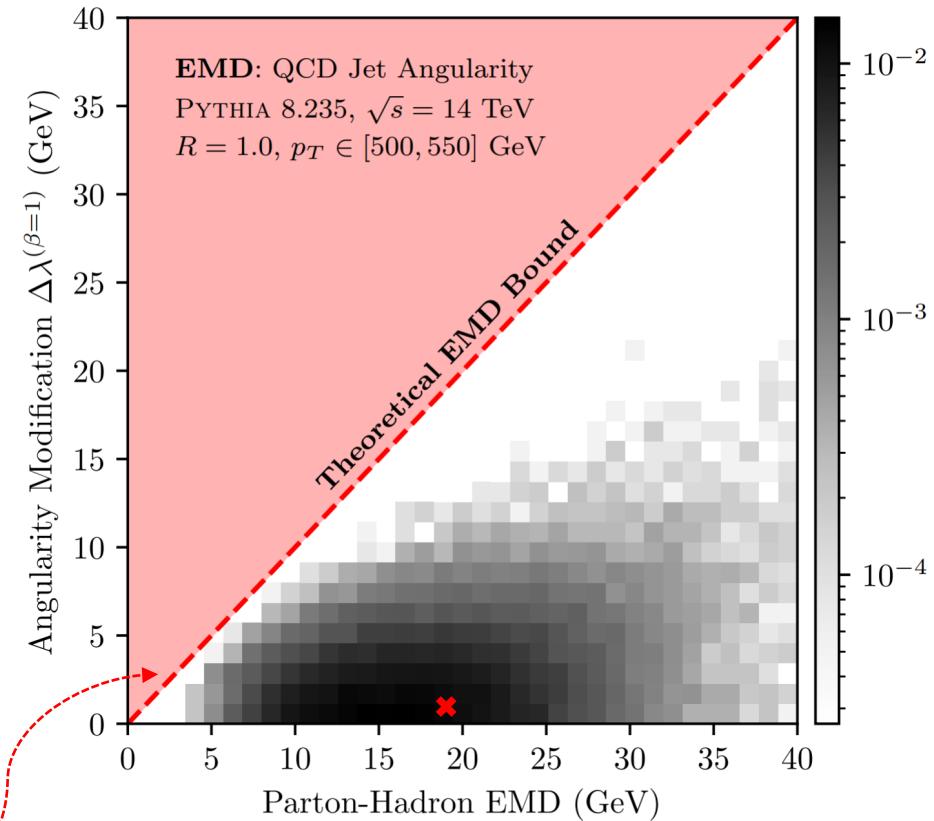
Quantifying event modifications: Hadronization

$$\lambda^{(\beta=1)} = \sum_{i=1}^M E_i \theta_i$$



$$\begin{aligned}\mathcal{E} &= \mathcal{E}_{\text{partons}} \\ \mathcal{E}' &= \mathcal{E}_{\text{hadrons}}\end{aligned}$$

$$|\lambda^{(\beta=1)}(\mathcal{E}) - \lambda^{(\beta=1)}(\mathcal{E}')| \leq \text{EMD}(\mathcal{E}, \mathcal{E}')$$



Exploring the Space of Events: Jet Classification

Classify W jets vs. QCD jets



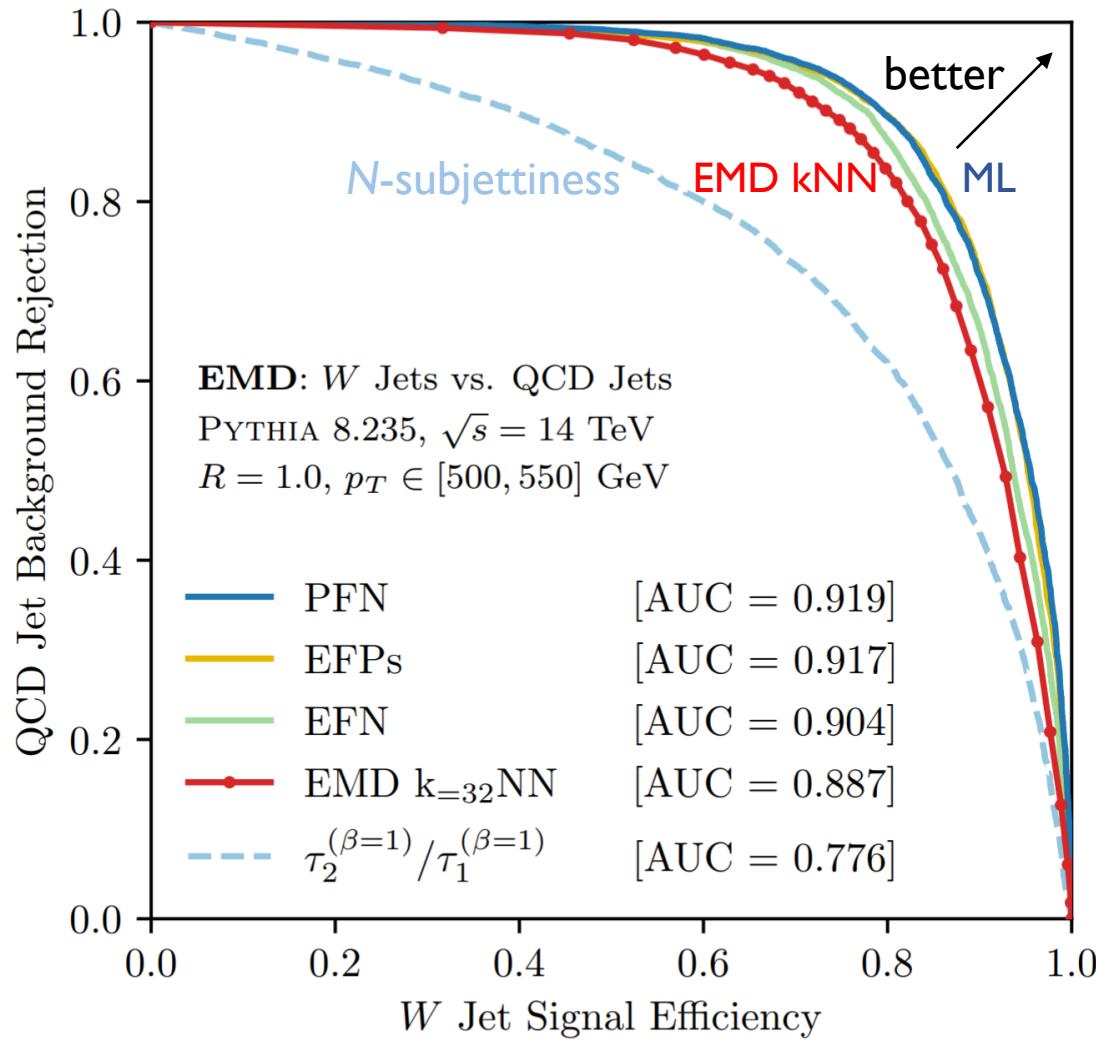
vs.



Look at a jet's nearest neighbors (kNN) to predict its class.

Optimal IRC-safe classifier with enough data.

Nearing performance of ML.



Exploring the Space of Events

Use EMD as a measure of event similarity

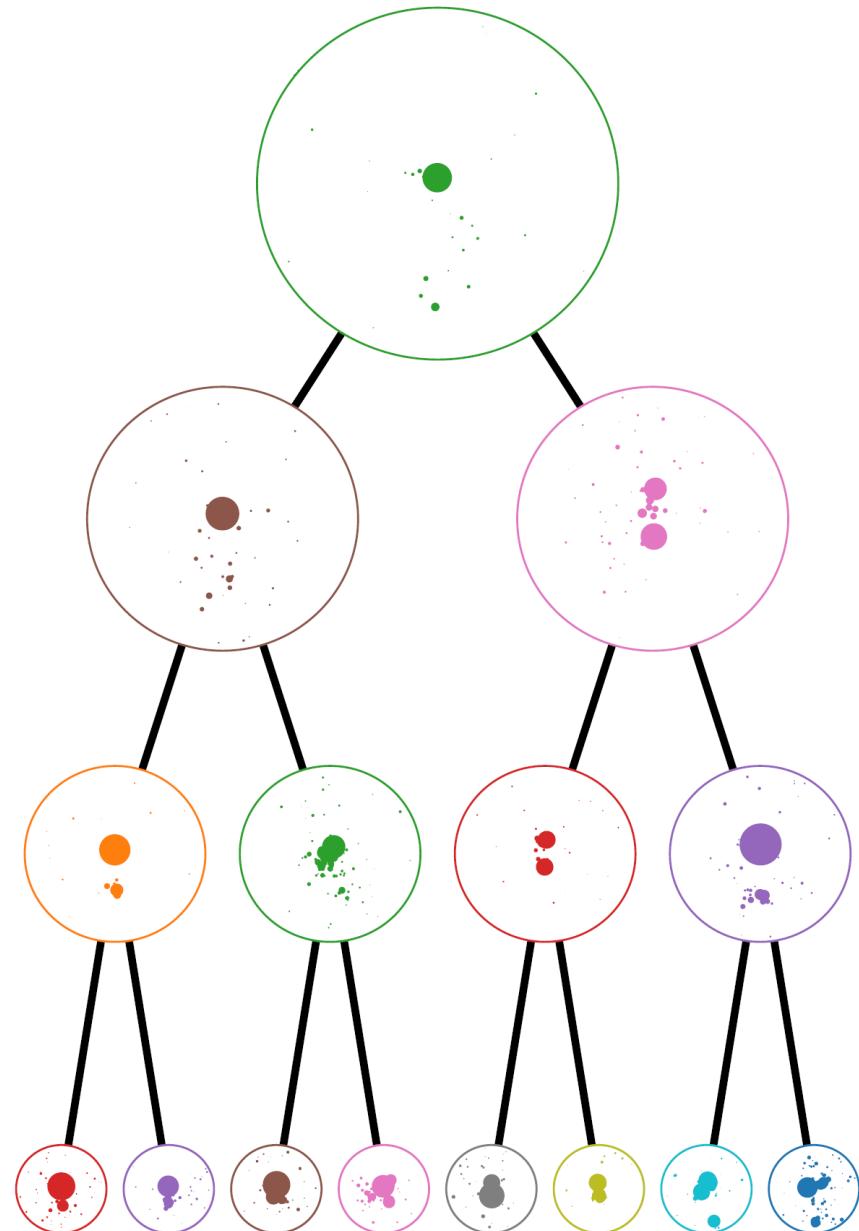
Unsupervised clustering algorithms can
be used to cluster events

Jets are clusters of particles
???? are clusters of jets

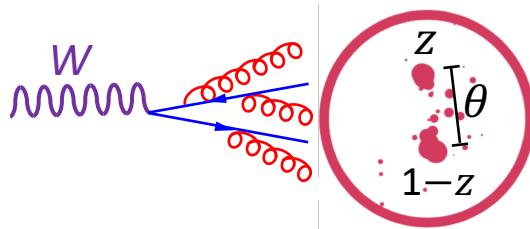
VP Tree: $O(\log(N))$ neighbor query time

Much more to explore.

Vantage Point (VP) Tree



Exploring the Space of Events: W jets



W jets are 2-pronged:

z : Energy Sharing of Prongs

θ : Angle between Prongs

φ : Azimuthal orientation

Constrained by W mass:

$$z(1 - z)\theta^2 = \frac{p_{\mu J}^2}{p_T^2} = \frac{m_W^2}{p_T^2}$$

Hence we expect a **two-dimensional** space of W jets.

After φ rotation: **one-dimensional**

t-SNE Manifold Dimension 2

“one pronged”

W jets, $R = 1.0$
 $p_T \in [500,510]\text{GeV}$

t-SNE Manifold Dimension 1

“top heavy”

“balanced”

“bottom heavy”

2x zoom

Exploring the Space of Jets: Correlation Dimension

Sketch of leading log (one emission) calculation:

$$\dim_{q/g}(Q) = Q \frac{\partial}{\partial Q} \ln \sum_{i=1}^N \sum_{j=1}^N \Theta[\text{EMD}(\varepsilon_i, \varepsilon_j) < Q]$$

$$= Q \frac{\partial}{\partial Q} \ln \Pr [\text{EMD} < Q]$$

$$= Q \frac{\partial}{\partial Q} \ln \Pr [\lambda^{(\beta=1)} < Q; C_{q/g} \rightarrow 2 C_{q/g}]$$

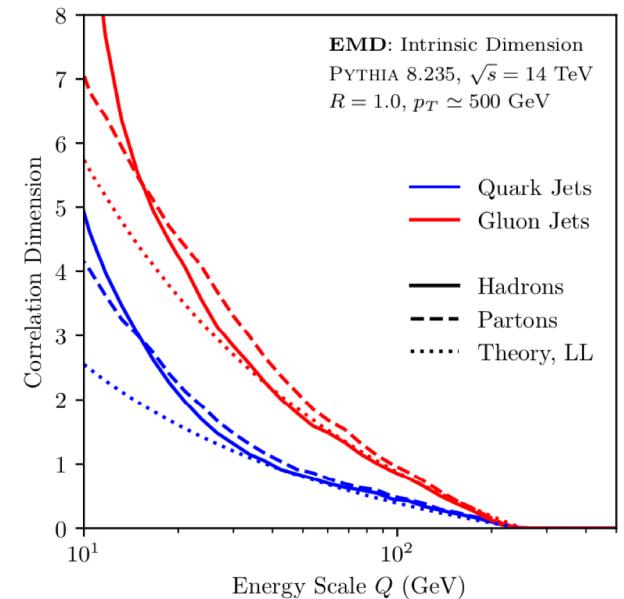
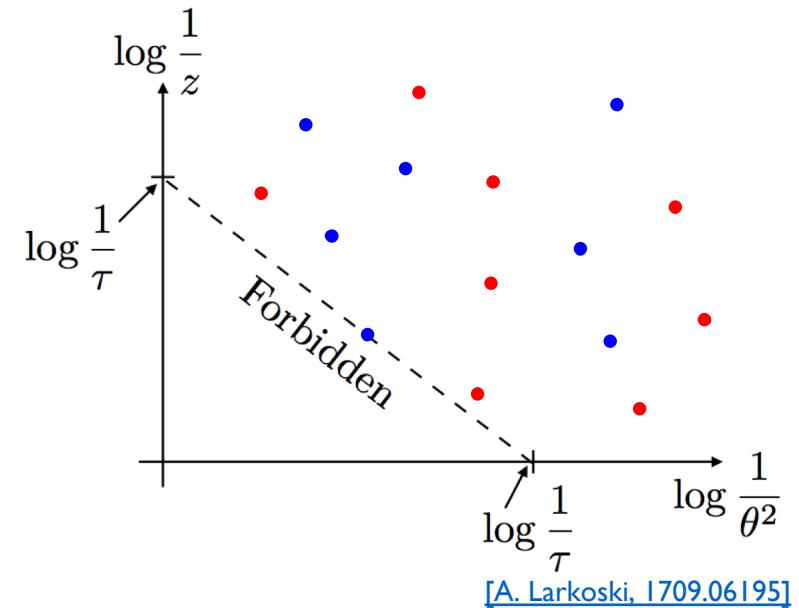
$$= Q \frac{\partial}{\partial Q} \ln \exp \left(- \frac{4\alpha_s C_{q/g}}{\pi} \ln^2 \frac{Q}{p_T/2} \right)$$

$$= - \frac{8\alpha_s C_{q/g}}{\pi} \ln \frac{Q}{p_T/2}$$

+ 1-loop running of α_s

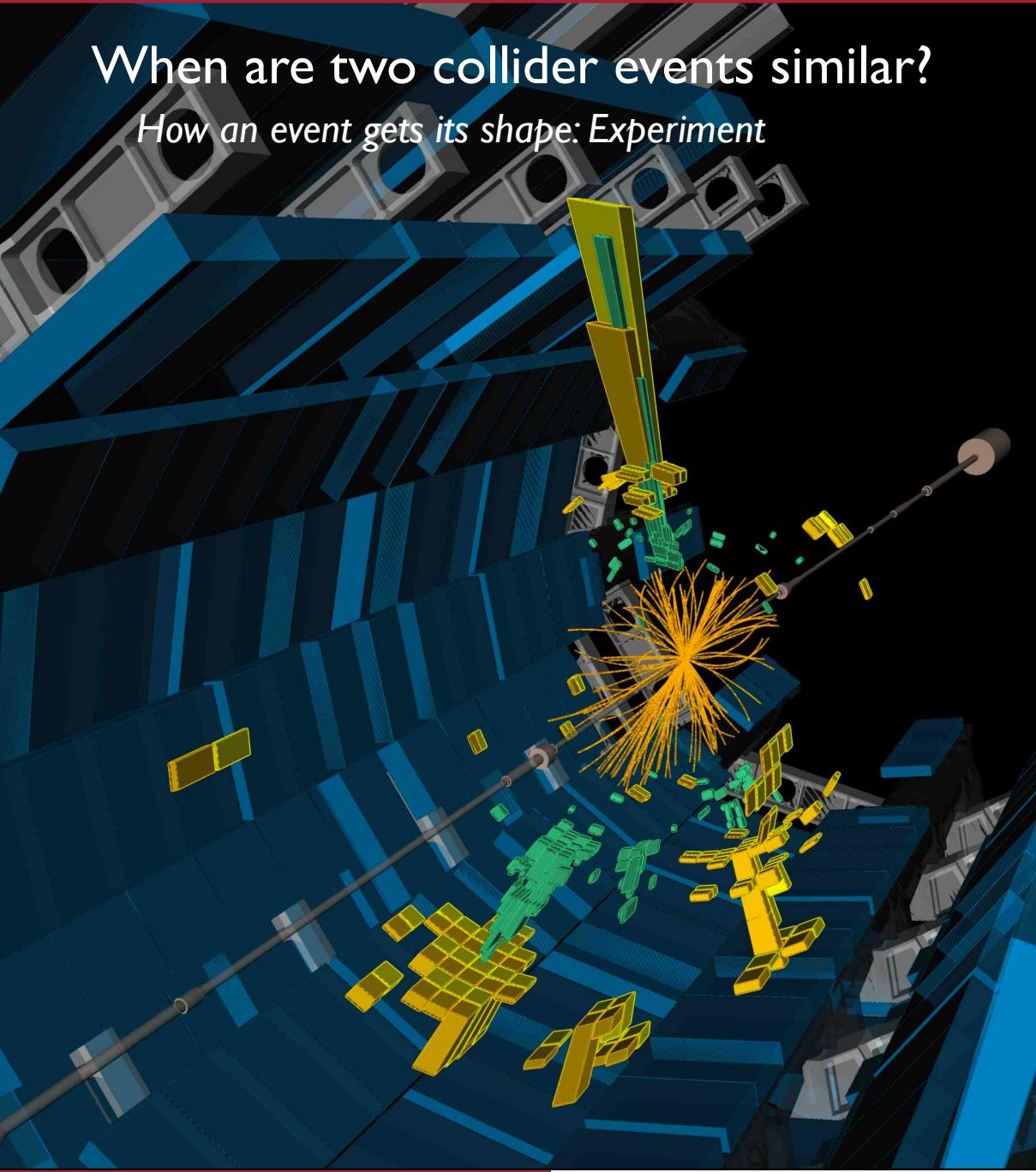
$$C_q = C_F = \frac{4}{3}$$

$$C_g = C_A = 3$$



When are two collider events similar?

How an event gets its shape: Experiment



tracker	ECAL	HCAL	
			γ photon
			e^\pm electron
			μ^\pm muon
			π^\pm pion
			K^\pm kaon
			K_L^0 K-long
			p/\bar{p} proton
			n/\bar{n} neutron

Pileup Mitigation with PUMML

