

For the following system of ODE's,

$$\begin{aligned}
c' &= -\frac{\mu}{y} \frac{cx}{k_1+c} - \frac{\eta}{y} \frac{cax}{k_2+c} + q(c_0 - c) \\
a' &= -\frac{\eta}{z} \frac{cax}{k_2+c} + q(a_0 - a) \\
x' &= \mu \frac{cx}{k_1+c} - \eta \frac{cax}{k_2+c} - qx \\
p' &= \mu \frac{cx}{k_1+c} + \eta \frac{cax}{k_2+c},
\end{aligned} \tag{1}$$

using the following parameter values,

Parameter	Value
$\mu$	2
$y$	1
$k_1$	1.5
$k_2$	1.5
$\eta$	20
$q$	0.3
$c_0$	1
$a_0$	1
$z$	1,

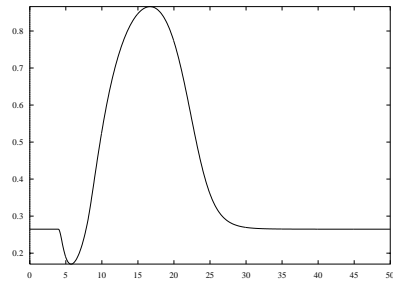
I graphed out how the system behaves when the antibiotics,  $a$ , are introduced. To simulate this effect, the equation for  $a'$  was changed to

$$\begin{aligned}
\alpha' &= -\frac{\eta}{z} \frac{cax}{k_2+c} + q(-a + a_0[h(t_{start}, t) - h(t_{end}, t)]), \\
\text{where } h(x, y) &= \frac{x^t}{y^n + x^n}, \\
t_{start} &= 4, \\
t_{end} &= 8, \\
\text{and } n &= 150.
\end{aligned} \tag{2}$$

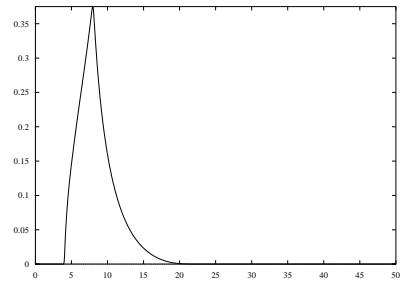
With this change we have that  $\alpha'(x) = a'(x), \forall x, 4 \leq x \leq 8$ . The graphs can be seen in Figure 1 and in Figure 2.

The effect of replacing  $\frac{cax}{k_2+c}$  with  $\frac{k_2cax}{(k_1+c)(k_2+c)}$  in (1) was again examined. As before, it was determined that there is no noticable effect from this change.

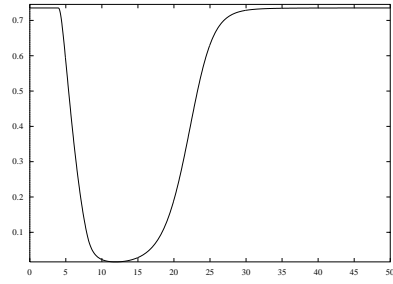
A plot of how  $p'$  changed with time can be seen in Figure 3.



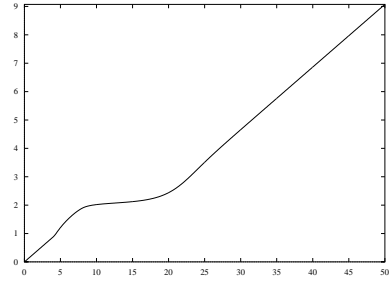
(a)



(b)



(c)



(d)

Figure 1: (a)  $c$  vs. time, (b)  $a$  vs. time, (c)  $x$  vs. time, and (d)  $p$  vs. time

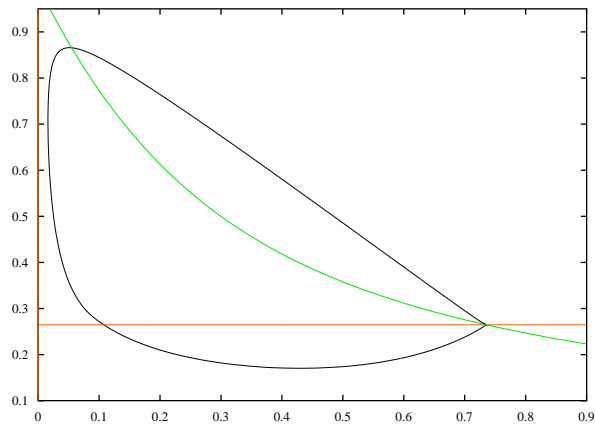


Figure 2: Phase portrait of  $x$  and  $c$

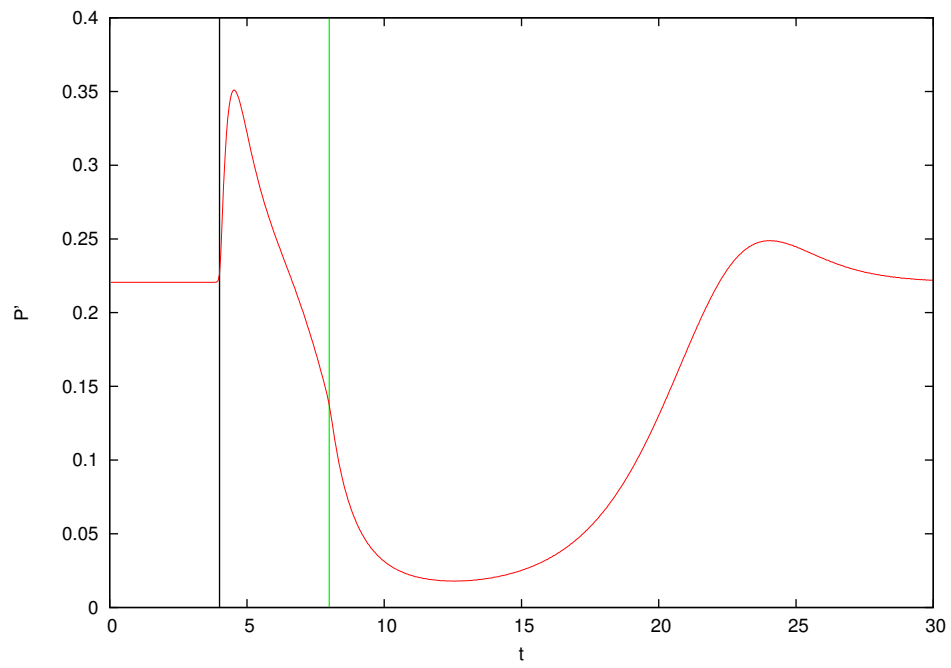


Figure 3: Plot of  $p'$  vs. time. The black line indicated when the antibiotics are introduced into the system, the green line shows the time they stopped.