Bayesian Phase Unwrapping with Factor Graphs

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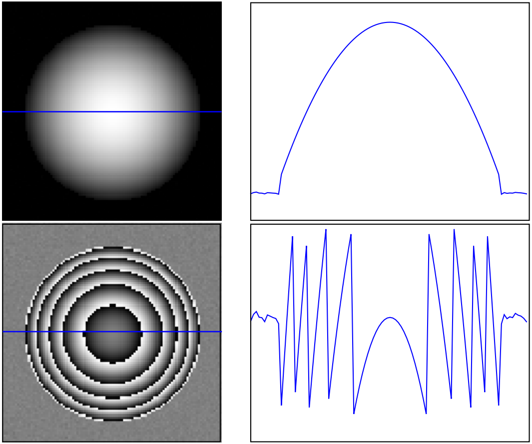
6.556 Final Project

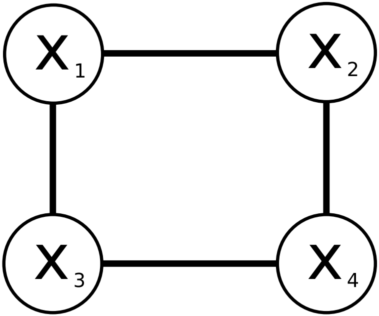
May 12, 2009

Phase Unwrapping for MR



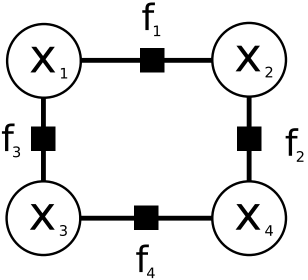
We know phase informaiton varies smoothly except at boundaries

Markov Random Fields



Markov random field, undirected graphical model, etc.

Factor Graphs



- Factor Graphs [3] express the same concepts as MRFs but make the **factors** explicit.

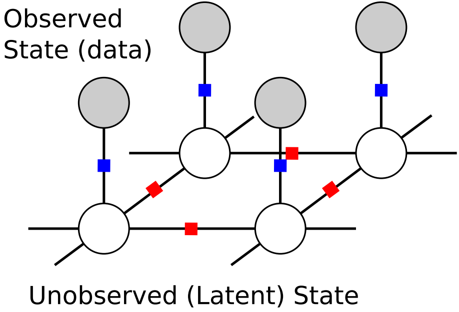
Factor Graph Probability

$$P(x_1, x_2, x_3, x_4) = f_1(x_1, x_2) \cdot f_2(x_2, x_3) \cdot f_3(x_3, x_4) \cdot f_4(x_4, x_1) \quad (1)$$

Ising Model: The original lattice MRF

Imagine you want to simulate a spin system Statistical physics people love doing this

Factor Graphs for Low-Level Vision



Properties of Image Factor Graphs [1]:

- Use observed state (data) to infer hidden state
- Have lattice structure like the ising
- Large number of vertices ($O(n)$ in number of pixels)
- $O(1)$ per-vertex connectivity
- Typically have homogeneous factors

Bayesian Factor Graphs

Bayes Rule

$$P(X|Y) = \frac{P(Y|X)P(X)}{\sum_X P(Y|X)P(X)}$$

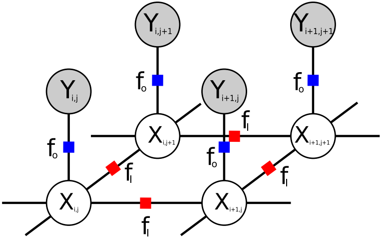
- $Y = y_{(i,j)}$: Observed Nodes
- $X = x_{(bi,j)}$: Hidden state we wish to estimate
- $P(Y|X)$: measurement model ("likelihood")
- $P(X)$: prior
-

Bayesian Factor Graphs For Low-Level vision

Our factor graph gives us P(X, Y)

MRFs for Phase: Frey's approach

In Ying and Frey's model [8] they formulate 2-D phase unwrapping as a low-level vision MRF problem.



- $Y_{(i,j)} \in \mathbb{R}$
- $x_{(i,j)} \in [0, 2\pi)$

Observation Potential

$$f_o = \delta((Y_{(i,j)} mod 2\pi) - X_{(i,j)})$$

Latent potential

$$f_l(X_1, X_2) = (X_1 - X_2)^2$$

discrete latent state, uniform factors

discrete latent state, unique factors

My formulation

Inference in MRFs

- The MRF tells us how to compute $P(X, Y)$. Bayes rule tells us how to compute $P(X|Y)$. But the sum is awful.
- But it's easy to compute $P^*(Y|X)$

Two generic approaches:

- draw samples from $p(x|D)$ to empirically estimate
- optimize to find MAP solution

Inference in MRFs

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We focus on sampling (Why?)

Markov-Chain Monte Carlo

Markov Property: next state only depends on current state

$$p(x_{t+1}|x_{1:t} = p(x_{t+1}|x_t)$$

Ergodic markov chains have stationary distributions
Set up a state space so that the asymptotic limit is the target distribution
Used in situations where you want to sample from $\pi(x)$ but only can compute $\pi^*(x)$

Metropolis Hastings

Consider a proposal distribution, $q(x \rightarrow x^*)$ We draw x^* as a *new target state* from this distribution. We compute the "Acceptance" ratio :

$$a = \min(1, \frac{p(x^*)}{p(x)} \cdot \frac{q(x \rightarrow x^*)}{q(x^* \rightarrow x)})$$

[5]

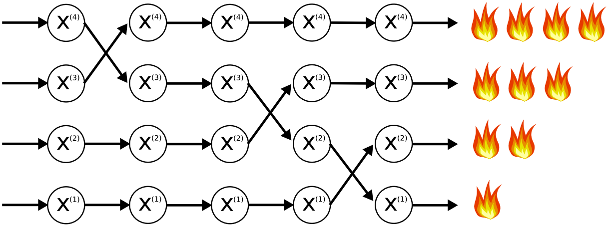
Gibbs Sampling

like MH but along an axis, useful when we can condition on other variables. Look, we can Gibbs sample in image MRFs with discrete state spaces [2]

Parallel Tempering

(aka "Replica Exchange Monte Carlo" [6], aka "Something to do with your 8 cores")

- Run N replicas of your chain, each at a different temperature
- Periodically propose MH-style swaps between adjacent chains
- Let's hot chains move around in flatter energy landscape



Swendsen-Wang

Work Through

Data-driven MCMC

- Any “move” is valid as long as it is reversable.
- We can cheat a little bit and construct moves based on the data to help the chains mix, without changing the target distribution

[7] Work Through

Partial Replica Exchange

[4]

MRFs and Parallelism

The conditional independence assumptions allow fine-grained parallelism

Our Implementation

use SW, etc. python, numpy, scipy, c++, boost, etc.
multithreaded C++

How to measure performance? I'm going to go for log-likelihood,

2-D Synthetic Data

3-D Synthetic Data

Div and Audrey

PRELUDE

Why, man, why?

Why go through all this effort for mediocre performance?

- Why be bayesian? - incorporate prior information explicitly - methods compose
- Why use MCMC? - MCMC “black boxes” that compose nicely [**?**]
- Why use factor graphs? - reasonable prior expressing local dependence - factors allow programmatic evaluation of conditional independencies, thus can easily identify opportunities for parallelism

Where to now?

Exact sampling using Systematic Stochsatic Search Better neighborhood connectivity / likelihood? GPU implementation Better visualization of posterior?

More information

Try it out

Source and presentation available for download on github

`https://github.com/ericmjonas/mrimrf/`

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