MTH 5051: Chapter 8 Homework

Due on October 4, 2019 at 11:59pm

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Chapter 8.1

Problem 5:

Determine the number of positive integers n, $1 \leq n \leq 2000$, that are

(a) Not divisible by 2, 3, or 5

(b) Not divisible by 2, 3, 5, or 7

(c) Not divisible by 2, 3, or 5, but are divisible by 7.

Solution:

(a) The way to do this is to subtract each divisble amount out of 2000, and add the values that 2,3, and 5 have in common as we double counted those. After that you have to subtract the values that all 3 share as we double counted those. By the laws of inclusion and exclusion you add all the even numbers and subtract all the odd number cases, so thats exactly what happnes. Because this is a problem of inclusion and exclusion we can denote 2 as c_1, 3 as c_2 and 5 as c_3 We can do this by:

$$N(c_1) = 2000/2 = 1000,\ N(c_2)2000/3 = 666,\ N(c_3)2000/5 = 400$$
$$N(c_1c_2) = 2000/(2*3) = 333,\ N(c_1c_3) = 2000/(2*5) = 200,\ N(c_2c_3) = 2000/(3*5) = 133$$
$$N(c_1c_2c_3)2000/(2*3*5) = 66$$
$$N(\bar{c_1}\bar{c_2}\bar{c_3}) = 2000 - [N(c_1) + N(c_2) + N(c_3)] + [N(c_1c_2) + N(c_1c_3) + N(c_2c_3)] - N(c_1c_2c_3) = \mathbf{534}$$

(b) We do essentially the same as we did before, except because there are 4 values instead of 3 there are many more shared values that we have to subtract. After this, we double subtracted the value that all 4 share, so we have to add those. We can now label 7 as c_4 This can be expressed in our equations:

$$N(c_1) = 2000/c_1 = 1000,\ N(c_2)2000/c_2 = 666,\ N(c_3)2000/c_3 = 400,\ N(c_4) = 2000/c_4 = 285$$
$$N(c_1c_2) = 2000/(c_1c_2) = 333,\ N(c_1c_3) = 2000/(c_1c_3) = 200,\ N(c_2c_3) = 2000/(c_2c_3) = 133,$$
$$N(c_1c_4) = 2000/(c_1c_4) = 142, N(c_2c_4)2000/(c_2c_4) = 95,\ N(c_3c_4) = 2000/(c_3c_4) = 57$$
$$N(c_1c_2c_3) = 2000/(c_1c_2c_3) = 66,\ N(c_1c_2c_4) = 2000/(c_1c_2c_4) = 47,\ N(c_2c_3c_4) = 2000/(c_2c_3c_4) = 19,$$
$$N(c_1c_3c_4) = 2000/(c_1c_3c_4) = 28$$
$$N(c_1c_2c_3c_4) = 2000/(2 \cdot 3 \cdot 5 \cdot 7) = 9$$
$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = 2000 - (1000 + 666 + 400 + 285) + (333 + 200 + 133 + 142 + 95 + 57)$$
$$- (66 + 47 + 19 + 28) + 9 = \mathbf{458}$$

(c) In order to easily do this you can get all the values from (a) and subtract them in (b). This would be:

$$534 - 458 = \mathbf{76}$$

Problem 7:

In how many ways can one arrange all of the letters in the word INFORMATION so that no pair of consecutive letters occurs more than once? [Here we want to count arrangements such as IINNOOFRMTA and FORTMAIINON but not INFORINMOTA (where "IN" occurs twice) or NORTFNOIAMI (where "NO" occurs twice).]

Solution:

First we have to find all the possible pairs that can occur more than once. The possible pairs are:

{IN, NI, NO, ON, IO, OI}

This is due to have 3 repeating letters {I, N, O}.

Let's now start by finding the total possible arrangements. The total possible arrangements is the 11 letters but making sure we don't count the repeat values. This would be counted by:

$$N = \frac{11!}{(2!)^3}$$

After this we need to find the criteria for single instances where IN, and alike letter combos afforementioned appear and where the possible combos of certain letters appear. We can describe the array mentioned above by letters c_1-c_6 in that order. We can get these equations by:

$$
\begin{aligned}
&N(c_1) = 9!/(2!)^2\\
&N(c_1) = N(c_2) = N(c_3) = N(c_4) = N(c_5) = N(c_6)\\
&N(c_1c_2) = 0 \text{ impossible to have pairs of IN and NI in one word}\\
&N(c_1c_2) = N(c_1c_4) = N(c_1c_5) = N(c_2c_3) = N(c_2c_6)\\
&\qquad\quad = N(c_3c_4) = N(c_3c_5) = N(c_4c_6) = N(c_5c_6)\\
&N(c_1c_3) = \frac{7!}{2!}\\
&N(c_1c_3) = N(c_1c_6) = N(c_2c_4) = N(c_2c_5) = N(c_3c_6) = N(c_4c_5)
\end{aligned}
$$

because 3, 4, 5, and 6 pairs do not exist in this case, these are all 0

$$S_0 = N = 4,989,600, \qquad S_1 = N(c_1)\cdot 6 = 544,320, \qquad S_2 = N(c_1c_3)\cdot 6 = 15,120$$
$$S_0 - S_1 + S_2 = 4,989,600 - 544,320 + 15,120 = \mathbf{4,460,400}$$

Problem 11:

At Flo's Flower Shop, Flo wants to arrange 15 different plants on five shelves for a window display. In how many ways can she arrange them so that each shelf has at least one, but no more than four, plants?

Solution:
Well, Initially, we have to understand that we have to place at least one of the plants on each window display, which gives us 10 "free plants" (free as in they can go to any other window display after that). However, we have to consider that there is a limit of 4 plants per display.

Problem 13:

Find the number of permutations of $a, b, c, ..., x, y, z$, in which none of the patterns *spin*, *game*, *path*, or *net* occurs.

Solution:

In this case, we can start by figuring out N which is pretty easy. It's just 26! for each letter in the alphabet. Now we have to calculate each instance of each work. Let's say *spin* is c_1, *game* is c_2, *path* is c_3, and *net* is c_4. c_1 is the same length as c_2 and c_3 are the same length they all are equal to each other. This can be explained by:

$$N(c_1) = (26 - 4 + 1)! = 23!$$
$$N(c_1) = N(c_2) = N(c_3)$$

Because c_4 is 3 letters it is instead:

$$N(c_4) = (26 - 3 + 1)! = 24!$$

Now, the only cases where 2 of the words can occur is with *spin* and *net* and *spin* and *game* However, *spinet* and some combo of *spin* and *game* use different amounts of letters. This would look like.

$$\begin{aligned}
&N(c_1c_2) = (26 - 8 + 2)! = 20! \\
&N(c_1c_4) = (26 - 6 + 1)! = 21! \\
&N(c_1c_3) = N(c_2c_3) = N(c_2c_4) = N(c_3c_4) = 0 \\
&N(c_1c_2c_3) = N(c_1c_2c_4) = N(c_1c_3c_4) = N(c_2c_3c_4) = 0 \\
&N(c_1c_2c_3c_4) = 0
\end{aligned}$$

When doing final calculations they look like:

$$26! - (3(23!) + 24!) + (20! + 21!)$$

Problem 16:

How many social security numbers (nine-digit sequences) have each of the digits 1,3, and 7 appearing at least once?

Solution:
a social security number consists of nine digits, so lets count every possible social security initially. This value would be:

$$10^9$$

Now by the laws of inclusion and exclusion we have to include the case scenarios where this value includes either the values of the 3 numbers 1,3,7. In order to do this we have to calculate which numbers have either, 1, 2, or 3 of these values. When we subtract the values we have to consider that we will not have 10 numbers available but 10 - num of digits appearing as we can only have 1,3, and 7 occur once. The calculation would look like:

$$10^9 - \binom{3}{1}9^9 + \binom{3}{2}8^9 - \binom{3}{3}7^9 = \mathbf{200,038,110}$$

Chapter 8.2

Problem 3:

In how many ways can one arrange the letters in CORRESPONDENTS so that

(a) There is no pair of consecutive identical letters?

(b) There are exactly two pairs of consecutive identical letters?

(c) There are at least three pairs of consecutive identical letters?

Solution:

(a) There are 4 letters that are repeated in the word CORRESPONDENTS. They are {O, R, E, N, S}. In order to find this we need to start with all possible combinations of the word. The word contains 14 letters with 5 repeats which would make it:

$$S_0 = N = \frac{14!}{(2!)^5} = 2,724,321,600$$

now we have to find the case scenario where each letter occurs twice, we will call them c_1-c_5 based on the array of letters mentioned above. From this we can get:

$$\begin{aligned}
&N(c_1) = \frac{13!}{(2!)^4}\\
&S_1 = \binom{5}{1}\frac{13!}{(2!)^4} = 1,945,944,000\\
&N(c_1c_2) = \frac{12!}{(2!)^3}\\
&S_2 = \binom{5}{2}\frac{12!}{(2!)^3} = 598,752,000\\
&N(c_1c_2c_3) = \frac{11!}{(2!)^2}\\
&S_3 = \binom{5}{3}\frac{11!}{(2!)^2} = 99,792,000\\
&N(c_1c_2c_3c_4) = \frac{10!}{2!}\\
&S_4 = \frac{10!}{2!}\binom{5}{4} = 9,072,000\\
&N(c_1c_2c_3c_4c_5) = 9!\\
&S_5 = 9!\binom{5}{5} = 362,880\\
&N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}\bar{c_5}) = S_0 - S_1 + S_2 - S_3 + S_5 - S_5 = \mathbf{1,286,046,720}
\end{aligned}$$

(b) If we want to find 2 pairs of consecutive letters we can use the values we found in part (a), except we will start at S_2 and work from there. We would result in an equation:

$$S_2 - S_3 + S_4 - S_5$$

Which will give you:

$$598,752,000 - 99,792,000 + 9,072,000 - 362,880 = \mathbf{507,669,120}$$

(c) This is very similar to the last problem, except what we instead have to do here is start at 3 consecutive letters. So the proper equation would look like:

$$S_3\binom{3}{1} - S_4\binom{4}{2} + S_5\binom{5}{3}$$

So this would be:

$$99,792,000 - 9,072,000 + 362,880 = \mathbf{91,082,880}$$

Problem 5:

In how many ways can one distribute ten distinct prizes among four students with exactly two students getting nothing? How many ways have at least two students getting nothing?

Solution:

We are essentially distributing 10 distinct prizes to 2 students, in the first part because exactly 2 students will get nothing. The second part of the prize means not only exactly 2, but also includes the chance where 3 of the students get nothing as well. To start with the first part of the problem, let's arbitrarily grab 2 students. Now we have to go through every possible way to distribute 10 prizes to 2 students. This would look like:

$$\left(\left(\binom{10}{1}+\binom{10}{2}+\binom{10}{3}+\binom{10}{4}\right)\times 2\right)+\binom{10}{5}$$
$$((10+45+120+210)*2)+252=1{,}022$$

We are able to multiply by two on the first part because of the symmetry between top and bottom values are similar (i.e $\binom{10}{4}=\binom{10}{6}, \binom{10}{3}=\binom{10}{7}$, etc.) after this we have to multiply this by $\binom{4}{2}$ in each way where some 2 students get the prizes. This becomes:

$$1{,}022\times\binom{4}{2}=\mathbf{6{,}132}$$

This answers the first question, now we have to move on to the second part, in which case at least 2 students receive nothing. We calculated every case where 2 students get nothing in the first part, now to answer completely where at least 2 gets nothing we have to include where 3 students get nothing. There are only 4 cases where 3 students get nothing (where student 1 gets all prizes, and same for students 2, 3, and 4). This would be:

$$6{,}132+4=\mathbf{6{,}136}$$

Problem 7:

If 13 cards are dealt from a standard deck of 52, what is the probability that these 13 cards include

(a) At least one card from each suit?

(b) Exactly one void (for example, no clubs)?

(c) Exactly two voids?

Solution:

(a) We can start by denoting each suit {clubs, diamonds, hearts, spades}as c_1 to c_4. If we use the laws of inclusion and exclusion we can say $N-\binom{4}{1}N(c_x)+\binom{4}{2}N(c_xc_y)-\binom{4}{3}N(c_xc_yc_z)$ You would multiply each value by the amount of suits you picked of the 4. The suit used is arbitrary, as they all have equal chances to be picked. These values would look like:

$$N = \binom{52}{13}$$
$$N(c_x) = \binom{39}{13},\ S_1 = \binom{39}{13}\binom{4}{1}$$
$$N(c_xc_y) = \binom{26}{13},\ S_2 = \binom{26}{13}\binom{4}{2}$$
$$N(c_xc_yc_z) = \binom{13}{13},\ S_3 = \binom{13}{13}\binom{4}{3}$$

So the final equation would look like:

$$N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4}) = \binom{52}{13} - \binom{4}{1}\binom{39}{13} + \binom{4}{2}\binom{26}{13} + \binom{4}{3}\binom{13}{13}$$
$$\boldsymbol{\frac{N(\bar{c_1}\bar{c_2}\bar{c_3}\bar{c_4})}{\binom{52}{13}}}$$

You have to divide the answer by $\binom{52}{13}$ in order to find the probability that you have that you have at least one card from each suit in the 13 you chose.

(b) We want to determine the E_1 value. We can use the values explained in the previous part. This can be determined as:

$$E_1 = S_1 - \binom{2}{1}S_2 + \binom{3}{2}S_3 - \binom{4}{3}$$
$$E_1 = \binom{39}{13}\binom{4}{1} - \binom{2}{1}\binom{26}{13}\binom{4}{2} + \binom{3}{2}\binom{13}{13}\binom{4}{3}$$

To get the probability you have to divide the E_1 value by the choice of all possible cards to pick of 13. This would be:

$$\boldsymbol{\frac{E_1}{\binom{52}{13}}}$$

(c) Two voids is similar in comparison to the last one, except we have to check the E_2 value instead of E_1. This would look like:

$$E_2 = S_2 - \binom{3}{1}S_3$$
$$E_2 = \binom{26}{13}\binom{4}{2} - \binom{3}{1}\binom{13}{13}\binom{4}{3}$$

We would get this value and divide it by $\binom{52}{13}$ so it would look like:

$$\boldsymbol{\frac{E_2}{\binom{52}{13}}}$$

Chapter 8.3

Problem 3:

How many derangements are there for 1, 2, 3, 4, 5?

Solution:

The number of derangements would look like:

$$d_5 = 5!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right] = \mathbf{44}$$

Problem 8:

Four applicants for a job are to be interviewed for 30 minutes each: 15 minutes with each of supervisors Nancy and Yolanda. (The interviews are in separate rooms, and interviewing starts at 9:00 a.m.)

(a) In how many ways can these interviews be scheduled during a one-hour period?

(b) One applicant, named Josephine, arrives at 9:00 A.M. What is the probability that she will have her two interviews one after the other?

(c) Regina, another applicant, arrives at 9:00 a.m. and hopes to be finished in time to leave by 9:50 a.m. for another appointment. What is the probability that Regina will be able to leave on time?

Solution:

(a) Since there are 4 applicants we have to find a derangement of 4 for the 4 people being interviewed. A derangement of 4 is:

$$d_4 = 4!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = 9$$

After this you need to multiply this by 4! in order to show the derangments multiplied by the amount of possible slots for the interviewed people. This would be:

$$\boldsymbol{d_4 \times 4!}$$

(b) To start initially I would like to see all the possibilities for Josephine. Josephine has the slots [1,2], [2,3], and [3,4] to be in back to back. This is 3 possibilites. More on this, she can also see either Nancy or Yolanda first, leaving 3 times multiplied by 2 possibilities. Now, after this, we have to consider the other applicants going while Josephine gets interviewed. During the process of Josephine getting interviewed only 2 other people can be interviewed in that time frame, and you would end up with a possibility of $\binom{3}{2}$ as you choose 2 applicants of the 3. After that there will be 2 possible ways for the last applicant to do the interview. We can also choose one person to interview while Josephine interviews, a $\binom{3}{1}$ problem, and this will leave 2 others after to go after, which can be arranged in 2 ways. which will result in the equation:

$$(3 \times 2) \times \left(\left(\binom{3}{2} \times 2\right) + \left(\binom{3}{1} \times 2 \times 2\right)\right) = 108$$

So the probability is:

$$\frac{\mathbf{108}}{\mathbf{d_4 \times 4!}}$$

(c) To start with this problem, This means that Regina can not have her interview in slot 4 of the 4 slots. This means that the possible interviews can occur such as: {[1,2], [1,3], [2,3]}. Now, because we have the same amount of arrangements as we do in (b) we can actually use the values from (b) for our answer. This means that our answer is the same as it was in (b).

$$\frac{\mathbf{108}}{\mathbf{d_4 \times 4!}}$$

Problem 12:

Ms. Pezzulo teaches geometry and then biology to a class of 12 advanced students in a classroom that has only 12 desks. In how many ways can she assign the students to these desks so that

(a) No student is seated at the same desk for both classes?

(b) There are exactly six students each of whom occupies the same desk for both classes?

Solution:

(a) We have to start by measuring out how many ways students can sit at the 12 seats. The amount of ways they can sit can be measured as is simply 12!. After this we have to find the amount of derangements. The derangement for 12 is:

$$d_{12} = 12!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} + \frac{1}{8!} - \frac{1}{9!} + \frac{1}{10!} - \frac{1}{11!} + \frac{1}{12!}\right]$$

In order to get the answer you have to multiply the total arrangements by the total derangements. This would be:

$$\mathbf{12!d_{12}}$$

(b) In this case we have to have 6 students occupy the same desks. In this case we have to have a derangement of 6 in order to change up our 6 students and we have to multiply by $\binom{12}{6}$ as we pick 6 students that are changing. We also still multiply all of this by 12! as the initial arrangement setup is still 12! amount of ways for 12 students to sit. This equation would look like:

$$\mathbf{12!d_6\binom{12}{6}}$$

Chapter 8.4 & 8.5

Problem 5:

(a) Find the rook polynomials for the shaded chessboards in Fig. 8.13.

(b) Generalize the chessboard (and rook polynomial) for Fig. 8.13(i)

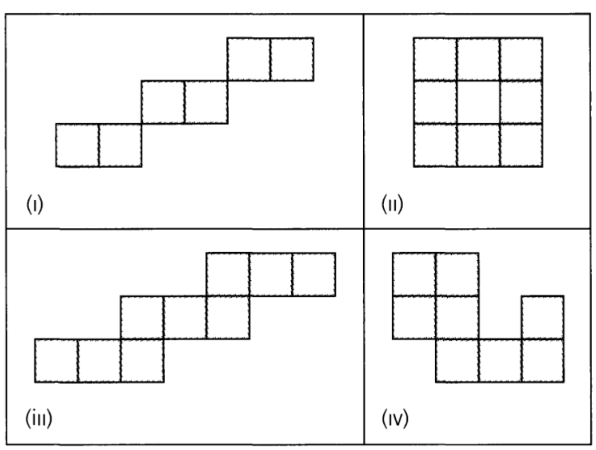

Figure 8.13

Solution:

(a) There are 4 different polynomials we have to look at, so lets look at those:

(i) If we do rook polynomials you'll find that for r_1 the possible positions (being numbered from boxes left to right) are the amount of boxes we have available to us, being 6. After this is r_2 which would be all the possible 2 rooks we could place in this board. This would the cardinality of the set: {[1,3], [1,4], [1,5], [1,6], [2,3], [2,4], [2,5], [2,6], [3,5], [3,6], [4,5], 4,6}. Lastly is all the possible 3 rooks we could place on the board in r_3. This would be the cardinality of the set: {[1,3,5], [1,3,6], [1,4,5], [1,4,6], [2,3,5], [2,3,6], [2,4,5], [2,4,6]} and would then result in the equation:

$$\mathbf{1 + 6x + 12x^2 + 8x^3}$$

(ii) Let's label each of the boxes in this chessboard from top right as 1 and follow right as 2,3. Underneath the first row would be 4,5,6, and underneath that would be 7,8,9. To start off r_1 is equal to the amount of blocks we have available to us which would be 9. Now for r_2, which would be the cardinality of the set: {[1,5], [1,6], [1,8], [1,9], [2,4], [2,6], [2,7], [2,9], [3,4], [3,5], [3,8], [3,9], [4,8], [4,9], [5,7], [5,9], [6,7], [6,9]} which would be 18. For r_3 it is the cardinality of the set {[1,5,9], [1,6,8], [2,4,9], [2,6,7], [3,5,7], [3,4,8]} which is 6. r_4 doesn't exist, so this then ends our rook polynomial, which leads to the equation:

$$\mathbf{1 + 9x + 18x^2 + 6x^3}$$

(iii) Lets number the third from the bottom left as 1,2,3, the row right above as 4,5,6, and the last row above that as 7,8,9. To start, the r_1 value is the amount of 9. The r_2, is is the cardinality of the set: {[1,4], [1,5], [1,6], [1,7], [1,8], [1,9], [2,4], [2,5], [2,6], [2,7], [2,8], [2,9], [3,5], [3,6], [3,7], [3,8], [3,9], [4,7], [4,8], [4,9], [5,7], [5,8], [5,9], [6,8], [6,9]}. Last is to find the cardinality of the set r_3 which is the items {[1,4,7], [1,4,8], [1,4,9], [1,5,7], [1,5,8], [1,5,9], [1,6,8], [1,6,9], [2,4,7], [2,4,8], [2,4,9], [2,5,7], [2,5,8], [2,5,9], [2,6,8], [2,6,9], [3,5,7], [3,5,8], [3,5,9], [3,6,8], [3,6,9]} which is 20 items. This would result in the equation:

$$\mathbf{1 + 9x + 25x^2 + 20x^3}$$

(iv) from left to right, starting at the top row, we will number the blocks. The top row is 1,2,the row below 3,4,5, and the last row below that is 6,7,8. To start, r_1 is 8. r_2 is the cardinality of the set: {[1,4], [1,5], [1,6], [1,7], [1,8], [2,3], [2,5], [2,7], [2,8], [3,6], [3,7], [3,8], [4,7], [4,8] [5,6], [5,7]} which is 15. The set r_3 is {[1,4,7], [1,4,8], [1,5,6], [1,5,7], [2,3,7] ,[2,3,8] [2,5,7], [2,5,8]}, and it has a cardinality of 8.

$$\mathbf{1 + 8x + 15x^2 + 8x^3}$$

(b) I think the easiest generalization for 8.13(i) is $\mathbf{(1 + 2x)^3}$

Problem 7:

Professor Ruth has five graders to correct programs in her courses in Java, C++, SQL, Perl, and VHDL. Graders Jeanne and Charles both dislike SQL, Sandra wants to avoid C++ and VHDL. Paul detests Java and C++, and Todd refuses to work in SQL and Perl. In how many ways can Professor Ruth assign each grader to correct programs in one language, cover all five languages, and keep everyone content?

Solution:

If we start by noting all 5 langauges {Java, C++, SQL, Perl, VHDL}, and think of this as the columns of our chessboard and the 5 people {Jeanne, Charles, Sandra, Paul, Todd} as the rows of our chessboard, we will get a board that looks like:

		X		
		X		
	X			X
X	X			
		X	X	

We can use the rook polynomials on this. Lets start with the shaded blocks at the top as 1, row below is 2, the row below, from left to right, is 3,4, the row below that is 5,6, and the row below that is 7,8. For the r_1 value it would be 8. The r_2 value would be: {[1,3], [1,4], [1,5], [1,6], [1,8], [2,3], [2,4], [2,5], [2,6], [2,8], [3,5], [3,7], [3,8], [4,5], [4,6], [4,7], [4,8], [5,7], [5,8], [6,7], [6,8]} and the cardinality is 21. For r_3 it is {[1,3,5], [1,3,7], [1,3,8], [1,4,5], [1,4,6], [1,4,7], [1,4,8], [2,3,5], [2,3,7], [2,3,8], [2,4,5], [2,4,6], [2,4,7], [2,4,8], [3,5,7], [3,5,8], [4,5,7], [4,5,8], [4,6,7], [4,6,8],} which has a cardinality of 20. Next is r_4 which has the set: {[1,3,5,8], [1,4,5,8], [1,4,6,8], [2,3,5,8], [2,4,5,8], [2,4,6,8]} which has a cardinality of 6. With This we can get the equation:

$$1 + 8x + 21x^2 + 20x^3 + 12x^4$$

Now, from here we have to understand that we have 5 languages that we are working with and need to use the rules of inclusion and exclusion in order to get a concrete answer. So we can replace x with the proper value that would be in its place (ex. N would be 5! N(c_1) would be 4!, etc, for the use of each language.). This would make the equation:

$$5! - (8 \times 4!) + (21 \times 3!) - (20 \times 2!) + 6 = \mathbf{20}$$

Problem 11:

A computer dating service wants to match each of four women with one of six men. According to the information these applicants provided when they joined the service, we can draw the following conclusions.

- Woman 1 would not be compatible with man 1, 3, or 6.
- Woman 2 would not be compatible with man 2 or 4.
- Woman 3 would not be compatible with man 3 or 6.
- Woman 4 would not be compatible with man 4 or 5.

In how many ways can the service successfully match each of the four women with a compatible partner?

Solution:

So immediately, I want to create a chessboard style table to describe this. The columns are the men ordered 1-6 and the rows are women ordered 1-4.

X		X			X
	X		X		
		X			X
			X	X	

from left to right, top to bottom, lets organize the blocks to numbers. The top rows blocks will be 1,2,3, the next row will be 4,5, the row after will be 6,7, the row after will be 8,9. The r_1 value is 9 to start off.

The r_2 value is the cardinality of set {[1,4], [1,5], [1,6], [1,7], [1,8], [1,9], [2,3], [2,4], [2,6], [2,7], [2,8], [3,4], [3,5], [3,6], [3,8], [3,9], [4,6], [4,7], [4,8], [4,9], [5,6], [5,7], [5,9], [6,8], [6,9], [7,8], [7,9]} which is 27. The r_3 set is {[1,4,6], [1,4,7], [1,4,8], [1,4,9], [1,5,6], [1,5,7], [1,5,9], [1,6,8], [1,6,9], [1,7,8], [1,7,9] [2,4,7], [2,4,8], [2,4,9], [2,5,7], [2,5,9], [2,7,8], [2,7,9], [3,4,6], [3,4,8], [3,4,9], [3,5,6], [3,5,9], [3,6,8], [3,6,9], [4,6,8], [4,6,9], [4,7,8], [4,7,9], [5,6,9], [5,7,9]} which has a massive cardinality of 31. After this is r_4 which is the cardinality of the set {[1,4,6,8], [1,4,6,9], [1,4,7,8], [1,4,7,9], [1,5,6,9], [1,5,7,9], [2,4,7,8], [2,4,7,9], [2,5,7,9], [3,4,6,8], [3,4,6,9], [3,5,6,9], [3,5,7,9]} which is 13. With all this I will get the equation:

$$1 + 9x + 27x^2 + 31x^3 + 13x^4$$

Now to get the amount of ways we have to follow a similar style to what we did in Problem 7, except slightly different because we have a differing number of horizontal rows and vertical rows. we have to then have $N = \frac{6!}{2!}$ and then $\frac{5!}{2!}$ as 2 is the difference between the col and row. count down the top factorial. This would look like:

$$\frac{6!}{2!} - \left(9 \times \frac{5!}{2!}\right) + \left(27 \times \frac{4!}{2!}\right) - \left(31 \times \frac{3!}{2!}\right) + \left(13 \times \frac{2!}{2!}\right) = \mathbf{64}$$