MTH 5051: Homework 6

Due on December 4, 2019 at 11:59pm

Dr. Jim Jones Section 01

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Chapter 12.1

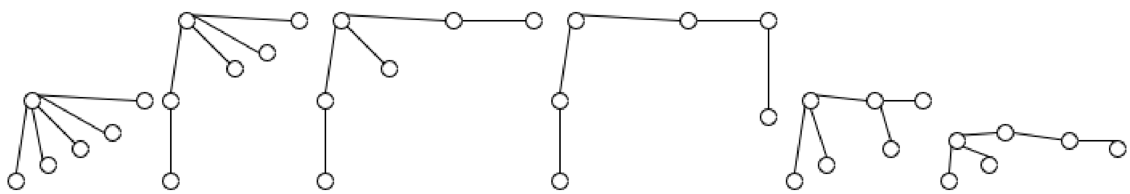
Problem 1:

(a) Draw the graphs of all nonisomorphic trees on six vertices

(b) How many Isomers does Hexane(C_6H_{14}) have.

Solution:

(a) The trees would look like:



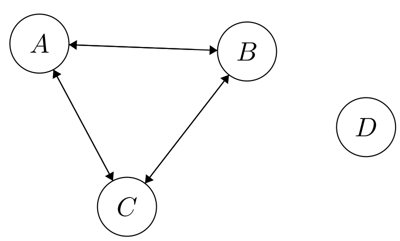
(b) There would be a total of 5 isomers.

Problem 7:

Give an example of an undirected graph $G = (V, E)$ where $|V| = |E| + 1$ but G is not a tree.

Solution:

The easy way to create an undirected graph that completes this is to create a small cycle, and a vertice on the outside of the connected graph. This would look like:



Problem 9:

If $G = (V, E)$ is a loop-free undirected graph, prove that G is a tree if there is a unique path between any two vertices of G.

Solution:

Chapter 12.2

Problem 5:

For the tree shown in Fig. 12.30, list the vertices according to a preorder traversal, an inorder traversal, and a postorder traversal.

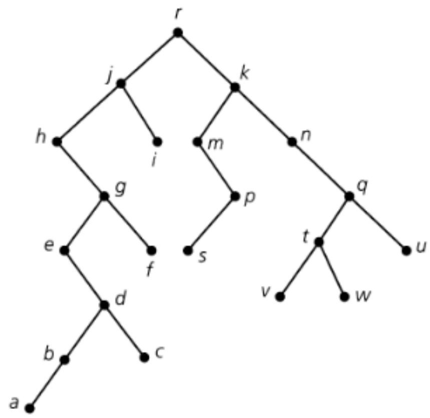


Figure 12.30

Solution:

The different traversals would look like this:

preorder:	r,j,h,g,e,d,b,a,c,f,i,k,m,p,s,n,q,t,v,w,u
inorder:	h,e,a,b,d,c,g,f,j,i,r,m,s,p,k,n,v,t,w,q,u
postorder:	a,b,c,d,e,f,g,h,i,j,s,p,m,v,w,t,u,q,n,k,r

Problem 7a:

Find the depth-first spanning tree for the graph shown in Fig. 11.72(a) if the order of the vertices is given as

(i) a,b,c,d,e,f,g,h;

(ii) h,g,f,e,d,c,b,a;

(iii) a,b,c,d,h,g,f,e;

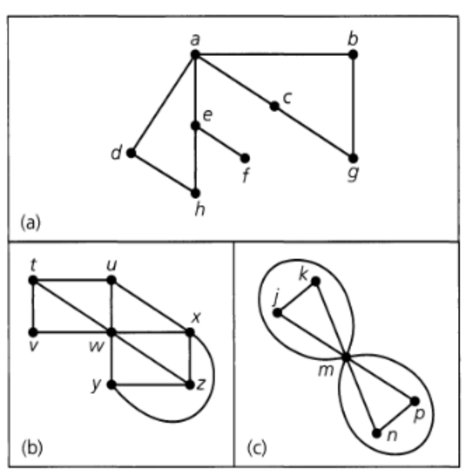
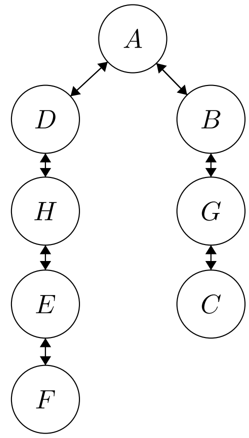


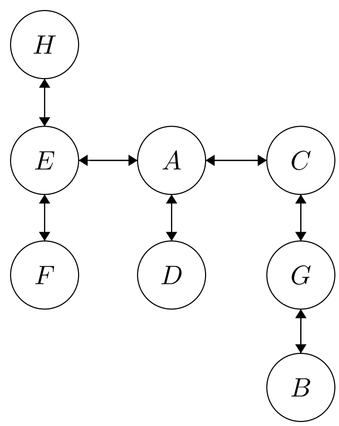
Figure 11.72

Solution:

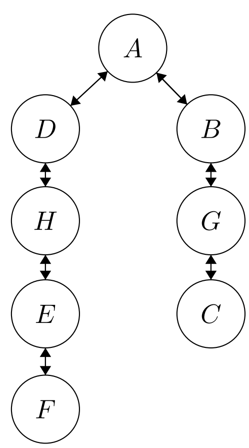
(i) For this patter, if you traverse (a) it would end up looking like:



(ii) if you instead go backwards it would end up looking like:



(iii) This actually ends being exactly the same as (i) spanning tree and would look like:



Problem 11:

Prove Theorom 12.6 and Corollary 12.1.

Theorom 12.6: Let $T = (V, E)$ be a complete m-ary tree with $|V| = n$. If T has ℓ leaves and i internal vertices, then

(a) $n = mi + 1$;

(b) $\ell = (m - 1)i + 1$;

(c) and $i = \dfrac{\ell - 1}{m - 1} = \dfrac{n - 1}{m}$

Corollary 12.1: Let T be a balanced complete m-ary tree with ℓ leaves. Then the height of T is $[\log_m \ell]$.

Solution:

Chapter 12.3

Problem 1:

(a) Give an example of two lists L_1, L_2, each of which is in ascending order and contains five elements, and where nine comparisons are needed to merge L_1, L_2 by the algorithm given in Lemma 12.1.

(b) Let $m, n \in \mathbb{Z}^+$ with $m < n$. Give an example of two lists L_1, L_2, each of which is in ascending order, where L_1 has m elements, L_2 has n elements, and $m + n|1$ comparisons are needed to merge L_1, L_2 by the algorithm given in Lemma 12.1.

Solution:

(a)

(b)

Problem 2:

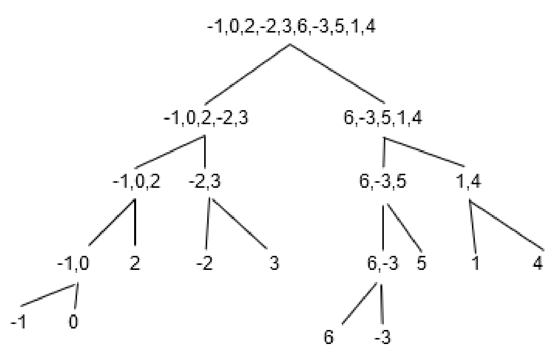
Apply the merge sort to each of the following lists. Draw the splitting and merging trees for each application of the procedure.

(a) -1, 0, 2, -2, 3, 6, -3, 5, 1,4

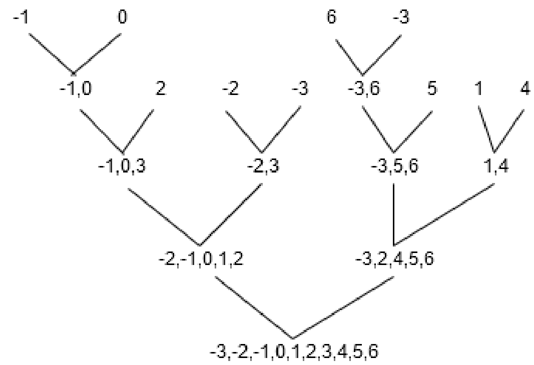
(b) -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3

Solution:

(a) Initially we need to start dividing each into individual pieces, so we are going to completely separate the the list. This initial separation would look like:



Now, it is separated so at this point we would perform the merge where the actual magic happens, this would look like:



(b)

Problem 3:

Related to the merge sort is a somewhat more efficient procedure called the quick sort. Here we start with a list $L : a_1, a_2, ..., a_n$, and use a_1 as a pivot to develop two sublists L_1 and L_2 as follows. For $i > 1$, if $a_i < a_1$, place a_i at the end of the first list being developed (this is L_1 at the end of the process); otherwise, place a, at the end of the second list L_2.

After all a_1, $i > 1$, have been processed, place a_1 at the end of the first list. Now apply quick sort recursively to each of the lists L_1 and L_2 to obtain sublists L_{11}, L_{12}, L_{21}, and L_{22}. Continue the process until each of the resulting sublists contains one element. The sublists are then ordered, and their concatenation gives the ordering sought for the original list L.

Apply quick sort to each list in Exercise 2.

Solution:

(a)

(b)

Chapter 12.4

Problem 3:

Construct an optimal prefix code for the symbols $a, b, c, ..., i, j$ that occur (in a given sample) with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3.

Solution:

Chapter 12.5

Problem 1:

Find the articulation points and biconnected components for the graph shown in Fig. 12.44

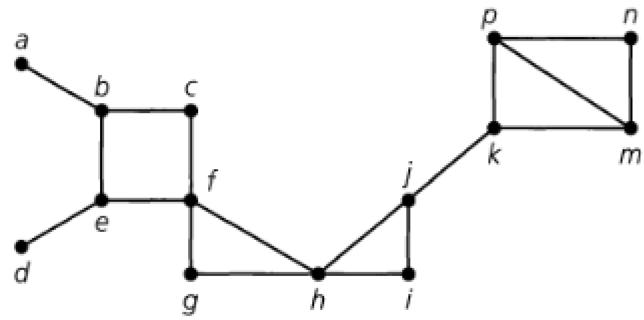


Figure 12.44

Solution:

Problem 3:

Let $T = (V, E)$ be a tree with $|V| = n > 3$.

(a) What are the smallest and the largest numbers of articulation points that T can have? Describe the trees for each of these cases.

(b) How many biconnected components does T have in each of the cases in part (a)?

Solution:

Problem 7:

Let $G = (V, E)$ be a loop-free connected undirected graph with $|V| > 3$. If G has no articulation points, prove that G has no pendant vertices.

Solution:

Problem 9:

Answer the questions posed in the previous exercise but this time order the vertices as h, g, f, e, d, c, b, a and let c be the root of T.

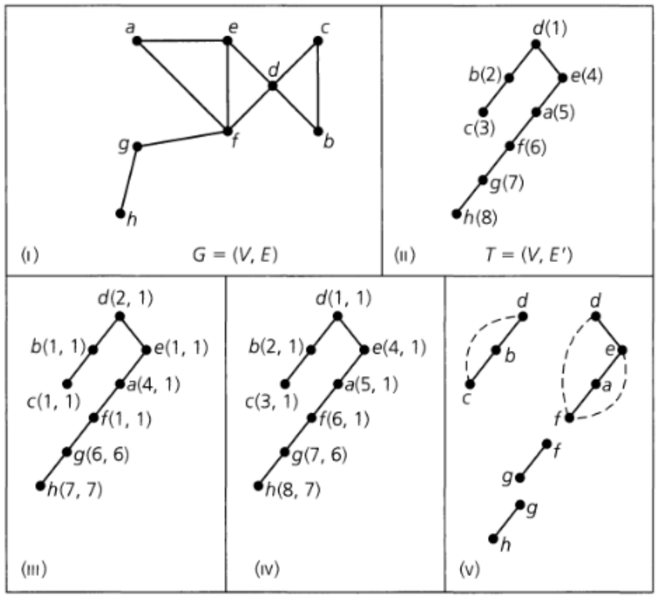


Figure 12.43

Solution:

Chapter 13.2

Problem 3:

Let $G = (V, E)$ be a loop-free weighted connected undirected graph with $T = (V, E)$,a minimal spanning tree for G. For $v, w \in V$, is the path from v to w in T a path of minimum weight in G?

Solution:

Problem 4:

Table 13.1 provides information on the distance (in miles) between pairs of cities in the state of Indiana.

A system of highways connecting these seven cities is to be constructed. Determine which highways should be constructed to that the cost of construction is minimal. (Assume that the cost of construction of a mile of highway is the same between every pair of cities.)

Table 13.1

	Bloomington	Evansville	Fort Wayne	Gary	Indianapolis	South Bend
Evansville	119	—	—	—	—	—
Fort Wayne	174	290	—	—	—	—
Gary	198	277	132	—	—	—
Indianapolis	51	168	121	153	—	—
South Bend	198	303	79	58	140	—
Terre Haute	58	113	201	164	71	196

Solution:

Primarily, it is important to understand the main focus. This can be done by connecting the cities:

Gary - South Bend (58 miles), South Bend - Fort Wayne (79 miles),
Fort Wayne - Indianapolis (121 miles), Indianapolis - Bloomington (51 miles),
Bloomington - Terre Haute (58 Miles), Terre Haute - Evansville (113 miles)

This would result in a total of 480 miles of road.

Problem 9:

Let $G = (V, E)$ be a loop-free weighted connected undirected graph, where for each pair of distinct edges $e_1, e_2 \in E$, $wt(e_1) \neq wt(e_2)$. Prove that G has only one minimal spanning tree.

Solution:

According to Kruskals algorithm earlier in the chapter, there are distinct