Linear Algebra: Homework #5-1

Due on March 28, 2019 at 11:59pm

Dr. Subasi Section 01

Eric Pereira

Problem 1

Consider the following 3×3 matrix:

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -1 & 4 & -3 \\ 2 & -3 & 4 \end{pmatrix}$$

(a) (10pts) Find eigenvalues of A.
Solution:

(b) (10pts) Find the eigenspaces of A corresponding to the eigenvalues found in Part (a). Justify your answer.
Solution:

(c) (5pts) Determine whether A is diagonalizable. Justify your answer.
Solution:

(d) (10pts) Find an invertible matrix P and a diagonal matrix D such that $P^{-1AP=D}$.
Solution:

(e) (10pts) Find a matrix S whose columns form an orthonormal basis for $\mathbb{R}^3$ obtained from the columns of matrix P found in part (d).

Gram-Schmidt Process: if $\{u_1, u_2, ..., u_k\}$ is a basis for a subspace W, then $\{v_1, v_2, ..., v_k\}$ is an orthogonal basis for W, where

$$v_1 = u_1,\ v_2 = u_2 - \frac{u_2 \cdot v_1}{||v_1||^2} v_1,\ ...,\ v_k = u_k - \frac{u_k \cdot v_1}{||v_1||^2} v_1 - \ ... \ - \frac{u_k \cdot v_{k-1}}{||v_{k-1}||^2} v_{k-1}$$

Solution:

(f) (5pts) Is S invertible? If yes, find S^{-1}. **Solution:**

(g) (5pts) Show that $S^{-1}AS = D$, where D is the diagonal matrix found in Part (d), that is, A is orthogonally diagonlizable.
Solution:

Problem 2

Consider the transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$,

$$T(x_1, x_2, x_3, x_4) = (x_2 - x_3 - x_4, x_1 + x_3 + 2x_4)$$

(a) (10pts) Show that T is a linear transformation.
Solution:

(b) (5pts) Find the standard matrix of T.
Solution:

(c) (10pts) Find a basis for the Kernel of T.
Solution:

(d) (10pts) Find a basis for the orthogonal complement of the kernel of T.
Hint: The orthogonal complement of *kernel* T is the set of all vectors that are orthogonal (perpendicular) to *kernel* T:

$$V^\perp = \{x \in \mathbb{R}^4 | x \cdot v = 0 \; \forall \; v \in kernel \; T\}$$

Solution:

(e) (5pts) Determine whether the set

$$B = \left\{ \begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ 1 \\ 2 \\ 1 \end{pmatrix} \right\}$$

is a basis for the kernel of T.
Solution:

(f) (5pts) Find a basis for the range of T.
Solution:

Problem 3

(20pts) Classify each of the following statements as True or False. Justify your answers.

(a) The determinant of an orthognal matrix A is equal to 1.
Solution:

(b) The projection matrix P defined as $P = A(A^T A)^{-1} A^T$ is symmetric for all A.
Solution:

(c) Every non-zero subspace W of $\mathbb{R}^n$ has an orthonormal basis.
Solution:

(d) Let A and B be two $n \times n$ orthogonal matrices. Then $B^{-1}AB$ is orthogonal.
Solution:

Problem 4

Suppose that A is an $n \times n$ orthogonal matrix.

(a) (10pts) Show that the matrix operator $T : \mathbb{R}^n \to \mathbb{R}^n, T(x) = Ax$ is an orthogonal operator.
Solution:

(b) (10pts) Prove that the only eigenvalues of A are 1 and -1.