

# Do you have Polleverywhere?

A) Yes B) No





# Norms

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#### Norms

For all  $a \in R$  and all  $u, v \in V$ ,

- $L_p(av) = |a| L_p(v)$
- $L_p(\mathbf{u} + \mathbf{v}) \leq L_p(\mathbf{u}) + L_p(\mathbf{v})$ 
  - triangle inequality or subadditivity
- If  $L_p(\mathbf{v}) = 0$  then  $\mathbf{v}$  is the zero vector
  - implies  $|\mathbf{v}| = 0$  iff  $\mathbf{v}$  is the zero vector

 $L_p$  norm:  $(\Sigma_j |x_j|^p)^{1/p}$ 

```
|(1,2,3)|_1?
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- A) 1
- B) 3
- C) sqrt(14)
- D) sqrt(14/3)
- E) none of the above



```
|(1,2,3)|_2?
```

- A) 1
- B) 3
- C) sqrt(14)
- D) sqrt(14/3)
- E) none of the above



```
|(1,2,3)|_{1/2}?
```

- A) 1
- B) 3
- **C)** sqrt(14)
- D) sqrt(14/3)
- E) none of the above



```
|(1,2,3)|_0?
```

- A) 1
- B) 3
- **C)** sqrt(14)
- D) sqrt(14/3)
- E) none of the above



# L<sub>0</sub> pseudo-norm

 $|\mathbf{x}|_0$  = number of  $x_j \neq 0$ 

How is this not a real norm?

## Norms

Is  $|x|_{1/2}$  convex?



## **Distance**

**♦** How do norms relate to distance?

### **Distance**

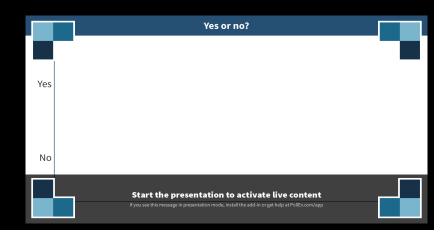
**♦** How do norms relate to distance?

$$d_{p}(\mathbf{x},\mathbf{y}) = |\mathbf{x}-\mathbf{y}|_{p}$$

#### Kernel

A symmetric function  $K: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}$  is a positive semi-definite (psd) kernel on  $\mathbf{X}$  if  $\Sigma_{i,j} c_i c_j K(\mathbf{x}_i, \mathbf{x}_j) \ge 0$ 

 $\Sigma_i$   $\mathbf{x}_i$  - the sum of the elements of x



 $\Sigma_i i x_i$  - the sum of the elements of x, each weighted by it's index, i



$$\Sigma_i \mathbf{x}_i^2$$



length(x) - number of the elements of x



d(x,y) - the Euclidean distance between x and some other (arbitrary, but fixed, vector y, also non-negative)



True or False: The only important thing you need to pick which doing k-nearest neighbors is k

