Eric_Hirsch_605_Assignment_13

Eric Hirsch

11/21/2021

1. ***Use integration by substitution to solve the integral below.***

$$\int 4e^{-7x}dx$$

We know that:

$$\int e^x dx = e^x + C$$

If we set u to -7x, then du = -7dx

So now we have:

$$\int 4e^{-7x}dx$$
$$4\int e^{-7x}dx$$
$$\frac{-4}{7}\int e^u du$$
$$\frac{-4}{7}e^u + c$$
$$\frac{-4}{7}e^{-7x} + c$$

2. ***Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of***

$$\frac{dN}{dt} = \frac{-3150}{t^4} - 220$$

bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

This is simply a matter of integrating the rate function. The level after one day will allow us to find the constant.

$$\int (-3150t^{-4} - 220)dt$$
$$= -1/3(-3150t)^{-3} - 220t + c$$
$$= 1050t^{-3} - 220t + c$$

Now we solve for c:

$$\frac{1050}{1^3} - 220(1) + c = 6350$$
$$c = 6530 - 1050 + 220$$
$$c = 5700$$

So here is the function:

$$N(t) = \frac{1050}{t^3} - 220t + 5700$$

3. Find the total area of the red rectangles in the figure below, where the equation of the line is f (x) = 2x - 9.

We can do this a numbeer of ways, but since it's a calculus assignment we'll integrate. We assume the interval is 4.5 to 8.5 from the graph (although this isn't 100% certain):

$$\int_{4.5}^{8.5} 2x - 9\ dx$$

Integral:

$$x^2 - 9x$$

Thus:

$$(8.5^2 - 9(8.5)) - (4.5^2 - 9(4.5)) = 16$$

4. ***Find the area of the region bounded by the graphs of the given equations.***

$$y = x^2 - 2x - 2$$
$$y = x + 2$$

For this question first we find the points of intersection:

$$x^2 - 2x - 2 = x + 2$$
$$x^2 - 3x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$

x=4 and x=-1.

So we need to subtract the area under the upside down parabola from all the area under the line - this will give us the area within the parabola.

There are many ways to get the area under the line - we could do 1*5 (the rectangular part of the area) plus 25/2 (The triangular part of the area.) This gives us 17.5.

Now we subtract:

```
int2 <- function(x) {(x^2-2*x-2)}
integrate(int2, lower = -1, upper = 4)
```

```
## -3.333333 with absolute error < 1.2e-13
```

17.5 - -3.33333 = 20.8333333

5. A beauty supply store expects to sell 110 flat irons during the next year. It costs 3.75 to store one flat iron for one year. There is a fixed cost of 8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

I think we need to make some assumptions here about rates of purchase. Setting n = # of orders, we will assume that storage costs are (3.75*110)/n - in other words, items bought for sale remain in the store until the next order and are bought on the day of the new order.

Thus,

$$T = 8.25n + \frac{3.75 * 110}{n}$$

This is a minimization problem, so we take the first derivative and set to 0:

$$T' = 8.25 + -1(\frac{3.75 * 110}{n^2}) = 0$$
$$8.25 * x^2 = (3.75 * 110)$$
$$x^2 = (3.75 * 110)/8.25$$
$$x = sqrt((3.75 * 110)/8.25) = \ 7$$

This gives us around 7 lots, with around (110/7 = 16) irons per lot.

6. ***Use integration by parts to solve the integral below***

$$\int ln(9x)x^6dx$$

To integrate by parts we use this formula:

$$uv - \int vdu$$

We will set u = ln(9x) and we can set dv = x^6. Now we need du and v:

$$u = ln(9x)$$
$$du = \frac{1}{x}dx$$

Now we calculate v by integrating dv:

$$dv = x^6$$
$$v = \frac{1}{7}x^7$$

This gives us:

$$\frac{x^7 ln(9x)}{7} - \int \frac{1}{7}x^7\frac{1}{x}dx$$

So now we simplify. To make the notation easier I'm going to set the first element to q:

$$q = \frac{x^7 ln(9x)}{7}$$

So now we have:

$$q - \int \frac{x^6}{7} dx$$

$$q - \frac{1}{7} \int x^6 dx$$

$$q - \frac{1}{7}(\frac{x^7}{7}) + C$$

$$q - \frac{x^7}{49} + C$$

$$\frac{x^7 ln(9x)}{7} - \frac{x^7}{49} + C$$

$$\frac{1}{7}(x^7 ln(9x) - \frac{x^7}{7}) + C$$

7. Determine whether f (x) is a probability density function on the interval 1, e^6

$$f(x) = \frac{1}{6x}$$

A probability density function will have an integral of 1 and be positive for all possible values - so that is what we are testing here. Positivity is guaranteed because the interval is positive and there are no negative signs in the function. So now we integrate:

$$\int_1^{e^6} \frac{1}{6x} dx$$

$$\frac{1}{6} \int_1^{e^6} \frac{1}{x} dx$$

Here we use R to generate the integral:

```
int2 <- function(x) {1/x}
integrate(int2, lower = 1, upper = exp(1)^6)
```

```
## 6 with absolute error < 0.00056
```

Because 1/6*6 = 1, then the distribution can be regarded as a probability density function.