```matlab
syms E I q0 L x
E = 210*10^9
```

E = 2.1000e+11

```matlab
I = (1/12)*10^(-8)
```

I = 8.3333e-10

```matlab
q0 = 100
```

q0 = 100

```matlab
L = 0.1
```

L = 0.1000

```matlab
F = 40
```

F = 40

c.

```matlab
% n = 1
displayFormula("q11 = int(q0*x, x, 0, L)")
```

$$\frac{1}{2} = \int_0^{\frac{1}{10}} 100\, x \, \mathrm{d}x$$

```matlab
q11 = int(q0*x, x, 0, L)
```

q11 =

$$\frac{1}{2}$$

```matlab
displayFormula("q12 = int((-q0/L)*x^2 + 2*q0*x^1, x, L, 2*L)")
```

$$\frac{2}{3} = \int_{\frac{1}{10}}^{\frac{2}{10}} \left(\frac{-100}{\frac{1}{10}} x^2 + 2\,100\, x \right) \mathrm{d}x$$

```matlab
q12 = int((-q0/L)*x^2 + 2*q0*x^1, x, L, 2*L)
```

q12 =

$$\frac{2}{3}$$

```matlab
q1 = q11 + q12
```

q1 =

$$\frac{7}{6}$$

```
F1 = q1 - 2*F*L - (7/6)*q0*L^2 + 2*F*L
```

F1 = 0

```
% n = 2
displayFormula("q21 = int(q0*x^2, x, 0, L)")
```

$$\frac{1}{30} = \int_0^{\frac{1}{10}} 100\, x^2\, \mathrm{d}x$$

```
q21 = int(q0*x^2, x, 0, L)
```

q21 =

$$\frac{1}{30}$$

```
displayFormula("q22 = int((-q0/L)*x^3 + 2*q0*x^2, x, L, 2*L)")
```

$$\frac{11}{120} = \int_{\frac{1}{10}}^{\frac{2}{10}} \left(\frac{-100}{\frac{1}{10}} x^3 + 2\,100\,x^2 \right) \mathrm{d}x$$

```
q22 = int((-q0/L)*x^3 + 2*q0*x^2, x, L, 2*L)
```

q22 =

$$\frac{11}{120}$$

```
q2 = q21 + q22
```

q2 =

$$\frac{1}{8}$$

```
F2 = q2 - F*(2*L)^2
```

F2 =

$$-\frac{59}{40}$$

```
A = (E*I) * [0 0; 0 int(4, x, 0, 2*L)]
```

A =

$$\begin{pmatrix} 0 & 0 \\ 0 & 140 \end{pmatrix}$$

```
B = [F1; F2]
```

B =

$$\begin{pmatrix} 0 \\ -\dfrac{59}{40} \end{pmatrix}$$

```
X = double(linsolve(A, B))
```

Warning: Solution is not unique because the system is rank-deficient.
X = 2×1
      0
  -0.0105

```
% n = 3
displayFormula("q31 = int(q0*x^3, x, 0, L)")
```

$$\frac{1}{400} = \int_0^{\frac{1}{10}} 100\, x^3 \, dx$$

```
q31 = int(q0*x^3, x, 0, L)
```

q31 =

$$\frac{1}{400}$$

```
displayFormula("q32 = int((-q0/L)*x^4 + 2*q0*x^3, x, L, 2*L)")
```

$$\frac{13}{1000} = \int_{\frac{1}{10}}^{\frac{2}{10}} \left(\frac{-100}{\frac{1}{10}}\, x^4 + 2 \cdot 100\, x^3 \right) dx$$

```
q32 = int((-q0/L)*x^4 + 2*q0*x^3, x, L, 2*L)
```

q32 =

$$\frac{13}{1000}$$

```
q3 = q31 + q32
```

q3 =

$$\frac{31}{2000}$$

```
F3 = q3 - F*(2*L)^3
```

F3 =

$$-\frac{609}{2000}$$

```
A = (E*I) * [int(4, x, 0, 2*L)        int(12*x, x, 0, 2*L);
              int(12*x, x, 0, 2*L)    int(36*x^2, x, 0, 2*L)]
```

A =

$$\begin{pmatrix} 140 & 42 \\ 42 & \dfrac{84}{5} \end{pmatrix}$$

```
B = [F2; F3]
```

B =

$$\begin{pmatrix} -\dfrac{59}{40} \\ -\dfrac{609}{2000} \end{pmatrix}$$

```
X = double(linsolve(A,B))
```

X = 2×1
    -0.0204
     0.0329

```
% n = 4
displayFormula("q41 = int(q0*x^4, x, 0, L)")
```

$$\frac{1}{5000} = \int_0^{\frac{1}{10}} 100\,x^4\,\mathrm{d}x$$

```
q41 = int(q0*x^4, x, 0, L)
```

q41 =

$$\frac{1}{5000}$$

```
displayFormula("q42 = int((-q0/L)*x^5 + 2*q0*x^4, x, L, 2*L)")
```

4

$$\frac{19}{10000} = \int_{\frac{1}{10}}^{\frac{2}{10}} \left(\frac{-100}{\frac{1}{10}} x^5 + 2\,100\, x^4 \right) dx$$

```
q42 = int((-q0/L)*x^5 + 2*q0*x^4, x, L, 2*L)
```

q42 =

$$\frac{19}{10000}$$

```
q4 = q41 + q42
```

q4 =

$$\frac{21}{10000}$$

```
F4 = q4 - F*(2*L)^4
```

F4 =

$$-\frac{619}{10000}$$

```
A = (E*I) * [int(4, x, 0, 2*L)      int(12*x, x, 0, 2*L)    int(24*x^2, x, 0, 2*L);
             int(12*x, x, 0, 2*L)   int(36*x^2, x, 0, 2*L)  int(72*x^3, x, 0, 2*L);
             int(24*x^2, x, 0, 2*L) int(72*x^3, x, 0, 2*L)  int(144*x^4, x, 0, 2*L)]
```

A =

$$\begin{pmatrix} 140 & 42 & \frac{56}{5} \\ 42 & \frac{84}{5} & \frac{126}{25} \\ \frac{56}{5} & \frac{126}{25} & \frac{1008}{625} \end{pmatrix}$$

```
B = [F2; F3; F4]
```

B =

$$\begin{pmatrix} -\frac{59}{40} \\ -\frac{609}{2000} \\ -\frac{619}{10000} \end{pmatrix}$$

```
X = double(linsolve(A,B))
```

X = 3×1
   -0.0196

```
    0.0248
    0.0201
```

```
% n = 5
displayFormula("q51 = int(q0*x^5, x, 0, L)")
```

$$\frac{1}{60000} = \int_{0}^{\frac{1}{10}} 100\, x^5\, \mathrm{d}x$$

```
q51 = int(q0*x^5, x, 0, L)
```

q51 =

$$\frac{1}{60000}$$

```
displayFormula("q52 = int((-q0/L)*x^6 + 2*q0*x^5, x, L, 2*L)")
```

$$\frac{1}{3500} = \int_{\frac{1}{10}}^{\frac{2}{10}} \left(\frac{-100}{\frac{1}{10}}\, x^6 + 2\,100\, x^5 \right) \mathrm{d}x$$

```
q52 = int((-q0/L)*x^6 + 2*q0*x^5, x, L, 2*L)
```

q52 =

$$\frac{1}{3500}$$

```
q5 = q51 + q52
```

q5 =

$$\frac{127}{420000}$$

```
F5 = q5 - F*(2*L)^5
```

F5 =

$$-\frac{5249}{420000}$$

```
A = (E*I) * [int(4, x, 0, 2*L)      int(12*x, x, 0, 2*L)      int(24*x^2, x, 0,
2*L)  int(40*x^3, x, 0, 2*L);
             int(12*x, x, 0, 2*L)   int(36*x^2, x, 0, 2*L)   int(72*x^3, x, 0,
2*L)  int(120*x^4, x, 0, 2*L);
             int(24*x^2, x, 0, 2*L) int(72*x^3, x, 0, 2*L)   int(144*x^4, x, 0,
2*L) int(240*x^5, x, 0, 2*L);
```

```
              int(40*x^3, x, 0, 2*L) int(120*x^4, x, 0, 2*L)  int(240*x^5, x, 0,
2*L) int(400*x^6, x, 0, 2*L)]
```

A =

$$\begin{pmatrix} 140 & 42 & \dfrac{56}{5} & \dfrac{14}{5} \\[2mm] 42 & \dfrac{84}{5} & \dfrac{126}{25} & \dfrac{168}{125} \\[2mm] \dfrac{56}{5} & \dfrac{126}{25} & \dfrac{1008}{625} & \dfrac{56}{125} \\[2mm] \dfrac{14}{5} & \dfrac{168}{125} & \dfrac{56}{125} & \dfrac{16}{125} \end{pmatrix}$$
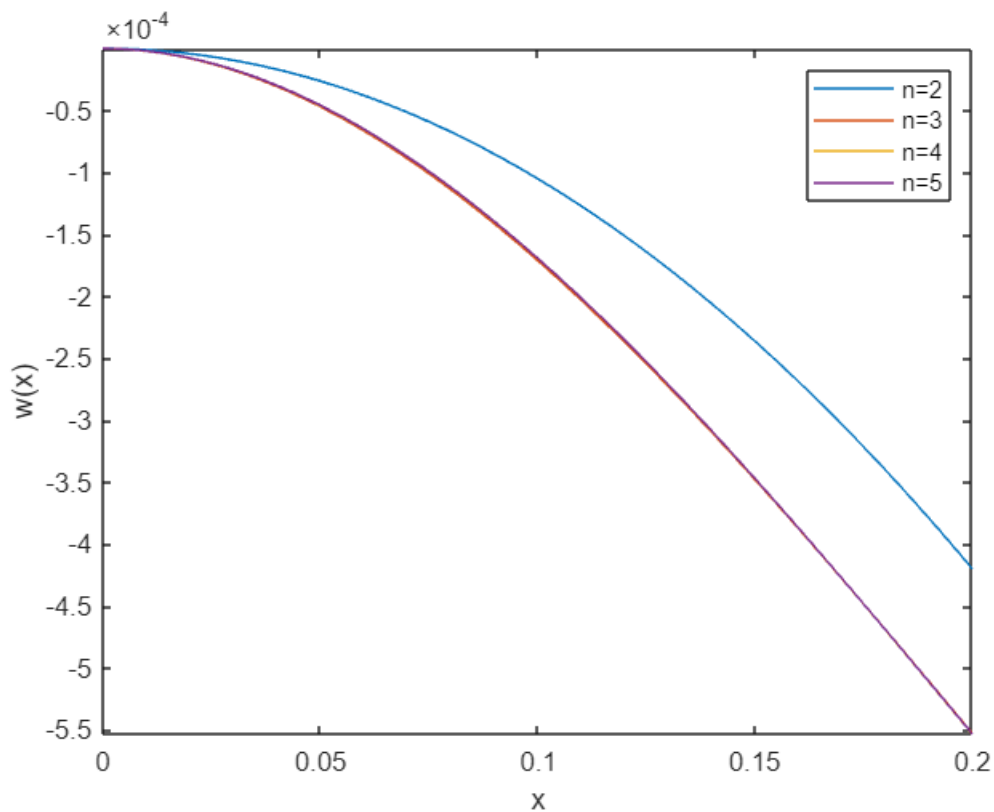
```
B = [F2; F3; F4; F5]
```

B =

$$\begin{pmatrix} -\dfrac{59}{40} \\[3mm] -\dfrac{609}{2000} \\[3mm] -\dfrac{619}{10000} \\[3mm] -\dfrac{5249}{420000} \end{pmatrix}$$

```
X = double(linsolve(A,B))
```

X = 4×1
   -0.0195
    0.0229
    0.0320
   -0.0238

```matlab
% plotting
fplot((-0.0105)*x^2, [0, 0.2])
hold on
fplot((-0.0204)*x^2 + (0.0329)*x^3, [0, 0.2])
fplot((-0.0196)*x^2 + (0.0248)*x^3 + (0.0201)*x^4, [0, 0.2])
fplot((-0.0195)*x^2 + (0.0229)*x^3 + (0.0320)*x^4 + (-0.0238)*x^5, [0, 0.2])
hold off

xlabel('x')
ylabel('w(x)')
legend('n=2', 'n=3', 'n=4', 'n=5')
```

```
% checking derivative
diff(diff(x^2))*diff(diff(x^2))
```

ans = 4

```
diff(diff(x^2))*diff(diff(x^3))
```

ans = $12\,x$

```
diff(diff(x^2))*diff(diff(x^4))
```

ans = $24\,x^2$

```
diff(diff(x^2))*diff(diff(x^5))
```

ans = $40\,x^3$

```
diff(diff(x^3))*diff(diff(x^2))
```

ans = $12\,x$

```
diff(diff(x^3))*diff(diff(x^3))
```

ans = $36\,x^2$

```
diff(diff(x^3))*diff(diff(x^4))
```

ans = $72\,x^3$

```
diff(diff(x^3))*diff(diff(x^5))
```

ans = $120\,x^4$

```
diff(diff(x^4))*diff(diff(x^2))
```

ans = $24\,x^2$

```
diff(diff(x^4))*diff(diff(x^3))
```

ans = $72\,x^3$

```
diff(diff(x^4))*diff(diff(x^4))
```

ans = $144\,x^4$

```
diff(diff(x^4))*diff(diff(x^5))
```

ans = $240\,x^5$

```
diff(diff(x^5))*diff(diff(x^2))
```

ans = $40\,x^3$

```
diff(diff(x^5))*diff(diff(x^3))
```

ans = $120\,x^4$

```
diff(diff(x^5))*diff(diff(x^4))
```

ans = $240\,x^5$

```
diff(diff(x^5))*diff(diff(x^5))
```

ans = $400\,x^6$

d.

```
% n = 2
syms a1 a2 u_h
```

```
u_h = a1*x + a2*x^2
```

u_h = $a_2\, x^2 + a_1\, x$

```
I2 = 0.5*E*I*int(diff(diff(u_h))^2, x, 0, 2*L) - int(u_h*q0, x, 0, L) - int(u_h*((-
q0/L)*x+2*q0), x, L, 2*L) + F*(a1*2*L+a2*((2*L)^2))
```

I2 =

$$70\, a_2{}^2 + \frac{59\, a_2}{40} + \frac{41\, a_1}{6}$$

```
diff(I2, a2)
```

ans =

$$140\, a_2 + \frac{59}{40}$$

```
% n = 3
syms a1 a2 a3 u_h
u_h = a1*x + a2*x^2 + a3*x^3
```

u_h = $a_3\, x^3 + a_2\, x^2 + a_1\, x$

```
I3 = 0.5*E*I*int(diff(diff(u_h))^2, x, 0, 2*L) - int(u_h*q0, x, 0, L) - int(u_h*((-
q0/L)*x+2*q0), x, L, 2*L) + F*(a1*2*L + a2*((2*L)^2) + a3*((2*L)^3))
```

I3 =

$$70\, a_2{}^2 + 42\, a_2\, a_3 + \frac{59\, a_2}{40} + \frac{42\, a_3{}^2}{5} + \frac{609\, a_3}{2000} + \frac{41\, a_1}{6}$$

```
diff(I3, a2)
```

ans =

$$140\, a_2 + 42\, a_3 + \frac{59}{40}$$

```
diff(I3, a3)
```

ans =

$$42\, a_2 + \frac{84\, a_3}{5} + \frac{609}{2000}$$

```
A = [140 42; 42 84/5]
```

A = 2×2
```
   140.0000   42.0000
    42.0000   16.8000
```

```
B = [-59/40; -609/2000]
```

B = 2×1

10

```
   -1.4750
   -0.3045
```

```
X = double(linsolve(A,B))
```

```
X = 2×1
   -0.0204
    0.0329
```

```
% n = 4
syms a1 a2 a3 a4 u_h
u_h = a1*x + a2*x^2 + a3*x^3 + a4*x^4
```

u_h = $a_4\,x^4 + a_3\,x^3 + a_2\,x^2 + a_1\,x$

```
I4 = 0.5*E*I*int(diff(diff(u_h))^2, x, 0, 2*L) - int(u_h*q0, x, 0, L) - int(u_h*((-
q0/L)*x+2*q0), x, L, 2*L) + F*(a1*2*L + a2*((2*L)^2) + a3*((2*L)^3) + a4*((2*L)^4))
```

I4 =

$$70\,a_2{}^2 + 42\,a_2\,a_3 + \frac{56\,a_2\,a_4}{5} + \frac{59\,a_2}{40} + \frac{42\,a_3{}^2}{5} + \frac{126\,a_3\,a_4}{25} + \frac{609\,a_3}{2000} + \frac{504\,a_4{}^2}{625} + \frac{619\,a_4}{10000} + \frac{41\,a_1}{6}$$

```
diff(I4, a2)
```

ans =

$$140\,a_2 + 42\,a_3 + \frac{56\,a_4}{5} + \frac{59}{40}$$

```
diff(I4, a3)
```

ans =

$$42\,a_2 + \frac{84\,a_3}{5} + \frac{126\,a_4}{25} + \frac{609}{2000}$$

```
diff(I4, a4)
```

ans =

$$\frac{56\,a_2}{5} + \frac{126\,a_3}{25} + \frac{1008\,a_4}{625} + \frac{619}{10000}$$

```
A = [140 42 56/5; 42 84/5 126/25; 56/5 126/25 1008/625]
```

```
A = 3×3
   140.0000   42.0000   11.2000
    42.0000   16.8000    5.0400
    11.2000    5.0400    1.6128
```

```
B = [-59/40; -609/2000; -619/10000]
```

```
B = 3×1
   -1.4750
   -0.3045
```

11

```
   -0.0619
```

```
X = double(linsolve(A,B))
```

```
X = 3×1
   -0.0196
    0.0248
    0.0201
```

```
% n = 5
syms a1 a2 a3 a4 a5 u_h
u_h = a1*x + a2*x^2 + a3*x^3 + a4*x^4 + a5*x^5
```

u_h = $a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x$

```
I5 = 0.5*E*I*int(diff(diff(u_h))^2, x, 0, 2*L) - int(u_h*q0, x, 0, L) - int(u_h*((-
q0/L)*x+2*q0), x, L, 2*L) + F*(a1*2*L + a2*((2*L)^2) + a3*((2*L)^3) + a4*((2*L)^4)
+ a5*((2*L)^5))
```

I5 =

$$70 a_2{}^2 + 42 a_2 a_3 + \frac{56 a_2 a_4}{5} + \frac{14 a_2 a_5}{5} + \frac{59 a_2}{40} + \frac{42 a_3{}^2}{5} + \frac{126 a_3 a_4}{25} + \frac{168 a_3 a_5}{125} + \frac{609 a_3}{2000} + \frac{504 a_4{}^2}{625} + \frac{56 a}{1\,}$$

```
diff(I5, a2)
```

ans =

$$140 a_2 + 42 a_3 + \frac{56 a_4}{5} + \frac{14 a_5}{5} + \frac{59}{40}$$

```
diff(I5, a3)
```

ans =

$$42 a_2 + \frac{84 a_3}{5} + \frac{126 a_4}{25} + \frac{168 a_5}{125} + \frac{609}{2000}$$

```
diff(I5, a4)
```

ans =

$$\frac{56 a_2}{5} + \frac{126 a_3}{25} + \frac{1008 a_4}{625} + \frac{56 a_5}{125} + \frac{619}{10000}$$

```
diff(I5, a5)
```

ans =

$$\frac{14 a_2}{5} + \frac{168 a_3}{125} + \frac{56 a_4}{125} + \frac{16 a_5}{125} + \frac{5249}{420000}$$

```
A = [140 42 56/5 14/5; 42 84/5 126/25 168/125; 56/5 126/25 1008/625 56/125; 14/5
168/125 56/125 16/125]
```

```
A = 4×4
```

```
    140.0000    42.0000    11.2000     2.8000
     42.0000    16.8000     5.0400     1.3440
     11.2000     5.0400     1.6128     0.4480
      2.8000     1.3440     0.4480     0.1280
```

```
B = [-59/40; -609/2000; -619/10000; -5249/420000]
```

```
B = 4×1
    -1.4750
    -0.3045
    -0.0619
    -0.0125
```

```
X = double(linsolve(A,B))
```

```
X = 4×1
    -0.0195
     0.0229
     0.0320
    -0.0238
```

f.

```
syms u2 u3 u4 u5
u2 = -0.0105*x^2
```

u2 =

$$-\frac{21\,x^2}{2000}$$

```
u3 = -0.0204*x^2 + 0.0329*x^3
```

u3 =

$$\frac{329\,x^3}{10000} - \frac{51\,x^2}{2500}$$

```
u4 = -0.0196*x^2 + 0.0248*x^3 + 0.0201*x^4
```

u4 =

$$\frac{201\,x^4}{10000} + \frac{31\,x^3}{1250} - \frac{49\,x^2}{2500}$$

```
u5 = -0.0195*x^2 + 0.0229*x^3 + 0.0320*x^4 - 0.0238*x^5
```

u5 =

$$-\frac{119\,x^5}{5000} + \frac{4\,x^4}{125} + \frac{229\,x^3}{10000} - \frac{39\,x^2}{2000}$$

```
% n=2
u25 = u2 - u5
```

u25 =

$$\frac{119\,x^5}{5000} - \frac{4\,x^4}{125} - \frac{229\,x^3}{10000} + \frac{9\,x^2}{1000}$$

```
I25 = double(0.5*E*I*int(diff(diff(u25))^2, x, 0, 2*L) - int(u25*q0, x, 0, L) -
int(u25*((-q0/L)*x+2*q0), x, L, 2*L) + F*((-0.0105*(2*L)^2) - (-0.0195*(2*L)^2 +
0.0229*(2*L)^3 + 0.0320*(2*L)^4 - 0.0238*(2*L)^5)))
```

I25 = 0.0069

```
% n=3
u35 = u3 - u5
```

u35 =

$$\frac{119\,x^5}{5000} - \frac{4\,x^4}{125} + \frac{x^3}{100} - \frac{9\,x^2}{10000}$$

```
I35 = double(0.5*E*I*int(diff(diff(u35))^2, x, 0, 2*L) - int(u35*q0, x, 0, L)
- int(u35*((-q0/L)*x+2*q0), x, L, 2*L) + F*((-0.0204*(2*L)^2 + 0.0329*(2*L)^3 -
(-0.0195*(2*L)^2 + 0.0229*(2*L)^3 + 0.0320*(2*L)^4 - 0.0238*(2*L)^5)))
```

I35 = 4.3308e-05

```
% n=4
u45 = u4 - u5
```

u45 =

$$\frac{119\,x^5}{5000} - \frac{119\,x^4}{10000} + \frac{19\,x^3}{10000} - \frac{x^2}{10000}$$

```
I45 = double(0.5*E*I*int(diff(diff(u45))^2, x, 0, 2*L) - int(u45*q0, x, 0, L)
- int(u45*((-q0/L)*x+2*q0), x, L, 2*L) + F*((-0.0196*(2*L)^2 + 0.0248*(2*L)^3
+ 0.0201*(2*L)^4) - (-0.0195*(2*L)^2 + 0.0229*(2*L)^3 + 0.0320*(2*L)^4 -
0.0238*(2*L)^5)))
```

I45 = -8.0235e-06