CEE 6513 Computational Methods in Mechanics

Homework 5

Student: Yu-Chen Pan GTID: 903918558

Problem 1.

- Differential equation: $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$

- Initial Conditions: $u(x,t) = 0, \quad t \le 0, \quad 0 \le x \le L$

- Boundary Conditions: $u(L,t) = 0$

- Excitation: $u(0,t) = f(t) = \begin{cases} 1 - \cos(2t) & 0 \le t \le \pi \\ 0 & otherwise. \end{cases}$

- Explicit Finite Difference scheme:

A finite number of mesh points represents the temporal domain [0, T].

$$0 = t_0 < t_1 < t_2 < \cdots < t_{N_t - 1} < t_{N_t} = T, \quad t_{n+1} - t_n = \Delta t$$

A finite number of mesh points represents the spatial domain [0, L].

$$0 = x_0 < x_1 < x_2 < \cdots < x_{N_x - 1} < x_{N_x} = L, \quad x_{i+1} - x_i = \Delta x$$

PDE:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} \approx \frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2}$$

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$\rightarrow \quad \frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2} = \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$\rightarrow \quad u_i^{n+1} = 2u_i^n - u_i^{n-1} + c^2(u_{i+1}^n - 2u_i^n + u_{i-1}^n)$$

For stability: $c = \frac{\Delta t}{\Delta x} \le 1$

For implementation:

Let $j = n + 1$

$$\rightarrow \quad u_i^j = 2u_i^{j-1} - u_i^{j-2} + c^2\left(u_{i+1}^{j-1} - 2u_i^{j-1} + u_{i-1}^{j-1}\right)$$

- Implementation:

Parameters for spatial discretization:

$L = 1$

$\Delta x = 0.01$

$$N_x = \frac{L}{\Delta x} + 1$$

Parameters for temporal discretization:

$T = 100$

$\Delta t = 0.01$

$$N_t = \frac{L}{\Delta t} + 1$$

Wave speed and Courant number:

$v = 1$

$$c = v\frac{\Delta t}{\Delta x} = 1$$

Solution matrix:

$U(i, n) = u(x, t)$

U has a size of $O(N_x N_t)$

Excitation:

$U(1, 1{:}S_1) = 1 - cos(2t(1{:}S_1));$

Set $U(x = 1, t) = 1 - \cos(2t)$, $1 \le t \le S_1 = \pi$

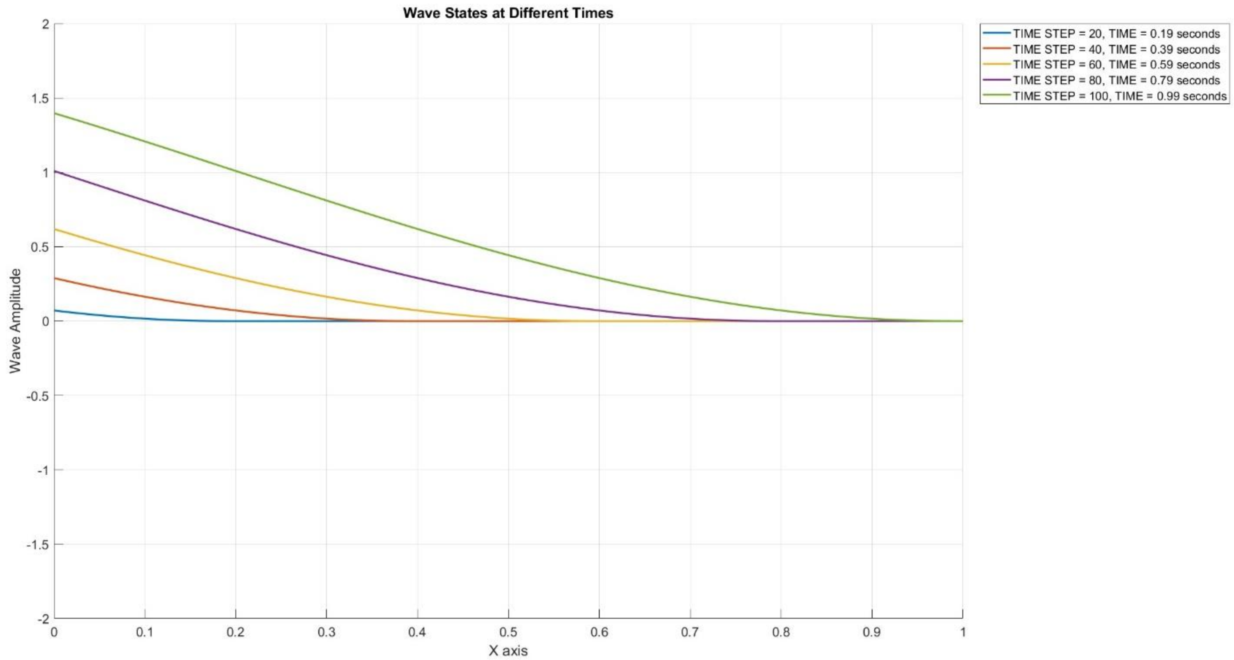
Finite difference scheme:

$U_1 = 2U(i, n-1) - U(i, n-2);$

$U_2 = U(i-1, n-1) - 2U(i, n-1) + U(i+1, n-1);$

$U(i, n) = U_1 + c^2 U_2;$

- Plot – u vs. time steps $= 20, 40, 60, 80, 100$ $(time\ steps)$:



- Plot – u vs. times $= 20, 40, 60, 80, 100$ (sec):