Problem Set 1:

1. Prove that $4^n + 15n - 1$ is divisible by 9 for all $n \geq 1$, using simple induction.

(3 marks)

2. Consider binary strings that start with a 1. By interpreting the 1's as specifying powers of 2, these strings are called binary representations of positive integers. For example:

   $10 = 2^1 = 2$
   $1011 = 2^3 + 2^1 + 2^0 = 11$
   $110001 = 2^5 + 2^4 + 2^0 = 49$

   a) Prove that every natural number $n \geq 1$ has a binary representation. (4 marks)
   b) Prove that the binary representation is unique. (4 marks)

3. Consider the Fibonacci-esque function g:

$$g(n) = \begin{cases} 1 & if\ n=0 \\ 3 & if\ n=1 \\ g(n-2) + g(n-1) & if\ n > 1 \end{cases}$$

   Use complete induction to prove that if n is a natural number greater than 1, then $2^{n/2} \leq g(n) \leq 2^n$.

(7 marks)

4. Given the function L(n) below:

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int L(n) :
        if n < 7 : return 0
        else : return 1 + L(n/7)
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   find a recurrence that expresses the time complexity, T(n) of L(n), in terms of n. Find a closed form for your recurrence and prove it is correct. Your expression should assign a constant to operations that do not depend on n.

(6 marks)

5. Let set F be recursively defined as follows:
   a. $7 \in F$
   b. If $u, v \in F$, then $u + v \in F$
   c. Nothing else is in F

   Use structural induction to prove that $\forall w \in F$, w mod 7 = 0, i.e., all elements in F are divisible by 7.

(6 marks)

Deadline: midnight on Saturday, 26 May 2018