CSC343: Introduction to Databases

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Database Design [40 points]

Question 1. [32 MARKS]

Consider a relation $R(A, B, C, D, E, F, G)$ and the set FD of functional dependencies

$$\{B \to D, BC \to A, E \to F, AB \to C, AC \to B, AD \to E\}$$

a. Compute all candidate keys for R given FD. Show your work.

The first step is the find the prime attributes. Since G is not in any functional dependency, we know that it must be in all all candidate keys. For all other attributes $X \in A, B, C, \ldots, F$, check the closure of $\{G, X\}^+$.

$$\begin{aligned}
\{G, A\}^+ &= \{G, A\} \\
\{G, B\}^+ &= \{G, B, D\} \\
&\vdots
\end{aligned}$$

After trying all six attributes with G, we need to try the closures with three attributes.

$$\begin{aligned}
\{G, A, B\}^+ &= \{G, A, B, C, D, E, F\} \\
\{G, A, C\}^+ &= \{G, A, C, B, \ldots\} && \text{(since we already know } \{G, A, B\} \text{ is a CK.)} \\
\{G, A, D\}^+ &= \{G, A, D, E, F\} \\
&\vdots \\
\{G, B, C\}^+ &= \{G, B, C, A, \ldots\} \\
&\vdots
\end{aligned}$$

Thus, after trying all pairs of combinations with G, we arrive at the three candidate keys: $[GAB, GAC, GBC]$.

b. Calculate a minimal cover for FD. Show your work.

The first step is to rewrite the FD's so that they have singleton RHS's.

Step two, we look for FD's with two or more attributes on the LHS and try to eliminate. To do this, we take proper subsets of the LHS to see if they contain the RHS or an attribute from the LHS.

$$\begin{aligned}
BC \to A &: \{B\}^+ = \{B, D\} \\
BC \to A &: \{C\}^+ = \{C\}
\end{aligned}$$

Since neither closures include the A or the other attribute on the LHS, we need this FD. We repeat this process for each FD with two or more attributes on the LHS.

$$\begin{aligned}
AB \to C &: \{A\}^+ = \{B\} \\
AB \to C &: \{B\}^+ = \{B, D\}
\end{aligned}$$

After completing this step, as it turns out, we cannot eliminate any FD's from R.

For step three, we need to determine if we have any redundant FD's. We need to go through every FD, assume that it is not present in the set of FD's and see if we can still obtain the RHS:

$$B \to D : \{B\}^+ = \{B\}$$

Hence, we cannot get to the D above without the FD $B \to D$. Therefore, this FD must be included in the minimal cover.

Repeating this process for each FD, we can see that there are no redundant FD's. We can also note that none of the FD's are redundant because the RHS's are all unique (i.e. the only way to get one of the RHS's is with that specific FD). Therefore, the given set of FD's is already a minimal cover.

c. Is R in BCNF (given the set FD)? Explain. If not, provide a BCNF decomposition. Show your work.

No, R is not in BCNF as there exists functional dependencies in the set FD that violate BCNF conditions. For all functional dependencies $X \to Y$, X must be a superkey in order for relation R to be in BCNF. Since the candidate keys are BCG, ABG and ACG, functional dependencies such as $B \to D$ and $E \to F$ violate BCNF as left hand side is not a superkey. Therefore, since there exists at least one functional dependency in the set FD that violates BCNF, the relation R is not in BCNF.

The first step in BCNF decomposition is to initialize a set $S = \{R\}$. Then while the set S has a relation S' that is not in BCNF, we pick a functional dependency $X \to Y$ in the set FD that violates BCNF and add relation XY to the set S. Then we update relation S'=S'-Y and project the FD's into the decomposed relations. We repeat this process until the set S no longer has a relation that violates BCNF.

$$S = \{ABCDEFG\}$$

Next, pick a functional dependency that violates BCNF. $B \to D$ violates BCNF as B is not a superkey

$$S = \{ABCEFG, BD\}$$

We project the FD's into relations R1(ABCEFG) and R2(BD). $AB \to C$ also violates BCNF as AB is not a superkey in relation R1.

$$S = \{ABEFG, ABC, BD\}$$

We project the FD's into relations R1(ABEFG) and R2(ABC) and R3(BD). $E \to F$ also violates BCNF as E is not a superkey in relation R1.

$$S = \{ABEG, EF, ABC, BD\}$$

The relation R is decomposed into a set of relations R1(A,B,E,G), R2(E,F), R3(A,B,C) and R4(B,D). Since the projected FD's into these relations no longer violate BCNF. R1(A,B,E,G), R2(E,F), R3(A,B,C) and R4(B,D) are in BCNF.

d. Is R in 3NF (given the set FD)? Explain. If not, provide a 3NF decomposition. Show your work.

No, R is not in 3NF because it can be decomposed into two or more relations such that each new relation contains a subset of the attributes of R, and every attribute of R appears as an attribute in at least one of the new relations.

The first two steps in the decomposition of R is to find all candidate keys of the relation, then find a minimal cover, which we have already done. Next, we need to create a relation for every FD in the minimal basis, and a relation for every candidate key not already included from the minimal basis, as is shown below.

$$R_1(B, D)$$
$$R_2(B, C, A)$$
$$R_3(E, F)$$
$$R_4(A, B, C)$$
$$R_5(A, C, B)$$
$$R_6(A, D, E)$$
$$R_7(G, A, B)$$
$$R_8(G, A, C)$$
$$R_9(G, B, C)$$

From this list, we can remove the relations with duplicate attributes, so we end up with the following 3NF decomposition:

$$R_1(B, D)$$
$$R_2(E, F)$$
$$R_3(A, B, C)$$
$$R_4(A, D, E)$$
$$R_5(G, A, B)$$

Question 2. [8 MARKS]

Prove that if a relation S has only one-attribute keys, S is in BCNF if and only if it is in 3NF.

Proof. Let S be a relation that has only one-attribute keys.

Let F^+ be the closure of FDs for R, assuming they are 3NF.

We show that, by BCNF, for every non-trivial FD $X \to Y$ in F^+, X is a superkey.

Suppose for contradiction that X is not a superkey.

Then 3NF states that Y is a member of a key, or is prime.

Since all keys are one-attribute keys, and given the above statement, Y is also a key, and $X \to Y$.

Then X is a superkey, which contradicts the statement. □

Entity-Relationship Model [20 points]

Question 3. [20 marks]

Draw an ER diagram based on the description above. Indicate the various attributes of each entity set and relationship and specify all key and participation constraints. Specify any other constraints, if necessary, in English.