FLOLAC'14

Type theory and logic

Lecture III: Heyting arithmetic

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Equality types

- Formation:
$$\frac{\Gamma \vdash A\ SET \qquad \Gamma \vdash t : A \qquad \Gamma \vdash u : A}{\Gamma \vdash t \equiv_A u\ SET}\ (\equiv\mathsf{F})$$
The subscript A, i.e., the type of t and u, is often omitted.
- Introduction:
$$\frac{\Gamma \vdash t : A}{\Gamma \vdash \mathtt{refl} : t \equiv t}\ (\equiv\mathsf{I})$$

Exercise. Assume $\Gamma \vdash t = u \in A$ and derive $\Gamma \vdash \mathtt{refl} : t \equiv u$.

Equality elimination

$$\frac{\begin{array}{l}\Gamma \ \vdash \ t : A \\ \Gamma, x : A, e : t \equiv x \ \vdash \ P_{xe}\ SET \\ \Gamma \ \vdash \ p : P_{xe}[t, \texttt{refl}/x, e] \\ \Gamma \ \vdash \ u : A \\ \Gamma \ \vdash \ q : t \equiv u\end{array}}{\Gamma \ \vdash \ \texttt{J}\,(x.\ e.\ P_{xe})\ p\ q : P_{xe}[u, q/x, e]}\ (\equiv\mathrm{E})$$

Exercise. Assuming $x : A \ \vdash \ P_x\ SET$, prove $\Pi(y : A)\ x \equiv y \to P_x \to P_x[y/x]$.

Equality computation

$$\frac{\begin{array}{l}\Gamma \vdash t : A \\ \Gamma, x : A, e : t \equiv x \vdash P_{xe}\ SET \\ \Gamma \vdash p : P_{xe}[t, \mathtt{refl}/x, e]\end{array}}{\Gamma \vdash \mathtt{J}\,(x.\,e.\,P_{xe})\ p\ \mathtt{refl} = p \in P_{xe}[t, \mathtt{refl}/x, e]}\ (\equiv\text{C})$$