Analysis and synthesis of inductive families

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Introduction

From intuitionistic type theory to dependently typed programming

We start with an introduction to intuitionistic type theory [Martin-Löf, 1984] and dependently typed programming [Altenkirch et al., 2005; McBride, 2004] using the Agda language [Norell, 2007, 2009; Bove and Dybjer, 2009]. Intuitionistic type theory was developed by Martin-Löf to serve as a foundation of intuitionistic mathematics like Bishop's renowned work on constructive analysis [Bishop and Bridges, 1985]. While originated from intuitionistic type theory, dependently typed programming is more concerned with mechanisation and practicalities, and is influenced by the program construction movement. It has thus departed from the mathematical traditions considerably, and deviations can be found from syntactic presentations to the underlying philosophy.

 $\mathbf{data}\ \mathsf{Nat}\ :\ \mathsf{Set}\ \mathbf{where}$

zero : Nat

 $\mathsf{suc}\ :\, \mathsf{Nat}\ \to\ \mathsf{Nat}$

4 CHAPTER~2.~~FROM~INTUITIONISTIC~TYPE~THEORY~TO~DEPENDENTLY~TYPED~PROGRAMMING

Ornaments, refinements, and upgrades

Chapter 2 McBride [2011] [McBride, 2011] McBride's work [2011] [Dagand and McBride, 2012] [Mu et al., 2009]

Categorical organisation of ornament-induced refinements

 $8 CHAPTER\ 4.\ CATEGORICAL\ ORGANISATION\ OF\ ORNAMENT-INDUCED\ REFINEMENTS$

Relational algebraic ornaments

Equivalence of ornaments and relational algebras

Conclusion

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