FLOLAC'14

Type theory and logic

Lecture III: Heyting arithmetic

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Equality types

- Formation:

$$\frac{\Gamma \ \vdash \ A : \mathcal{U} \qquad \Gamma \ \vdash \ t \ : \ A \qquad \Gamma \ \vdash \ u \ : \ A}{\Gamma \ \vdash \ t \equiv_A u : \mathcal{U}} \ (\equiv\mathsf{F})$$

  The subscript A, i.e., the type of t and u, is often omitted.

- Introduction:

$$\frac{\Gamma \ \vdash \ t \ : \ A}{\Gamma \ \vdash \ \mathtt{refl} \ : \ t \equiv t} \ (\equiv\mathsf{I})$$

Exercise. Assume $\Gamma \ \vdash \ t = u \ \in A$ and derive $\Gamma \ \vdash \ \mathtt{refl} \ : \ t \equiv u$.

Equality elimination

$$\frac{\begin{array}{l}\Gamma \vdash t : A \\ \Gamma \vdash P : \Pi[x : A]\ t \equiv x \to \mathcal{U} \\ \Gamma \vdash p : P\,t\,\mathtt{refl} \\ \Gamma \vdash u : A \\ \Gamma \vdash q : t \equiv u\end{array}}{\Gamma \vdash \mathtt{J}\,p\,q : P\,u\,q}\ (\equiv\mathrm{E})$$

Exercise. Derive

$$P : A \to \mathcal{U} \vdash \_ : \Pi[x : A]\ \Pi[y : A]\ x \equiv y \to P\,x \to P\,y$$

Equality computation

$$\frac{\begin{array}{l}\Gamma \ \vdash \ t : A \\ \Gamma \ \vdash \ P : \Pi[x:A]\ t \equiv x \to \mathcal{U} \\ \Gamma \ \vdash \ p : P\, t\, \mathtt{refl}\end{array}}{\Gamma \ \vdash \ \mathtt{J}\, p\, \mathtt{refl} = p \ \in \ P\, t\, \mathtt{refl}}\ (\equiv\mathsf{C})$$

Natural numbers

- Formation:

$$\frac{}{\Gamma \ \vdash \ \mathbb{N} : \mathcal{U}} \ (\mathbb{N}\mathsf{F})$$

- Introduction:
- Elimination:
- Computation: