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Filtered Latent Dirichlet Allocation: Variational Algorithm

Introduction

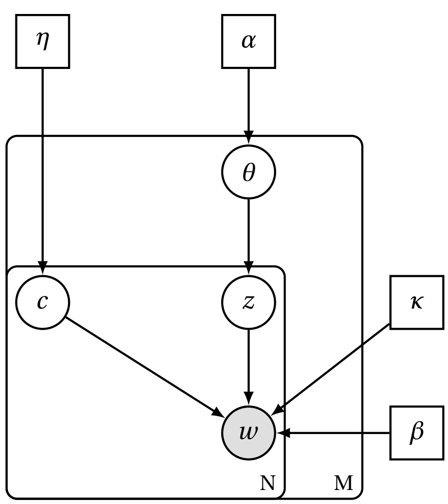
Filtered latent Dirichlet allocation (fLDA) is a natural extension of latent Dirichlet allocation (Blei et al. 2003) which organically captures and filters out corpus-specific stop words that are not otherwise caught by generic stop word lists.

Several variants of the fLDA probabilistic model can be found in the existing literature, however, to our knowledge, no mean-field variational algorithm has ever been explicitly derived.

fLDA attaches β and κ to a Bernoulli switch c, which controls whether a term w is sampled from a topic distribution β_z, or the stop word distribution κ. Those terms with the highest probability in κ are those most likely to be corpus-specific stop words, and their appearance in the topic distributions will be suppressed.

Probabilistic graphical model:

$$\begin{aligned}
\theta_{d=1\ldots M} &\sim \text{Dirichlet}_K(\alpha) \\
c_{d=1\ldots M,n=1\ldots N_d} &\sim \text{Bernoulli}(\eta) \\
z_{d=1\ldots M,n=1\ldots N_d} &\sim \text{Categorical}_K(\theta_d) \\
w_{d=1\ldots M,n=1\ldots N_d} &\sim \text{Categorical}_V((1-c_{d_n})\kappa + c_{d_n}\beta_{z_{d_n}})
\end{aligned}$$



Variational Inference

The marginal log-likelihood for fLDA is,

$$\log p(w|\beta,\kappa,\alpha,\eta) = \sum_{d=1}^{M} \int \sum_{c_d} \sum_{z_d} p(z_d, c_d, \theta_d | w_d, \beta, \kappa, \alpha, \eta) \log \frac{p(w_d, z_d, c_d, \theta_d|\beta,\kappa,\alpha,\eta)}{p(z_d, c_d, \theta_d|w_d,\beta,\kappa,\alpha,\eta)} d\theta_d$$

The mean-field approximation for the distribution over latent variables is,

$$\prod_{d=1}^{M} p(z_d, c_d, \theta_d | w_d, \beta, \kappa, \alpha, \eta) \approx \prod_{d=1}^{M} q(z_d|\phi_d) q(c_d|\tau_d) q(\theta_d|\gamma_d)$$

$$\begin{aligned} q(z_{d_n}|\phi_{d_n}) &= \text{Categorical}(\phi_{d_n}) \\ q(c_{d_n}|\tau_{d_n}) &= \text{Bernoulli}(\tau_{d_n}) \\ q(\theta_d|\gamma_d) &= \text{Dirichlet}(\gamma_d) \end{aligned}$$

Using the KL-divergence and the mean-field approximation, the evidence lower bound for the fLDA marginal log-likelihood, in expected value form, is,

$$\log p(w|\beta,\kappa,\alpha,\eta) \geq \sum_{d=1}^{M} \mathrm{E}_q[\log p(w_d, z_d, c_d, \theta_d|\beta,\kappa,\alpha,\eta)] - \mathrm{E}_q[\log q(z_d, c_d, \theta_d|\phi_d, \tau_d, \gamma_d)]$$

Breaking up the expected values in this lower bound, we obtain,

$$\begin{aligned} \sum_{d=1}^{M} \Big(& \mathrm{E}_q[\log p(w_d|z_d, c_d, \beta, \kappa)] + \mathrm{E}_q[\log p(z_d|\theta_d)] + \mathrm{E}_q[\log p(c_d|\eta)] + \mathrm{E}_q[\log p(\theta_d|\alpha)] \\ & - \mathrm{E}_q[\log q(z_d|\phi_d)] - \mathrm{E}_q[\log q(c_d|\tau_d)] - \mathrm{E}_q[\log q(\theta_d|\gamma_d)] \Big) \end{aligned}$$

In particular,

$$\mathrm{E}_q[\log p(w_d|z_d, c_d, \beta, \kappa)] = \sum_{n=1}^{N_d} \tau_{d_n} \big(-\log \kappa_{w_{d_n}} + \sum_{i=1}^{K} \phi_{d_{in}} \log \beta_{i w_{d_n}} \big) + \log \kappa_{w_{d_n}}$$

$$\mathrm{E}_q[\log p(c_d|\eta)] = N_d \log(1-\eta) + \log\left(\frac{\eta}{1-\eta}\right) \sum_{n=1}^{N_d} \tau_{d_n}$$

$$\mathrm{E}_q[\log q(c_d|\tau_d)] = \sum_{n=1}^{N_d} \tau_{d_n} \log \tau_{d_n} + (1-\tau_{d_n}) \log(1-\tau_{d_n})$$

Calculations for the remaining expectations may be found in (Blei et al. 2003).

The update formulas distinct from those found in (Blei et al. 2003) are as follows,

1. $$\eta \longleftarrow \frac{\sum_{d=1}^{M} \sum_{n=1}^{N_d} \tau_{d_n}}{\sum_{d=1}^{M} N_d}$$

2. $$\kappa_j \underset{\propto}{\longleftarrow} \sum_{d=1}^{M} \sum_{n=1}^{N_d} (1 - \tau_{d_n}) w_{d_n}^{j}$$

3. $$\tau_{d_n} \longleftarrow \frac{\eta}{\eta + (1 - \eta)\kappa_{w_{d_n}} \prod_{i=1}^{K} \beta_{iw_{d_n}}^{-\phi_{d_{in}}}}$$

4. $$\phi_{d_{in}} \underset{\propto}{\longleftarrow} \beta_{iw_{d_n}}^{\tau_{d_n}} \exp\left(\phi(\gamma_i) - \phi(\sum_{l=1}^{K} \gamma_l)\right)$$

5. $$\beta_{ij} \underset{\propto}{\longleftarrow} \sum_{d=1}^{M} \sum_{n=1}^{N_d} \tau_{d_n} \phi_{d_{in}} w_{d_n}^{j}$$

References

1. Blei, David M.; Ng, Andrew Y.; Jordan, Michael I (January 2003). Lafferty, John (ed.). "Latent Dirichlet Allocation". Journal of Machine Learning Research. 3 (45): pp. 9931022. doi:10.1162/jmlr.2003.3.4-5.993. Archived from the original on 2012-05-01. Retrieved 2006-12-19.