Name:

University of Illinois

Spring 2020

CS 446/ECE 449 Machine Learning
Homework 1: Linear Regression

Due on Thursday February 6 2020, noon Central Time
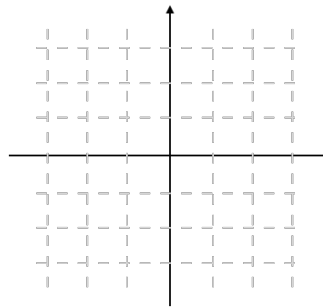
1. [**17 points**] Linear Regression

   We are given a dataset $\mathcal{D} = \{(1,1), (2,1)\}$ containing two pairs (x, y), where each $x \in \mathbb{R}, y \in \mathbb{R}$ denotes a real-valued number.

   We want to find the parameters $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \in \mathbb{R}^2$ of a linear regression model $\hat{y} = w_1 x + w_2$ using

   $$\min_w \frac{1}{2} \sum_{(x,y) \in \mathcal{D}} \left(y - w^\top \begin{bmatrix} x \\ 1 \end{bmatrix} \right)^2. \tag{1}$$

   (a) (2 points) Plot the given dataset and find the optimal w^* by inspection.

   Your answer: **Mark the axis**

   (b) (4 points) Using general matrix vector notation, the program in Eq. (1) is equivalent to

   $$\min_w \frac{1}{2} \|\mathbf{y} - \mathbf{X}w\|_2^2. \tag{2}$$

   Specify the dimensions of the introduced matrix $\mathbf{X}$ and the introduced vector $\mathbf{y}$. Also write down explicitly the matrices and vectors using the values in the given dataset $\mathcal{D}$.

   Your answer:

(c) (3 points) **Derive** the general analytical solution for the program given in Eq. (2). Also plug in the values for the given dataset $\mathcal{D}$ and compute the solution numerically.
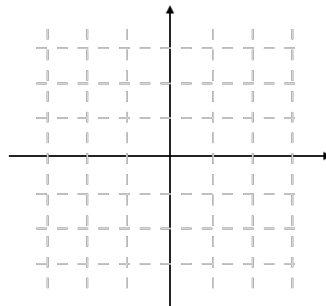
Your answer:

(d) (1 point) Numerous ways exist to compute this solution via PyTorch. Read the docs for the functions 'torch.gels', 'torch.gesv', and 'torch.inverse'. Use all three approaches when completing the file `A1_LinearRegression.py` and verify your answer. Which solution provides the most accurate value for w_1 for our dataset?

Your answer:

(e) (6 points) We are now given a dataset $\mathcal{D} = \{(0,0), (1,1), (2,1)\}$ of pairs (x,y) with $x, y \in \mathbb{R}$ for which we want to fit a quadratic model $\hat{y} = w_1 x^2 + w_2 x + w_3$ using the program given in Eq. (2). Specify the dimensions of the matrix $\mathbf{X}$ and the vector $\mathbf{y}$. Also write down explicitly the matrix and vector using the values in the given dataset. Find the optimal solution w^* and draw it together with the dataset into a plot.

Your answer: **Mark the axis**

(f) (1 points) Complete `A2_LinearRegression2.py` and verify your reply for the previous answer. How did you specify the matrix $\mathbf{X}$?

Your answer: