ECE4122 Assignment 1

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1. Question 1

$x = 26935449\ Hz, A = 26\frac{V}{m}$

i. For a non-magnetic lossless material, the Brewster is given as:

$$\sin(\theta_{BA}) = \frac{1}{\sqrt{1+\frac{\varepsilon_1}{\varepsilon_2}}}$$

Thus:

$$\theta_{BA} = \sin^{-1}\left(\frac{1}{\sqrt{1+\frac{(1)}{(20)}}}\right)$$

$$\theta_{BA} = 77.40°$$

ii. Phase constants:

$$\beta_1 = \frac{\omega}{c} = \frac{26935449 * 2\pi}{3 * 10^8} = 0.56\ rad/m$$

$$\beta_2 = \frac{\omega\sqrt{\varepsilon}}{c} = \frac{26935449 * 2\pi}{3 * 10^8} * \sqrt{20} = 2.50\ rad/m$$

iii. Intrinsic Wave Impedances:

$$\eta_1 = \sqrt{\frac{\mu_0\mu_1}{\varepsilon_0\varepsilon_1}} = \sqrt{\frac{1.257 * 10^{-6}}{8.85 * 10^{-12}}} = 376.87\ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_0\mu_2}{\varepsilon_0\varepsilon_2}} = \frac{376.87}{\sqrt{20}} = 84.27\ \Omega$$

iv. TE polarized reflection coefficient. At Brewster's angle of incidence, we can determine the angle of transmission using Snell's law:

$$\theta_t = \sin^{-1}\left(\frac{\beta_1}{\beta_2}\sin(\theta_i)\right) = \sin^{-1}\left(\frac{0.56}{2.50}\sin(77.40)\right) = 12.62°$$

$$\Gamma_{TE} = \frac{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}{\eta_2\cos(\theta_i) - \eta_1\cos(\theta_t)}$$

$$= \frac{84.27\cos(77.40) - 376.87\cos(12.62)}{84.27\cos(77.40) + 376.87\cos(12.62)} = -0.90$$

Also calculating transmission coefficient:

$$\tau_{TE} = 1 + \Gamma_{TE} = 0.1$$

v. Time Domain E fields for TE polarization:

$$E^i = 26e^{-j\beta_1(xsin\theta_i + zcos\theta_i)}$$

$$= 26e^{-j0.55x}e^{-j0.12z}a_y\frac{V}{m}$$

$$E^r = \Gamma_{TE} * E^i$$

$$= -23.4e^{-j0.55x}e^{j0.12z}a_y\frac{V}{M}$$

$$E^t = \tau_{TE} * 26e^{-j\beta_2(xsin\theta_t+zcos\theta_t)}a_y$$
$$= 2.6e^{-j0.55x}e^{-j2.44z}a_y\frac{V}{M}$$

vi. TM reflection coefficient:
Since this is at the Brewster angle, $\Gamma_{TM} = 0$.
For transmission coefficient:

$$\tau_{TM} = \frac{\cos(\theta_i)}{\cos(\theta_t)} = 0.22$$

vii. Time Domain E fields for TM polarization:

$$E^i = 26e^{-j0.55x}e^{-j0.12z}(\cos(\theta_i)\,a_x - \sin(\theta_i)\,a_z)$$
$$= (5.67a_x - 25.37a_z)e^{-j0.55x}e^{-j0.12z}\frac{V}{m}$$

$$E^r = 0$$

$$E^t = 0.22 * 26e^{-j\beta_2(xsin\theta_t+zcos\theta_t)}(\cos(\theta_t)\,a_x - \sin(\theta_t)\,a_z)$$
$$= 0.22 * (25.37a_x - 5.68a_z)e^{-j0.55x}e^{-j2.44z}$$
$$= (5.58a_x - 1.25a_z)e^{-j0.55x}e^{-j2.44z}\frac{V}{M}$$

viii. Combining:

$$E^i(z,t) = \left(5.67a_x + 26a_y - 25.37a_z\right)\cos(\omega t - 0.55x - 0.12z)\frac{V}{m}$$
$$E^r(z,t) = -23.4\cos(\omega t - 0.55x + 0.12z)\,a_y\frac{V}{m}$$
$$E^t(z,t) = \left(5.58a_x + 2.6a_y - 1.25a_z\right)\cos(\omega t - 0.55x - 2.44z)\frac{V}{m}$$

2. Question 2

i.

$$Z_A = 134.5 + 179.3j$$
$$Z_0 = 50$$
$$\Gamma = \frac{Z_A + Z_0}{Z_A - Z_0}$$

ii. Using the Smith Chart:

Normalized load impedance:

$$Z_{norm} = \frac{Z_L}{Z_0} = \frac{134.5 + 179.3j}{50} = 2.69 + 3.59j$$

Translating through the center of Smith Chart:

$$y_L = 0.16 + 0.19j$$

Using a compass to draw a circle from y_L until you hit the unit circle:

$$y_A = 1 + 2.4j$$

WTG distance:

$$y_L = 0.47\lambda$$
$$y_A = 0.195\lambda$$
$$d = (0.5 - 0.47) + 0.195 = 0.225\lambda$$

From the frequency and permittivity, the wavelength is:

$$\lambda = \frac{c}{f\sqrt{\varepsilon}} = \frac{3 * 10^8}{2.2 * 10^9 * \sqrt{4.2}} = 0.0665m$$

Hence the through line length is

$$0.167 * 0.0665 = 0.0149m$$
$$= 15\ mm$$

iii.

$$y_A = 1 + 2.4j$$
$$y_S = -jx = -2.4j$$
$$WTG = 0.187\lambda$$
$$l = 0.5 - 0.187 = 0.313\lambda$$
$$= 0.0208m$$
$$= 20.81mm$$