# HW7

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# 1 HW7

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```
In [1]: import pandas as pd
        import numpy as np
        import matplotlib.pyplot as plt
        import statsmodels.api as sm
        from sklearn.model_selection import train_test_split
        from sklearn.metrics import classification_report, confusion_matrix
        from sklearn import preprocessing
        from sklearn.linear_model import LogisticRegression
        from sklearn.linear_model import LinearRegression
        from sklearn.model_selection import train_test_split, LeaveOneOut, KFold, cross_val_sco
        from sklearn import metrics
        from sklearn.metrics import classification_report, mean_squared_error
        from scipy.interpolate import LSQUnivariateSpline
```

# 3 Question1: Multinomial logistic regression and cross validation

### 3.0.1 (a) Use a multinomial logistic regression model of the following form with the following linear predictor

(1) Estimate the model on a 75% sample training set using the following command. (2) Report your two sets of estimated coecients and intercepts forj = 1 and j = 2 (not the coecients for j = 3). Report your error rates (1 - precision) on the test set using the code below. (3) Which category(ies) of cultivar is the model best at predicting? (4) Is (are) the most accurately predicted category(ies) the one(s) with the most observations? (5) Report the MSE from the test set.

```
In [2]: # read in data
        df=pd.read_csv("data/strongdrink.txt")
        df.describe()

Out[2]:          cultivar        alco       malic         ash         alk        magn  \
        count  176.000000  176.000000  176.000000  176.000000  176.000000  176.000000
        mean     1.926136   13.006534    2.327159    2.367386   19.492045   99.840909
        std      0.771047    0.814431    1.117747    0.275617    3.355821   14.329499
```

```
min       1.000000   11.030000    0.740000    1.360000   10.600000   70.000000
25%       1.000000   12.362500    1.597500    2.210000   17.175000   88.000000
50%       2.000000   13.050000    1.845000    2.360000   19.500000   98.000000
75%       3.000000   13.682500    3.047500    2.560000   21.500000  107.250000
max       3.000000   14.830000    5.800000    3.230000   30.000000  162.000000

         tot_phen        flav  nonfl_phen     proanth   color_int         hue  \
count  176.000000  176.000000  176.000000  176.000000  176.000000  176.000000
mean     2.298920    2.043352    0.359545    1.597727    5.031761    0.961000
std      0.627333    0.995579    0.123046    0.571958    2.317965    0.227225
min      0.980000    0.340000    0.130000    0.410000    1.280000    0.480000
25%      1.747500    1.242500    0.267500    1.250000    3.200000    0.790000
50%      2.380000    2.155000    0.340000    1.560000    4.640000    0.975000
75%      2.800000    2.882500    0.430000    1.952500    6.147500    1.120000
max      3.880000    5.080000    0.660000    3.580000   13.000000    1.710000

          OD280rat      proline
count  176.000000   176.000000
mean     2.623409   748.477273
std      0.705369   316.208737
min      1.270000   278.000000
25%      1.990000   500.000000
50%      2.780000   673.500000
75%      3.172500   986.250000
max      4.000000  1680.000000
```

In [3]:
```
df.cultivar.value_counts()
```

Out[3]:
```
2    71
1    59
3    46
Name: cultivar, dtype: int64
```

In [4]:
```
# Training and testing sets
y=df["cultivar"]
X=df[["alco","malic","tot_phen","color_int"]]
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25,random_state
y_test.count(), y_train.count()
```

Out[4]:
```
(44, 132)
```

## 4 Estimate and Report the results & error rates

In [5]:
```
Mlogit = LogisticRegression(solver='newton-cg',multi_class='multinomial')
result = Mlogit.fit(X_train, y_train)
pd.DataFrame({"j = 1":np.append(result.intercept_[0],result.coef_[0]),
              "j = 2":np.append(result.intercept_[1],result.coef_[1])},
               index=["beta0","beta1",'beta2','beta3','beta4'])
```

```
Out[5]:              j = 1      j = 2
        beta0 -24.010989  22.802446
        beta1   1.700403  -1.468044
        beta2  -0.265605  -0.333053
        beta3   1.223894   0.664012
        beta4   0.022756  -0.922712
```

```
In [6]: y_pred=result.predict(X_test)
        confusion_matrix = confusion_matrix(y_test, y_pred)
        print(confusion_matrix)
```

```
[[13  0  0]
 [ 2 19  0]
 [ 0  0 10]]
```

```
In [7]: print(classification_report(y_test, y_pred, digits=3))
        print('The error rate (class 1) is', 1 - 0.867,
              '\nThe error rate (class 2) is', 1 - 1,
              '\nThe error rate (class 3) is', 1 - 1)
```

```
              precision    recall  f1-score   support

           1      0.867     1.000     0.929        13
           2      1.000     0.905     0.950        21
           3      1.000     1.000     1.000        10

   micro avg      0.955     0.955     0.955        44
   macro avg      0.956     0.968     0.960        44
weighted avg      0.961     0.955     0.955        44

The error rate (class 1) is 0.133
The error rate (class 2) is 0
The error rate (class 3) is 0
```

The model is best at predicting third group with the highest f1 score And the most accurately predicted category is not the one with the most observations. However, the results here are not robust. When we try different random states, we can find that the results are not always robust. In this case, it is not sutible to make a conclusion since the test set is small.

```
In [8]: print('The test MSE is', (y_test != y_pred).astype(int).mean())
```

```
The test MSE is 0.045454545454545456
```

## 5 (b) Perform a leave-one-out cross validation (LOOCV)

1. Report your error rates (1 - precision) for each category?

2. How do your error rates compare to those from part (a)?
3. Report your LOOCV estimate for the test MSE as the average MSE, where yi is the left out-observation from each test set.

```
In [9]: Xvals = df[['alco', 'malic', 'tot_phen', 'color_int']].values
        yvals = df['cultivar'].values
        N = Xvals.shape[0]
        loo = LeaveOneOut()
        loo.get_n_splits(Xvals)
        MSE = pd.DataFrame({'index': np.zeros(N),
                            'error': np.zeros(N),
                            'y_pred': np.zeros(N),
                            'y_test': np.zeros(N)})

        for train_index, test_index in loo.split(Xvals):
            X_train, X_test = Xvals[train_index], Xvals[test_index]
            y_train, y_test = yvals[train_index], yvals[test_index]
            Logit = LogisticRegression(multi_class='multinomial', solver='newton-cg')
            Logit.fit(X_train, y_train)
            y_pred = Logit.predict(X_test)
            MSE['index'][test_index] = test_index
            MSE['error'][test_index] = (y_test != y_pred).astype(int)
            MSE['y_pred'][test_index] = y_pred
            MSE['y_test'][test_index] = y_test

        error_1 = MSE[MSE['y_pred'] == 1]['error'].mean()
        error_2 = MSE[MSE['y_pred'] == 2]['error'].mean()
        error_3 = MSE[MSE['y_pred'] == 3]['error'].mean()
        error_MSE = MSE['error'].mean()
```

```
In [10]: print(classification_report(MSE['y_test'], MSE['y_pred'], digits=3))
         print('')
         print('The error rate for class 1 is', error_1, ', std is', MSE[MSE['y_pred'] == 1]['
         print('The error rate for class 2 is', error_2, ', std is', MSE[MSE['y_pred'] == 2]['
         print('The error rate for class 3 is', error_3, ', std is', MSE[MSE['y_pred'] == 3]['
         print('The average MSE (for all classes) is', error_MSE,', std is', MSE['error'].value
```

```
             precision    recall  f1-score   support

        1.0      0.902     0.932     0.917        59
        2.0      0.914     0.901     0.908        71
        3.0      0.956     0.935     0.945        46

  micro avg      0.920     0.920     0.920       176
  macro avg      0.924     0.923     0.923       176
weighted avg     0.921     0.920     0.921       176
```

```
The error rate for class 1 is 0.09836065573770492 , std is 0.29780167417352377
The error rate for class 2 is 0.08571428571428572 , std is 0.27994168488950605
The error rate for class 3 is 0.044444444444444446 , std is 0.20608041101101562
The average MSE (for all classes) is 0.07954545454545454 , std is 0.2705882022679695
```

(a) The error rate (class 1) is 0.133 The error rate (class 2) is 0 The error rate (class 3) is 0 The test MSE is 0.045

(b) The error rate for class 1 is 0.098 The error rate for class 2 is 0.085 The error rate for class 3 is 0.044 The average MSE (for all classes) is 0.079

WE can see that error in (a) is smaller than error in (b) except class 1. Average MSE of (a) is also smaller compared to (b)

```
In [11]: MSE.iloc[50:70]
```

Out[11]:

| | index | error | y_pred | y_test |
|---|---|---|---|---|
| 50 | 50.0 | 0.0 | 1.0 | 1.0 |
| 51 | 51.0 | 0.0 | 1.0 | 1.0 |
| 52 | 52.0 | 0.0 | 1.0 | 1.0 |
| 53 | 53.0 | 0.0 | 1.0 | 1.0 |
| 54 | 54.0 | 0.0 | 1.0 | 1.0 |
| 55 | 55.0 | 0.0 | 1.0 | 1.0 |
| 56 | 56.0 | 0.0 | 1.0 | 1.0 |
| 57 | 57.0 | 0.0 | 1.0 | 1.0 |
| 58 | 58.0 | 0.0 | 1.0 | 1.0 |
| 59 | 59.0 | 0.0 | 2.0 | 2.0 |
| 60 | 60.0 | 0.0 | 2.0 | 2.0 |
| 61 | 61.0 | 1.0 | 3.0 | 2.0 |
| 62 | 62.0 | 1.0 | 1.0 | 2.0 |
| 63 | 63.0 | 0.0 | 2.0 | 2.0 |
| 64 | 64.0 | 0.0 | 2.0 | 2.0 |
| 65 | 65.0 | 0.0 | 2.0 | 2.0 |
| 66 | 66.0 | 1.0 | 1.0 | 2.0 |
| 67 | 67.0 | 0.0 | 2.0 | 2.0 |
| 68 | 68.0 | 0.0 | 2.0 | 2.0 |
| 69 | 69.0 | 0.0 | 2.0 | 2.0 |

```
In [12]: # Double check with model_selection.cross_val_score
         resloo=cross_val_score(Mlogit, X, y, cv=loo)
         print("LOOCV MSE: %.4f (%.2f%%)" % (1-resloo.mean(), resloo.std()*100.0))
```

```
LOOCV MSE: 0.0795 (27.06%)
```

```
In [13]: # Check whether two are same
         (1-resloo)-MSE.error.values
```

```
Out[13]: array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
                0., 0., 0., 0., 0., 0.])
```

# 6 (c) Perform a k-fold cross validation in which the data are divided into k = 4 groups.

1. Report your error rates (1 - precision) for each category.
2. How do your error rates compare to those from parts (a) and (b)?
3. Report your k-fold estimate for the test MSE as the average MSE.

```
In [15]: k = 4
         KF = KFold(n_splits=k, random_state=10, shuffle=True)
         KF.get_n_splits(Xvals)
         General_error_1 = np.zeros(k)
         General_error_2 = np.zeros(k)
         General_error_3 = np.zeros(k)
         General_MSE = np.zeros(k)

         k_ind = int(0)
         for train_index, test_index in KF.split(Xvals):
             print('When k index=', k_ind)
             X_train, X_test = Xvals[train_index], Xvals[test_index]
             y_train, y_test = yvals[train_index], yvals[test_index]
             Loggit = LogisticRegression(multi_class='multinomial',
                                         solver='newton-cg')
             Logit.fit(X_train, y_train)
             y_pred = Logit.predict(X_test)
             MSE = pd.DataFrame({'error': (y_test != y_pred).astype(int),'y_pred': y_pred})
             error_1 = MSE[MSE['y_pred'] == 1]['error'].mean()
             error_2 = MSE[MSE['y_pred'] == 2]['error'].mean()
             error_3 = MSE[MSE['y_pred'] == 3]['error'].mean()
             error_all_class = MSE['error'].mean()
             General_error_1[k_ind] = error_1
             General_error_2[k_ind] = error_2
             General_error_3[k_ind] = error_3
             General_MSE[k_ind] = error_all_class
             print('\n',classification_report(y_test, y_pred, digits=3))
             print('error rate (class 1) is', error_1)
```

```
        print('error rate (class 2) is', error_2)
        print('error rate (class 3) is', error_3)
        print('MSE (for all classes) is', error_all_class)
        print('The stds are respectively \n', MSE[MSE['y_pred'] == 1]['error'].values.std
              ',\n', MSE[MSE['y_pred'] == 2]['error'].values.std(),
              ',\n', MSE[MSE['y_pred'] == 3]['error'].values.std(),
              ',\n', MSE['error'].values.std(),'\n')
        k_ind += 1

    print('\nk-fold resuts:')
    print('The average error rate (class 1) is', General_error_1.mean(), ', std is', Gene
    print('The average error rate (class 2) is', General_error_2.mean(), ', std is', Gene
    print('The average error rate (class 3) is', General_error_3.mean(), ', std is', Gene
    print('The average MSE (for all classes) is', General_MSE.mean(), ', std is', General
```

```
When k index= 0

              precision    recall  f1-score   support

           1      0.706     1.000     0.828        12
           2      1.000     0.750     0.857        24
           3      0.778     0.875     0.824         8

   micro avg      0.841     0.841     0.841        44
   macro avg      0.828     0.875     0.836        44
weighted avg      0.879     0.841     0.843        44

error rate (class 1) is 0.29411764705882354
error rate (class 2) is 0.0
error rate (class 3) is 0.2222222222222222
MSE (for all classes) is 0.1590909090909091
The stds are respectively
 0.4556450995538137 ,
 0.0 ,
 0.41573970964154905 ,
 0.36576083953252464

When k index= 1

              precision    recall  f1-score   support

           1      1.000     0.765     0.867        17
           2      0.688     1.000     0.815        11
           3      1.000     0.938     0.968        16

   micro avg      0.886     0.886     0.886        44
   macro avg      0.896     0.901     0.883        44
weighted avg      0.922     0.886     0.890        44
```

```
error rate (class 1) is 0.0
error rate (class 2) is 0.3125
error rate (class 3) is 0.0
MSE (for all classes) is 0.11363636363636363
The stds are respectively
 0.0 ,
 0.46351240544347894 ,
 0.0 ,
 0.31736909190383955

When k index= 2

              precision    recall  f1-score   support

           1      0.938     1.000     0.968        15
           2      0.938     0.938     0.938        16
           3      1.000     0.923     0.960        13

   micro avg      0.955     0.955     0.955        44
   macro avg      0.958     0.954     0.955        44
weighted avg      0.956     0.955     0.954        44

error rate (class 1) is 0.0625
error rate (class 2) is 0.0625
error rate (class 3) is 0.0
MSE (for all classes) is 0.045454545454545456
The stds are respectively
 0.24206145913796356 ,
 0.24206145913796356 ,
 0.0 ,
 0.20829889522526548

When k index= 3

              precision    recall  f1-score   support

           1      0.882     1.000     0.938        15
           2      1.000     0.900     0.947        20
           3      1.000     1.000     1.000         9

   micro avg      0.955     0.955     0.955        44
   macro avg      0.961     0.967     0.962        44
weighted avg      0.960     0.955     0.955        44

error rate (class 1) is 0.11764705882352941
error rate (class 2) is 0.0
error rate (class 3) is 0.0
```

```
MSE (for all classes) is 0.045454545454545456
The stds are respectively
 0.32218973970892123 ,
 0.0 ,
 0.0 ,
 0.20829889522526548


k-fold resuts:
The average error rate (class 1) is 0.11856617647058823 , std is 0.10956792179947901
The average error rate (class 2) is 0.09375 , std is 0.1288470508005519
The average error rate (class 3) is 0.05555555555555555 , std is 0.09622504486493763
The average MSE (for all classes) is 0.09090909090909091 , std is 0.04821182598999188
```

In [17]:
```python
# Double check with model_selection.cross_val_score
reskf=cross_val_score(Mlogit, X, y, cv=KF)
print("4-fold MSE: ", (1-reskf.mean(), reskf.std()))
```

```
4-fold MSE:  (0.09090909090909094, 0.04821182598999189)
```

# 7 Question 2 - Splines and interpolation

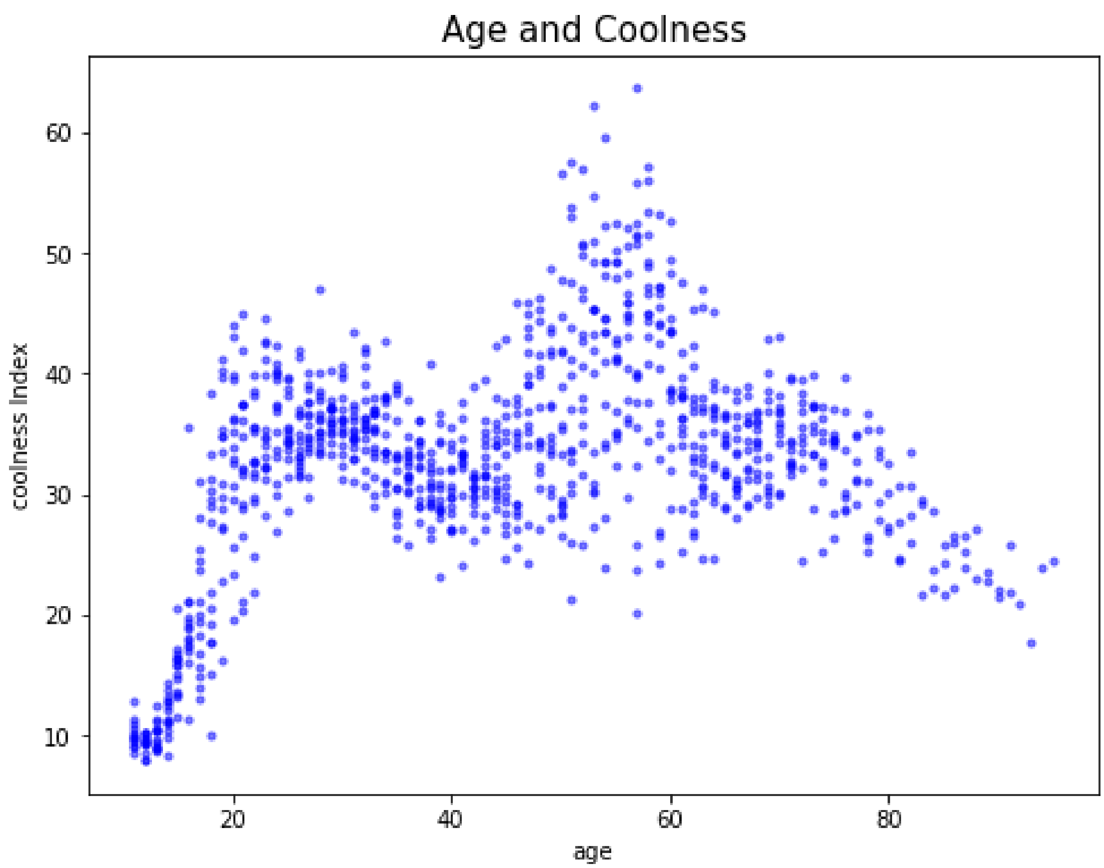
# 8 (a) Create a scatterplot of the data with age on the x-axis and Coolness Index on the y-axis. Label your axes, and give the plot a title.

In [18]:
```python
df=pd.read_csv("data/CoolIndex.txt",header = None)
df.columns = ['age', 'coolness']
df.head()
```

Out[18]:

| | age | coolness |
|---|---|---|
| 0 | 11.0 | 10.981602 |
| 1 | 11.0 | 11.364925 |
| 2 | 11.0 | 10.190227 |
| 3 | 11.0 | 9.903725 |
| 4 | 11.0 | 8.997918 |

In [19]:
```python
fig = plt.figure(figsize = (8,6))
plt.scatter(df.age,df.coolness,s=10,c='blue',alpha=0.5)
plt.xlabel("age")
plt.ylabel("coolness Index")
plt.title('Age and Coolness', fontsize = 15)
plt.show()
```




## 9 (b) Use ordinary least squares (OLS) regression to t a stepwise function

Use 5 bins [11; 22), [22; 40), [40; 59), [59; 77), [77; 95]. 1. Plot this step function on top of the scatterplot of the data from part (a). 2. Report your estimated step function values for each bin. 3. What is the predicted coolness of a 73-year old from the stepwise function?

```
In [20]: df['G1'] = ['1' if (x>=11) & (x<22) else '0' for x in df['age']]
         df['G2'] = ['1' if (x>=22) & (x<40) else '0' for x in df['age']]
         df['G3'] = ['1' if (x>=40) & (x<59) else '0' for x in df['age']]
         df['G4'] = ['1' if (x>=59) & (x<77) else '0' for x in df['age']]
         df['G5'] = ['1' if (x>=77) & (x<=95) else '0' for x in df['age']]
         df.head(), df.shape
```

```
Out[20]: (     age   coolness G1  G2  G3  G4  G5
          0  11.0  10.981602  1   0   0   0   0
          1  11.0  11.364925  1   0   0   0   0
          2  11.0  10.190227  1   0   0   0   0
          3  11.0   9.903725  1   0   0   0   0
          4  11.0   8.997918  1   0   0   0   0, (956, 7))
```

```
In [21]: #X=df.iloc[:,2:7]
         X=df[["G1","G2","G3","G4","G5"]]
         X.head()
```

```
Out[21]:    G1  G2  G3  G4  G5
         0   1   0   0   0   0
         1   1   0   0   0   0
         2   1   0   0   0   0
         3   1   0   0   0   0
         4   1   0   0   0   0
```

```
In [22]: reg1 = sm.OLS(endog=df.coolness, exog=X.astype(float), missing='drop') # .astype(floa
         reg_results = reg1.fit()
         print(reg_results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:               coolness   R-squared:                       0.429
Model:                            OLS   Adj. R-squared:                  0.427
Method:                 Least Squares   F-statistic:                     178.7
Date:                Tue, 26 Feb 2019   Prob (F-statistic):          3.73e-114
Time:                        21:16:13   Log-Likelihood:                -3214.5
No. Observations:                 956   AIC:                             6439.
Df Residuals:                     951   BIC:                             6463.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
G1            20.1025      0.562     35.746      0.000      18.999      21.206
G2            34.4758      0.431     80.006      0.000      33.630      35.321
G3            37.6351      0.424     88.814      0.000      36.804      38.467
G4            35.2254      0.485     72.560      0.000      34.273      36.178
G5            27.2964      0.936     29.175      0.000      25.460      29.132
==============================================================================
Omnibus:                       80.102   Durbin-Watson:                   1.236
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              101.718
Skew:                           0.714   Prob(JB):                     8.17e-23
Kurtosis:                       3.719   Cond. No.                         2.21
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
In [23]: b1, b2, b3, b4, b5 = reg_results.params
         print('b1 =', b1, '\nb2 =', b2,'\nb3 =', b3, '\nb4 =', b4,'\nb5 =', b5)
```

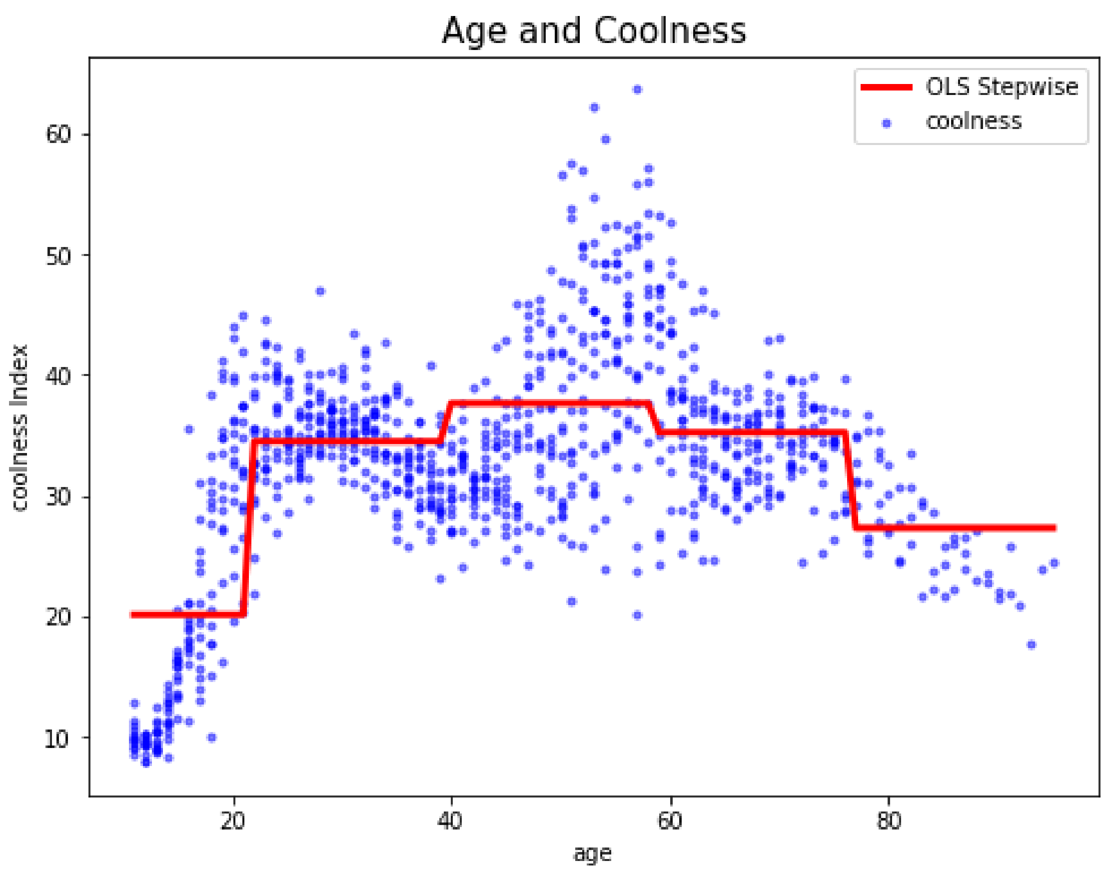
```
b1 = 20.102457252090748
b2 = 34.47578807755938
```

```
b3 = 37.63510549244961
b4 = 35.22540004024275
b5 = 27.296378244321282
```

```
In [24]: print('Predicted Coolness of 73-year old is', reg_results.params[3])
         print('Predicted Coolness of 73-year old is', reg_results.predict([0,0,0,1,0]))
```

```
Predicted Coolness of 73-year old is 35.22540004024275
Predicted Coolness of 73-year old is [35.22540004]
```

```
In [25]: fig = plt.figure(figsize = (8,6))
         plt.scatter(df.age,df.coolness,s=10,c='blue',alpha=0.5)
         plt.plot(df.age,reg_results.predict(),c="red",label = "OLS Stepwise",linewidth='3')
         plt.legend()
         plt.xlabel("age")
         plt.ylabel("coolness Index")
         plt.title('Age and Coolness', fontsize = 15)
         plt.show()
```



## 10 (c) Fit a linear spline to the data over the 5 age bins from part (b).

Plot your continuous linear spline against a scatterplot of the data from part (a) and the estimated step function from part (b). Label your axes, include a legend, and give the plot a title. What is the predicted coolness of a 73-year old from the linear spline?

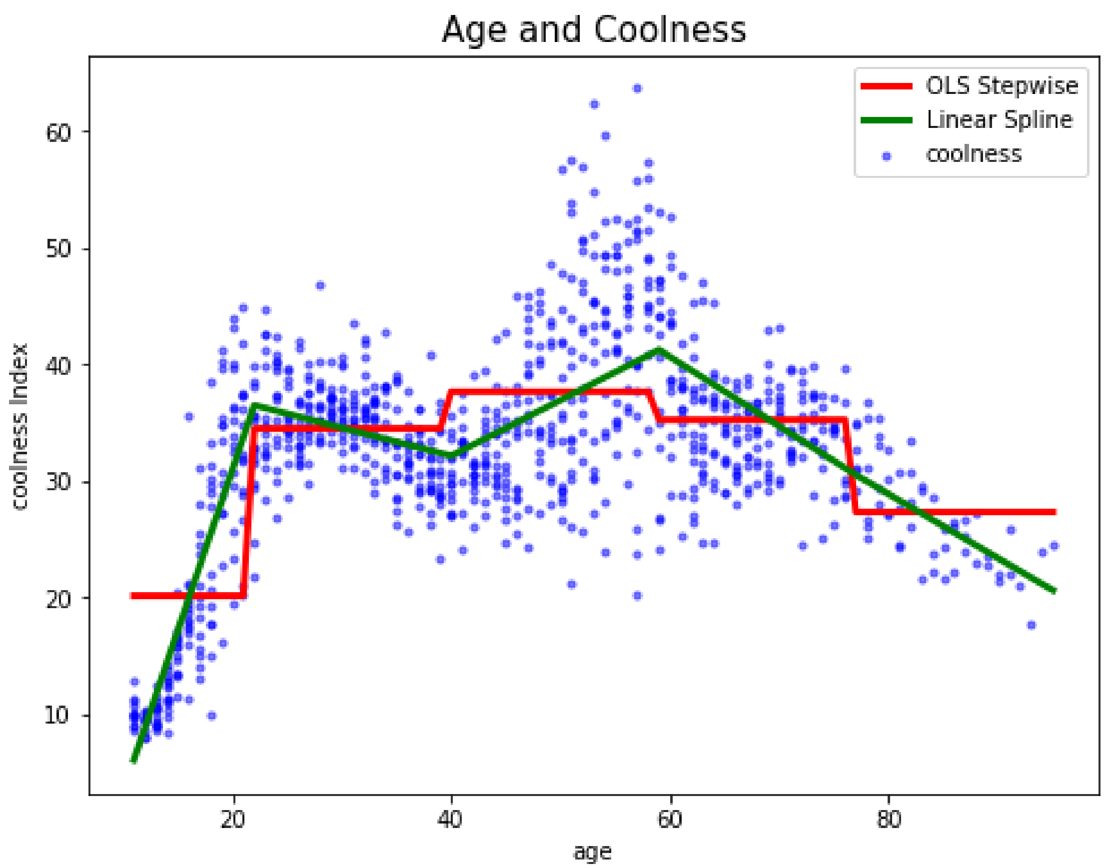
In [26]:
```
#Group data by age as LSQUnivariateSpline requires x to be strictly increasing
df2=df.groupby('age').mean()
df2['age']=df2.index
df2.head()
```

Out[26]:

| age | coolness | age |
|---|---|---|
| 11.0 | 10.110237 | 11.0 |
| 12.0 | 9.365623 | 12.0 |
| 13.0 | 10.015882 | 13.0 |
| 14.0 | 11.747109 | 14.0 |
| 15.0 | 15.434739 | 15.0 |

In [27]:
```
knots=[22,40,59,77]
spline1=LSQUnivariateSpline(df2.age.values, df2.coolness.values, t=knots, k=1)
```

In [28]:
```
fig = plt.figure(figsize = (8,6))
plt.scatter(df.age,df.coolness,s=10,c='blue',alpha=0.5)
plt.plot(df.age,reg_results.predict(),c="red",label = "OLS Stepwise",linewidth = 3)
plt.plot(df.age,spline1(df.age),"green",label = "Linear Spline",linewidth = 3)
plt.legend()
plt.xlabel("age")
plt.ylabel("coolness Index")
plt.title('Age and Coolness', fontsize = 15)
plt.show()
```



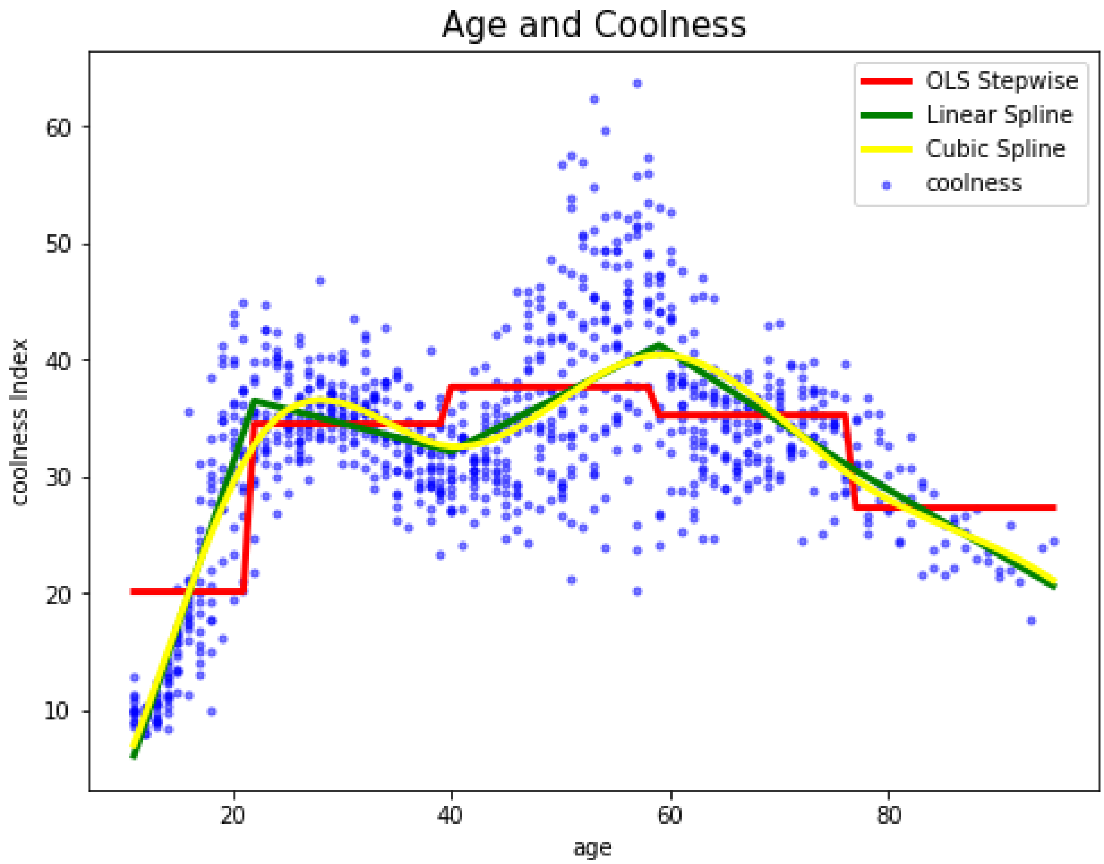

```
In [29]: print('Predicted Coolness of 73-year old is', spline1(73))

Predicted Coolness of 73-year old is 32.86784862349653
```

## 11 (d) Fit a cubic spline (continuous) to the data over the 5 age bins

```
In [30]: spline2=LSQUnivariateSpline(df2.age.values, df2.coolness.values, t=knots, k=3)

In [31]: fig = plt.figure(figsize = (8,6))
         plt.scatter(df.age,df.coolness,s=10,c='blue',alpha=0.5)
         plt.plot(df.age,reg_results.predict(),c="red",label = "OLS Stepwise",linewidth = 3)
         plt.plot(df.age,spline1(df.age),c="green",label = "Linear Spline",linewidth = 3)
         plt.plot(df.age,spline2(df.age),c="yellow",label ="Cubic Spline",linewidth=3)
         plt.legend()
         plt.xlabel("age")
         plt.ylabel("coolness Index")
         plt.title('Age and Coolness', fontsize = 15)
         plt.show()
```




```
In [32]: print('Predicted Coolness of 73-year old is', spline2(73))

Predicted Coolness of 73-year old is 32.642301066279764


In [ ]:

In [ ]:

In [ ]:
```