

Project 4: Perturbed Harmonic Oscillators

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Hamiltonian

$$H = \frac{1}{2} (p_1^2 + p_2^2 + x_1^2 + x_2^2) + x_1^4 + 2Cx_1^2x_2^2 + x_2^4$$

$$\dot{x}_1 = \frac{\partial H}{\partial p_1} = p_1 \quad \dot{p}_1 = -\frac{\partial H}{\partial x_1} = -x_1 - 4x_1^3 - 4Cx_1^2x_2^2$$

$$\dot{x}_2 = \frac{\partial H}{\partial p_2} = p_2 \quad \dot{p}_2 = -\frac{\partial H}{\partial x_2} = -x_2 - 4x_2^3 - 4Cx_1x_2^2$$

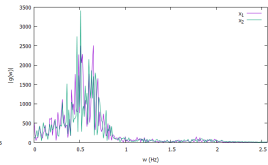
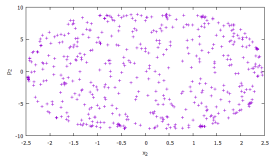
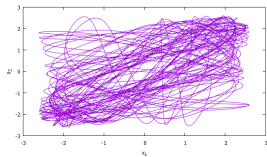
Parameters

- ▶ Initial conditions:

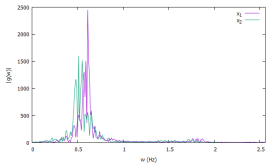
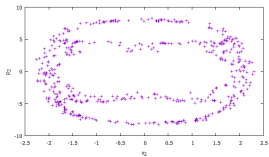
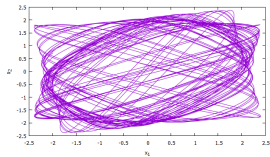
$$x_1 = 0.0 \quad x_2 = 0.0 \quad p_1 = 4.0 \quad p_2 = 8.0$$

- ▶ Initial Energy: $H = 40$
- ▶ Warm-up: 1000 time steps
- ▶ Phase space and FFT data points: 4096 points sampled over 100 time steps
- ▶ Poincare section: 500 total points sampled, one every period ($x_1 = 0$)

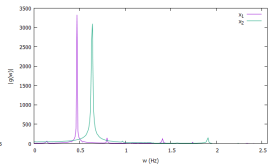
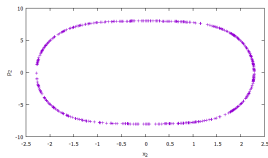
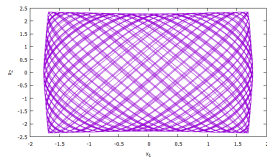
$C=-0.5$



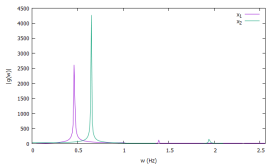
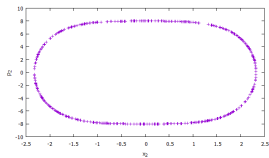
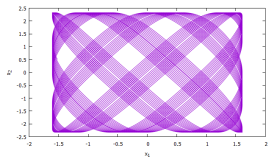
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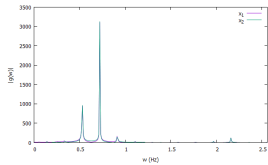
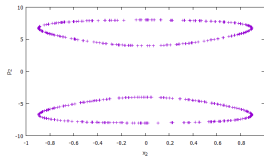
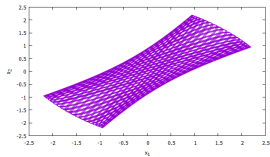
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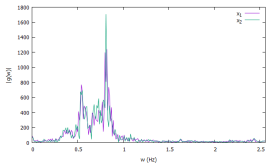
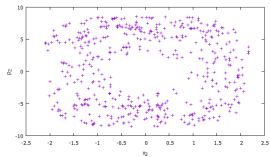
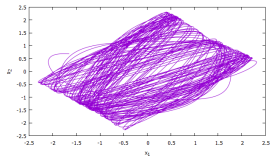
$C=0.0$



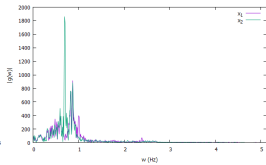
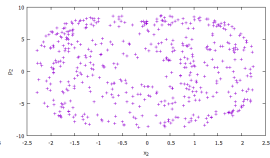
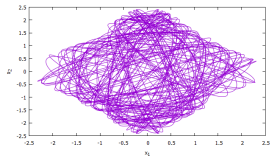
$C=1.5$



$C=4.5$



$C=5.2$



Thank You