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# Optimisation multi-objectifs appliquée aux liners acoustiques

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Workshop "design" de matériaux absorbants acoustiques aéronautiques
ONERA, 14 juin 2021

# Reducing aircraft noise

france bleu

Toulouse Changer

Infos Sports Culture Vie quotidienne

Transports

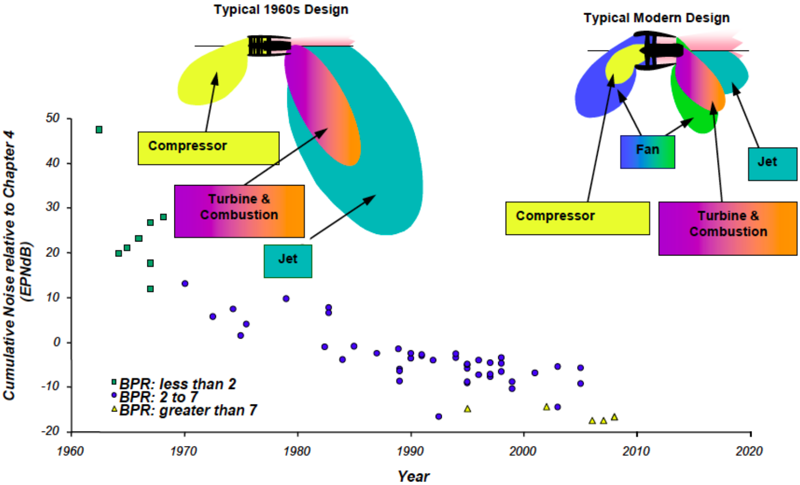
## Aéroport Toulouse-Blagnac : les plaintes pour nuisances sonores se multiplient

Lundi 24 février 2020 à 4:46 - Mis à jour le lundi 24 février 2020 à 11:13 - Par Alexandre Berthaud, Clémence Fulleda, France Bleu Occitanie, France Bleu

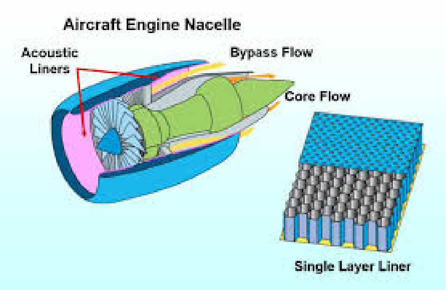
Toulouse

**En à peine deux mois, le CCNAAT (collectif contre les nuisances aériennes dans l'agglomération toulousaine) a recensé près de 5.000 plaintes de riverains sur son site internet. Cette forte augmentation serait liée à la mise en place de nouveaux couloirs aériens à Toulouse-Blagnac.**

# Reducing aircraft noise



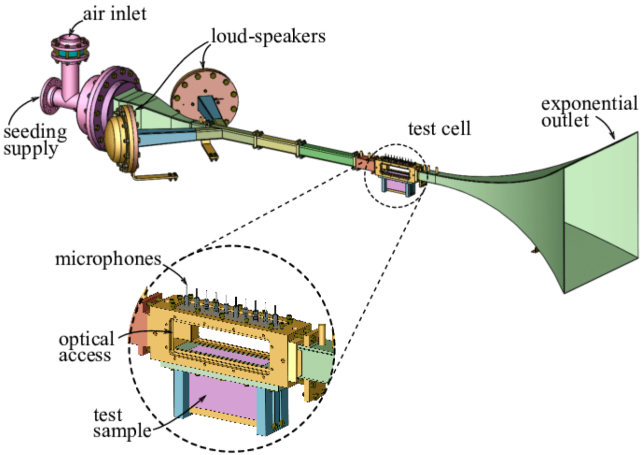
# Reducing aircraft noise


*Use of liners in nacelle aircraft engine to reduce fan, turbine and combustion noises*

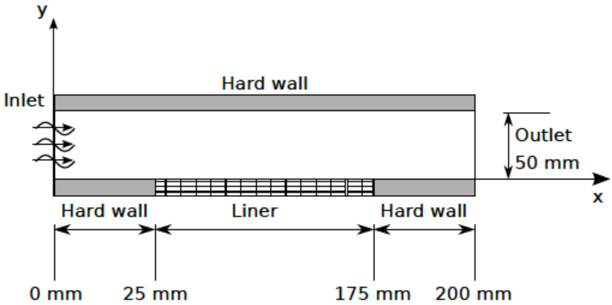
ACARE, Clean Sky 2, ACNUSA ...

# Acoustic liners in ducts




*ONERA's aero-thermo-acoustic test bench B2A*

# Acoustic liners in ducts




*ONERA's aero-thermo-acoustic test bench B2A*

J. Primus, F. Simon, E. Piot, AIAA Paper #2011-2869 (2011)

O. Léon, E. Piot, D. Sebbane, F. Simon, *Exp. Fluids* **58**, 72 (2017)

# Fluid flow models

- Navier-Stokes equations for a compressible fluid flow: mass, momentum, and energy conservation equations,

$$\begin{aligned} \frac{d\varrho}{dt} &= -\varrho\boldsymbol{\nabla}\cdot\boldsymbol{v} + m\,, \\ \varrho\frac{d\boldsymbol{v}}{dt} &= \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} + \varrho\boldsymbol{f}\,, \\ \varrho T\frac{ds}{dt} &= \boldsymbol{\tau} : \boldsymbol{\nabla}\boldsymbol{v}^{\mathsf{T}} - \boldsymbol{\nabla}\cdot\boldsymbol{q} + \varrho Q\,. \end{aligned} \qquad \text{[NS]}$$

- $\varrho(\boldsymbol{r},t)$ is the fluid density, $\boldsymbol{v}(\boldsymbol{r},t)$ the velocity, $T(\boldsymbol{r},t)$ the temperature, $s(\boldsymbol{r},t)$ the specific entropy, and $\boldsymbol{q}(\boldsymbol{r},t)$ the heat flux vector at the position $\boldsymbol{r}=(x,y,z)$ and time $t$;
- $\boldsymbol{\sigma}(\boldsymbol{r},t) = -p(\boldsymbol{r},t)\boldsymbol{I} + \boldsymbol{\tau}(\boldsymbol{r},t)$ is the stress tensor, $\boldsymbol{I}$ is the identity matrix, $p(\boldsymbol{r},t)$ is the (static) fluid pressure, $\boldsymbol{\tau}(\boldsymbol{r},t)$ is the viscous stress tensor;
- $m(\boldsymbol{r},t)$ is the specific mass source per unit time, $\boldsymbol{f}(\boldsymbol{r},t)$ is the force per unit mass exerted on the fluid (neglecting gravity), and $Q(\boldsymbol{r},t)$ is the heat production per unit mass;
- Convective derivative following the particle paths:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla}\,.$$

# Fluid flow models

- Euler equations for an ideal compressible fluid flow: mass, momentum, and energy conservation equations,

$$\begin{aligned} \frac{d\varrho}{dt} &= -\varrho \boldsymbol{\nabla} \cdot \boldsymbol{v} + m \,, \\ \varrho \frac{d\boldsymbol{v}}{dt} &= -\boldsymbol{\nabla} p + \varrho \boldsymbol{f} \,, \\ \frac{ds}{dt} &= 0 \,. \end{aligned} \qquad [E]$$

- $\varrho(\boldsymbol{r}, t)$ is the fluid density, $\boldsymbol{v}(\boldsymbol{r}, t)$ the velocity, $p(\boldsymbol{r}, t)$ the (static) pressure, and $s(\boldsymbol{r}, t)$ the specific entropy at the position $\boldsymbol{r} = (x, y, z)$ and time $t$;
- $m(\boldsymbol{r}, t)$ is the specific mass source per unit time, $\boldsymbol{f}(\boldsymbol{r}, t)$ is the force per unit mass exerted on the fluid (neglecting gravity);
- Ideal fluid: no friction $\boldsymbol{\tau} = \mathbf{0}$, no heat conduction $\boldsymbol{q} = \mathbf{0}$, and no heat production $Q = 0$;
- Isentropic flow (the "adiabatic equation"): from the equation of state $p = p(\varrho, s)$ where $c$ is the speed of sound:

$$\frac{dp}{dt} = c^2 \frac{d\varrho}{dt} \,, \quad c^2(\varrho, s) = \left( \frac{\partial p}{\partial \varrho} \right)_s \,.$$

# Fluid flow models

- Stationary Euler equations for a steady background flow: mass, momentum, and state equations,

$$
\begin{aligned}
(\boldsymbol{v}_0 \cdot \boldsymbol{\nabla})\varrho_0 &= -\varrho_0 \boldsymbol{\nabla} \cdot \boldsymbol{v}_0 \,, \\
(\varrho_0 \boldsymbol{v}_0 \cdot \boldsymbol{\nabla})\boldsymbol{v}_0 &= -\boldsymbol{\nabla} p_0 \,, \\
(\boldsymbol{v}_0 \cdot \boldsymbol{\nabla})p_0 &= c_0^2 (\boldsymbol{v}_0 \cdot \boldsymbol{\nabla})\varrho_0 \,.
\end{aligned}
$$

- Linearization: the actual flow is a perturbation $(\varrho', \boldsymbol{v}', p')$ of a stationary flow $(\varrho_0, \boldsymbol{v}_0, p_0)$ generated by the mass $m$ and force $\boldsymbol{f}$ injected to the fluid:

$$
\begin{aligned}
\varrho(\boldsymbol{r}, t) &= \varrho_0(\boldsymbol{r}) + \varrho'(\boldsymbol{r}, t) \,, \\
\boldsymbol{v}(\boldsymbol{r}, t) &= \boldsymbol{v}_0(\boldsymbol{r}) + \boldsymbol{v}'(\boldsymbol{r}, t) \,, \\
p(\boldsymbol{r}, t) &= p_0(\boldsymbol{r}) + p'(\boldsymbol{r}, t) \,.
\end{aligned}
$$

- The primed quantities $\varrho' \ll \varrho_0$, $\boldsymbol{v}' \ll \boldsymbol{v}_0$, and $p' \ll p_0$ are the acoustic density, velocity, and pressure, respectively, of which non-linear contributions to the Euler equations are assumed negligible.

# Fluid flow models

- Linearized Euler equations for the acoustic disturbances: mass, momentum, and state equations,

$$
\begin{aligned}
\frac{d\varrho'}{dt} &= -\varrho' \boldsymbol{\nabla} \cdot \boldsymbol{v}_0 - \boldsymbol{\nabla} \cdot (\varrho_0 \boldsymbol{v}') + m\,, \\
\varrho_0 \frac{d\boldsymbol{v}'}{dt} &= \frac{\varrho'}{\varrho_0} \boldsymbol{\nabla} p_0 - \boldsymbol{\nabla} p' - \varrho_0 (\boldsymbol{v}' \cdot \boldsymbol{\nabla}) \boldsymbol{v}_0 + \varrho_0 \boldsymbol{f}\,, \\
\frac{dp'}{dt} &= c_0^2 \left( \frac{d\varrho'}{dt} + \boldsymbol{v}' \cdot \boldsymbol{\nabla} \varrho_0 \right) - \boldsymbol{v}' \cdot \boldsymbol{\nabla} p_0 + (c^2)' \boldsymbol{v}_0 \cdot \boldsymbol{\nabla} \varrho_0\,.
\end{aligned}
\qquad \text{[LEE]}
$$

- Convective derivative following the particle paths in the background flow:

$$
\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{v}_0 \cdot \boldsymbol{\nabla}\,.
$$

- Convected wave equation: taking $\frac{d}{dt}$(mass conservation) $-\boldsymbol{\nabla}\cdot$(momentum conservation),

$$
\frac{d}{dt} \left( \frac{1}{c_0^2} \frac{dp'}{dt} \right) - \varrho_0 \boldsymbol{\nabla} \cdot \left( \frac{1}{\varrho_0} \boldsymbol{\nabla} p' \right) = \frac{dm}{dt} - \varrho_0 \boldsymbol{\nabla} \cdot \boldsymbol{f}\,. \qquad \text{[CWE]}
$$

D. Blokhintzev, *J. Acoust. Soc. Am.* **18**, 322 (1946)

# Convected acoustics in a duct

- Assume $\boldsymbol{v}_0 = (u_0, 0, 0)$ and $u_0$ is constant. Then $\frac{d}{dt} = \frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}$, and the homogeneous ($m = 0$ and $\boldsymbol{f} = \boldsymbol{0}$) convected wave equation reads:

$$\frac{\partial^2 p'}{\partial t^2} + 2u_0 \frac{\partial^2 p'}{\partial x \partial t} + u_0^2 \frac{\partial^2 p'}{\partial x^2} - c_0^2 \Delta p' = 0 \,.$$

- Convected Helmholtz equation: seeking for harmonic solutions $p'(\boldsymbol{r}, t) = \mathrm{e}^{\mathrm{i}\omega t} \widehat{p}(\boldsymbol{r}, \omega)$, the complex amplitude $\widehat{p}$ satisfies:

$$\Delta \widehat{p} + k_0^2 \left(1 - \mathrm{i} \frac{M_0}{k_0} \frac{\partial}{\partial x}\right)^2 \widehat{p} = 0 \,, \qquad \text{[CHE]}$$

with $M_0 = \frac{u_0}{c_0}$ the Mach number, and $k_0 = \frac{\omega}{c_0}$ the wave number without flow.

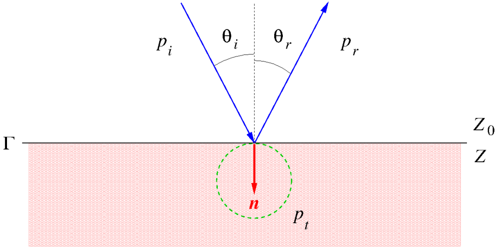
- Boundary condition on the duct wall $\Gamma$:

$$Z \frac{\partial \widehat{p}}{\partial \boldsymbol{n}} \bigg|_{\Gamma} + \mathrm{i} k_0 Z_0 \left(1 - \mathrm{i} \frac{M_0}{k_0} \frac{\partial}{\partial x}\right)^2 \widehat{p}|_{\Gamma} = 0 \,,$$

where $Z_0 = \varrho_0 c_0$ is the fluid impedance, and $\boldsymbol{n}$ is the outward unit normal to the fluid domain.

K. Ingard, *J. Acoust. Soc. Am.* **31**, 1035 (1959)
M. Myers, *J. Sound Vib.* **71**, 429 (1980)

# Liner impedance




- The liner impedance $Z(\omega) = R(\omega) + \mathrm{i}X(\omega)$ characterizes the response of a surface Γ to plane waves, representing both resistive ($R > 0$) and inertial ($X \in \mathbb{R}$) effects occurring within a (porous) material lining the duct. A hard wall is $Z = \infty$.
- The reflection coefficient of a plane wave $p_i$ impinging the liner from the fluid medium at incidence $\theta_i = \theta_r$ (Descartes' law):

$$\mathcal{R}(\theta_i) = \frac{Z\cos\theta_i - Z_0}{Z\cos\theta_i + Z_0}$$

where $Z = \frac{p_t}{\boldsymbol{v}\cdot\boldsymbol{n}}|_\Gamma$. The absorption coefficient is $\beta(\theta_i) = 1 - |\mathcal{R}(\theta_i)|^2$.

# Convected waves in a duct

- Modal solution: $\widehat{p}(\boldsymbol{r}, \omega) = \mathrm{e}^{-\mathrm{i}kx} P(\boldsymbol{x}_\perp; k, \omega)$ with $\boldsymbol{r} = (x, \boldsymbol{x}_\perp)$, then

$$\Delta_\perp P + k_\perp^2 P = 0\,,$$

with $\Delta_\perp := \Delta - \partial_x^2$, $(1 - M_0^2)k^2 + 2M_0 k_0 k + k_\perp^2 - k_0^2 = 0$, and boundary condition:

$$\left.\frac{\partial P}{\partial \boldsymbol{n}}\right|_\Gamma + \mathrm{i}\frac{k_0 Z_0}{Z(\omega)}\left(1 - \frac{kM_0}{k_0}\right)^2 P|_\Gamma = 0\,.$$

- Modal solution in a cylindrical duct: $P(\boldsymbol{x}_\perp; k, \omega) = \mathrm{e}^{-\mathrm{i}m\theta} P_m(r; k, \omega)$ with $\boldsymbol{x}_\perp = (r, \theta)$, then

$$r^2 \frac{d^2 P_m}{dr^2} + r\frac{dP_m}{dr} + (k_\perp^2 r^2 - m^2)P_m = 0\ ,$$

with $(1 - M_0^2)k^2 + 2M_0 k_0 k + k_\perp^2 - k_0^2 = 0$ and boundary condition:

$$\left.\frac{dP_m}{dr}\right|_{r=R} + \mathrm{i}\frac{k_0 Z_0}{Z(\omega)}\left(1 - \frac{kM_0}{k_0}\right)^2 P_m(R; k, \omega) = 0\,.$$

# Convected waves in a duct

- The overall solution reads as the summation of Bessel functions of the first kind $z \mapsto J_m(z)$:

$$\boxed{\widehat{p}(\boldsymbol{r}, \omega) = \sum_m \sum_n P_{mn}(x, r, \theta; \omega) = \sum_m \sum_n \mathrm{e}^{-\mathrm{i}k_{mn}x - \mathrm{i}m\theta} A_{mn} J_m(k_\perp r)},$$

where $k_\perp(\omega)$ solves:

$$k_\perp J'_m(k_\perp R) + \mathrm{i}\frac{k_0 Z_0}{Z(\omega)} \left(1 - \frac{kM_0}{k_0}\right)^2 J_m(k_\perp R) = 0\,,$$

and the axial wave number $k_{mn}(\omega) \in \mathbb{C}$ as a function of the $n$-th $k_\perp$ is:

$$k_{mn} = \frac{k_0}{1 - M_0^2} \left(\pm\sqrt{1 - (1 - M_0^2)\frac{k_\perp^2}{k_0^2}} - M_0\right)\,.$$

- **Remark**: 2D harmonic solutions for arbitrary duct cross-sections, flow profiles, and liner arrangements (with *e.g.* impedance discontinuities) are available with `PyOPAL`.

# Optimization problem

$$
\begin{cases}
\min\limits_{\boldsymbol{X} \in \mathbb{R}^d} \boldsymbol{f}(\boldsymbol{X}) \quad \text{such that} \\
\mathbf{A}\boldsymbol{X} \leq \mathbf{b}\,, \\
\boldsymbol{X}_{\text{min}} \leq \boldsymbol{X} \leq \boldsymbol{X}_{\text{max}}\,.
\end{cases}
$$

- The optimization variables $\boldsymbol{X} = (X_1, \dots X_d) \in \mathbb{R}^d$ are physical parameters pertaining to the liner constitutive materials and/or the background flow (*e.g.* $M_0$);
- The matrix $\mathbf{A}$ imposes linear constraints to the optimization variables (*e.g.* internal constraints on the parameters for the constitutive materials or global constraints for the whole liner);
- The optimization variables are sought for in a bounded region $\mathcal{D} := \prod_{i=1}^{d} [X_i^{\text{min}}, X_i^{\text{max}}]$;
- The objective functions $\boldsymbol{f}$ are formed with:
  - observables of the acoustic field $\widehat{p}(\boldsymbol{r}, \omega)$ if one aims at qualifying the efficiency of the liner in operational conditions,
  - observables of the liner impedance $Z(\omega)$ if one aims at identifying the physical parameters of the liner with reference to a target impedance $Z_{\text{ref}}(\omega)$,

  among other possible optimization strategies.

# Examples of objective functions

- A modal transmission loss factor at some fixed frequency $\omega$:

$$f(\boldsymbol{X}) \equiv \mathrm{TL}_{mn}(\boldsymbol{X};\omega) = \frac{1}{|x_{\mathrm{out}} - x_{\mathrm{in}}|} 20 \log_{10} \frac{P_{mn}(x_{\mathrm{out}}, r, \theta; \omega)}{P_{mn}(x_{\mathrm{in}}, r, \theta; \omega)} = -20 \log_{10}(\mathrm{e}) \Im\mathfrak{m}\{k_{mn}(\boldsymbol{X};\omega)\} \, ;$$

- A global transmission loss factor at some fixed frequency $\omega$:

$$f(\boldsymbol{X}) \equiv \mathrm{TL}(\boldsymbol{X};\omega) = \frac{1}{|x_{\mathrm{out}} - x_{\mathrm{in}}|} 20 \log_{10} \frac{\widehat{p}(x_{\mathrm{out}}, r, \theta; \omega)}{\widehat{p}(x_{\mathrm{in}}, r, \theta; \omega)} \, ;$$

- A weighted transmission loss factor over a set of frequencies of interest $W$:

$$f(\boldsymbol{X}) = -\sum_{\omega \in W} \beta(\omega) \, \mathrm{TL}(\boldsymbol{X};\omega) \, ,$$

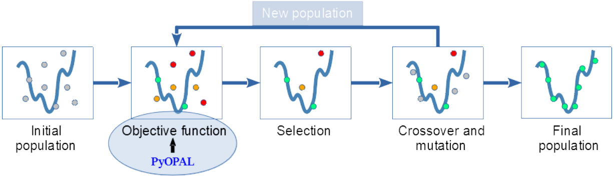
or:

$$f(\boldsymbol{X}) = -\max_{\omega \in W} \beta(\omega) \, \mathrm{TL}(\boldsymbol{X};\omega) \, ,$$

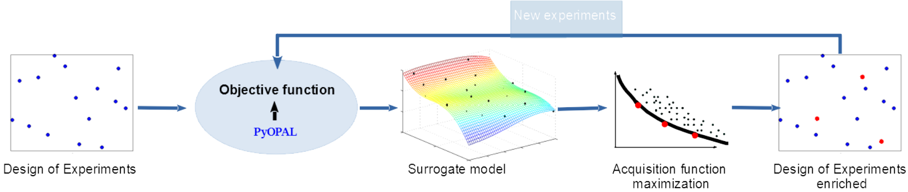
for some weight function $\omega \mapsto \beta(\omega)$.

# Mono and multi-objective optimization techniques

- Evolutionary algorithms
  - Based on operators applied on a population evolving during generations;
  - Very efficient but need to evaluate many candidate solutions.

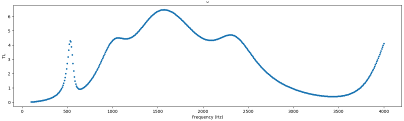



- Bayesian optimization
  - Based on a surrogate model iteratively enriched by new experiments;
  - Parsimonious approach adapted to costly simulations, uncertainties...

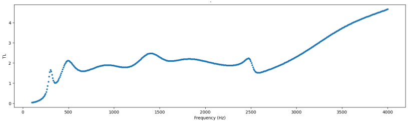


# Optimization problem formulations

- Mono-objective formulation with $d = 36$ optimization variables

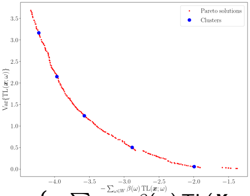



$$-\min_{\boldsymbol{X}\in\mathcal{D}} \max_{\omega\in W} \beta(\omega)\, \mathrm{TL}(\boldsymbol{X};\omega)$$

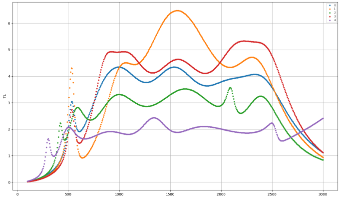



$$\min_{\boldsymbol{X}\in\mathcal{D}} \mathrm{Var}\{\mathrm{TL}(\boldsymbol{X};\omega)\}$$

- Multi-objective formulation with $d = 36$ optimization variables



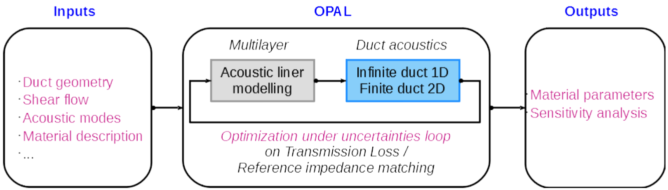

$$\min_{\boldsymbol{X}\in\mathcal{D}} \begin{cases} -\sum_{\omega\in W} \beta(\omega)\, \mathrm{TL}(\boldsymbol{X};\omega) \\ \mathrm{Var}\{\mathrm{TL}(\boldsymbol{X};\omega)\} \end{cases}$$




$\omega \mapsto \mathrm{TL}(\boldsymbol{X}_P;\omega)$ at several points $\boldsymbol{X}_P$ of the Pareto front

# Optimization under uncertainties

Design or environmental parameters can be affected by uncertainties:




- Sensitivity analysis to extract important parameters:

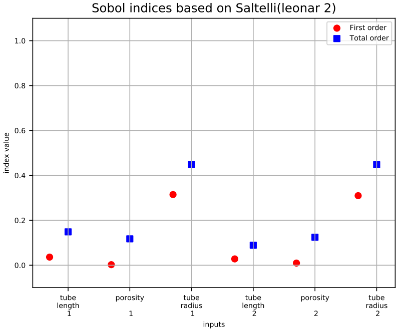
$$S_i = \frac{\mathrm{Var}\{\mathbb{E}\{f \mid X_i\}\}}{\mathrm{Var}\{f\}}, \quad S_{Ti} = 1 - \frac{\mathrm{Var}\{\mathbb{E}\{f \mid \boldsymbol{X}_{\sim i}\}\}}{\mathrm{Var}\{f\}} = \frac{\mathbb{E}\{\mathrm{Var}\{f \mid \boldsymbol{X}_{\sim i}\}\}}{\mathrm{Var}\{f\}};$$

- Optimization-based robustness measures to select optimal solutions ensuring performance in the presence of uncertainty:

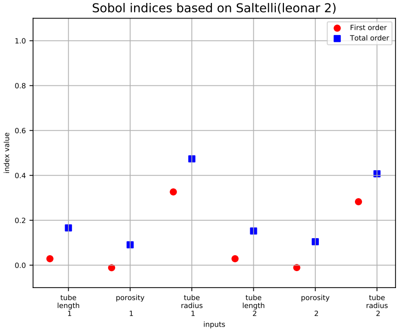
$$\min_{\boldsymbol{X} \in \mathcal{D}} \rho(\boldsymbol{X}) \quad \text{with} \quad \rho(\boldsymbol{X}) = \alpha \mathbb{E}_{\boldsymbol{\xi}}\{f(\boldsymbol{X}, \boldsymbol{\xi})\} + (1-\alpha)\mathrm{Var}_{\boldsymbol{\xi}}\{f(\boldsymbol{X}, \boldsymbol{\xi})\}.$$

# Sensitivity analysis

Study of LEONAR liner with two layers, described by $d = 6$ parameters.
Sobol' indices are estimated by Monte Carlo methods (*e.g.* Saltelli 2002). The total order indices inform on how the TL is affected by liner parameters uncertainties.
Evaluation of the interest of a surrogate model for Sobol' indices estimation:




Sensitivity indices estimated by Monte Carlo with $10^5$ PyOPAL simulations.



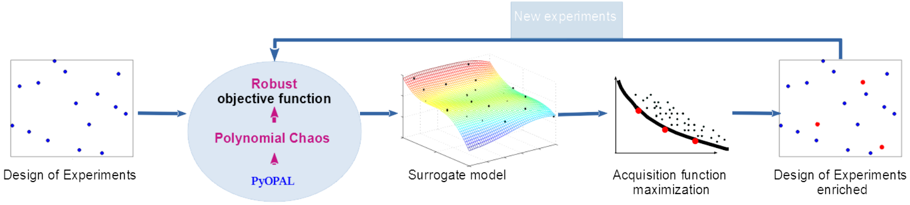

Sensitivity indices estimated by Monte Carlo with Kriging model from $2\,10^3$ PyOPAL simulations.

A. Saltelli, *Comput. Phys. Comm.* **145**(2), 280 (2002)

# Robust optimization with an uncertain parameter

Liner parameters optimization with $d = 4$ variables in presence of uncertainty on $\mathrm{SPL_{in}}$:

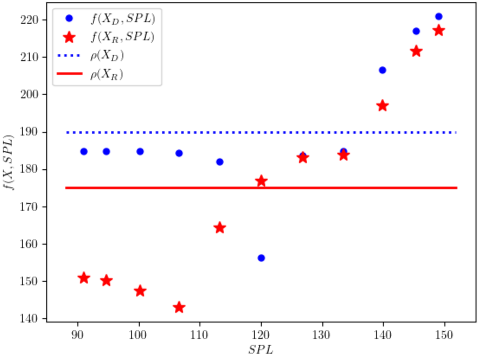
1. Uncertainty quantification: incident Sound Pressure Level $\xi = \mathrm{SPL_{in}} \sim \mathcal{N}(120; 10)$;
2. Bayesian optimization for minimization of robustness measure with $f = \mathrm{SPL_{out}}$;
3. Choice of deterministic robustness measure: $\rho(\boldsymbol{X}) = \mathbb{E}_{\mathrm{SPL_{in}}}\{\mathrm{SPL_{out}}(\boldsymbol{X}, \mathrm{SPL_{in}})\}$ $(\alpha = 1)$;
4. Polynomial Chaos : degree $= 20$ estimated from 11 Gauss points evaluated with `PyOPAL`.



# Robust optimization with an uncertain parameter

Comparison between deterministic: $\boldsymbol{X}_D = \arg\min_{\boldsymbol{X}\in\mathcal{D}} f(\boldsymbol{X}, \mathrm{SPL}_{\mathrm{in}} = 120)$ and robust: $\boldsymbol{X}_R = \arg\min_{\boldsymbol{X}\in\mathcal{D}} \rho(\boldsymbol{X})$ optimal solutions considering $\mathrm{SPL}_{\mathrm{in}}$ variability:

| | Objective value $f$ | Robust value $\rho$ |
|---|---|---|
| Deterministic solution $\boldsymbol{X}_D$ | 156 | 190 |
| Robust solution $\boldsymbol{X}_R$ | 177 | 175 |



# Perspectives

- 2D solutions for arbitrary duct cross-sections, sheared flows, spatially varying liner impedance, non linear effects... with `PyOPAL`: see next talk by R. Roncen;
- 3D solutions with `SPACE` (C. Peyret);
- Robust optimization with several random inputs;
- Surrogates in higher dimensions (*e.g.* multi-layer liners);
- Parallel and high performance computing for optimization.