Diffusion d'ondes acoustiques dans des écoulements hétérogènes aléatoires

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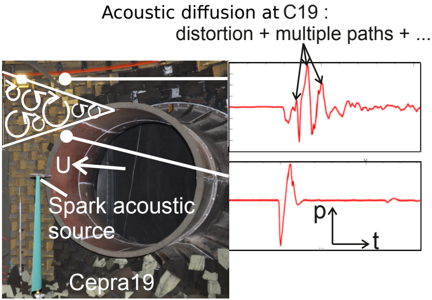
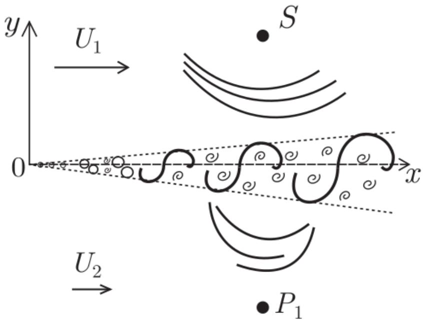
Acoustic scattering by turbulent shear layers





Anechoic wind tunnel with open-jet test section (nozzle exit Ø $= 2$-3 m, $U_1 \leq 100$ m/s)

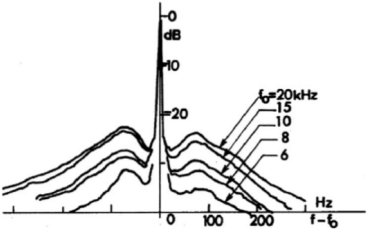
Applications:

- Analysis: spectral broadening and phase shift of acoustic waves;
- Identification: localization of sources through shear layers using diffuse waves.

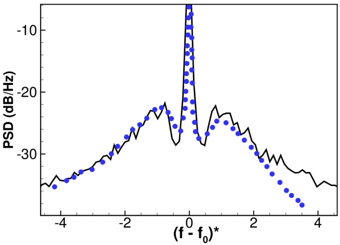
Acoustic scattering by turbulent shear layers

Results:

- Model of the acoustic pressure field transmitted by an horizontally stratified random flow (Gay-Garnier-Savin 2018);
- Model of multiple scattering of waves by an heterogeneous, unsteady flow in the high-frequency limit and weak coupling regime (Akian-Savin 2018): **THIS TALK!**



$M \approx 0.2$, $Re \approx 1200$, $f_0 = 6 - 20\ kHz$



Candel-Guédel-Julienne AIAA Paper 76-544 (1976)

Bennaceur *et al.* *Comput. Fluids* 138, 83 (2016)

Outline

- Full nonlinear Euler equations for an ideal fluid flow in the absence of friction, heat conduction, or heat production:

$$\frac{\mathrm{d}\varrho}{\mathrm{d}t} + \varrho \boldsymbol{\nabla}_{\boldsymbol{x}} \cdot \boldsymbol{v} = 0\,,$$
$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} + \frac{1}{\varrho} \boldsymbol{\nabla}_{\boldsymbol{x}} p = \mathbf{0}\,,$$
$$\frac{\mathrm{d}s}{\mathrm{d}t} = 0\,,$$

  where ϱ: the fluid density; $\boldsymbol{v}$: the particle velocity, s: the specific entropy; and p: the thermodynamic pressure given by the equation of state $p = p(\varrho, s)$.

- The usual convective derivative following the particle paths:

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}}\,.$$

- Isentropic flow (the "adiabatic equation"):

$$\frac{\mathrm{d}p}{\mathrm{d}t} = c^2 \frac{\mathrm{d}\varrho}{\mathrm{d}t}\,, \quad c^2(\varrho, s) = \left(\frac{\partial p}{\partial \varrho}\right)_s\,,$$

  where c: speed of sound.

Acoustic waves in heterogeneous unsteady flow
Linearized Euler equations

- Linearization about an ambient flow (subscript 0):

$$\begin{aligned}\varrho(\boldsymbol{x},t) &= \varrho_0(\boldsymbol{x},t)+\varrho'(\boldsymbol{x},t)\,,\\ \boldsymbol{v}(\boldsymbol{x},t) &= \boldsymbol{v}_0(\boldsymbol{x},t)+\boldsymbol{v}'(\boldsymbol{x},t)\,,\\ s(\boldsymbol{x},t) &= s_0(\boldsymbol{x},t)+s'(\boldsymbol{x},t)\,,\\ p(\boldsymbol{x},t) &= p_0(\boldsymbol{x},t)+p'(\boldsymbol{x},t)\,.\end{aligned}$$

- The ambient quantities satisfy the nonlinear Euler equations:

$$\begin{aligned}&\frac{\mathrm{d}\varrho_0}{\mathrm{d}t}+\varrho_0\boldsymbol{\nabla}_{\boldsymbol{x}}\cdot\boldsymbol{v}_0=0\,,\\ &\frac{\mathrm{d}\boldsymbol{v}_0}{\mathrm{d}t}+\frac{1}{\varrho_0}\boldsymbol{\nabla}_{\boldsymbol{x}}p_0=\mathbf{0}\,,\\ &\frac{\mathrm{d}s_0}{\mathrm{d}t}=0\,,\end{aligned}$$

where here and throughout:

$$\frac{\mathrm{d}}{\mathrm{d}t}=\frac{\partial}{\partial t}+\boldsymbol{v}_0\cdot\boldsymbol{\nabla}_{\boldsymbol{x}}\,.$$

- The acoustic perturbations $\varrho', \boldsymbol{v}', s'$ and p' are such that:

$$p' = c_0^2 \varrho' + \left(\frac{\partial p}{\partial s}\right)_{\varrho_0} s' \,,$$

  where $c_0(\boldsymbol{x}, t) > 0$: speed of sound in the ambient flow.
- Velocity quasi-potential $\phi(\boldsymbol{x}, t)$:

$$p' = \varrho_0 \frac{\mathrm{d}\phi}{\mathrm{d}t} \,, \quad \boldsymbol{v}' = \boldsymbol{\nabla}_{\boldsymbol{x}} \phi + \mathrm{O}(L^{-1}) + \mathrm{O}(T^{-1}) \,,$$

  where L/T: (large) length/time scales over which the ambient quantities have significant spatial variations.
- Then it satisfies the convected wave equation valid for $\lambda \ll L$:

$$\boxed{\frac{1}{\varrho_0} \boldsymbol{\nabla}_{\boldsymbol{x}} \cdot (\varrho_0 \boldsymbol{\nabla}_{\boldsymbol{x}} \phi) - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{c_0^2} \frac{\mathrm{d}\phi}{\mathrm{d}t}\right) = 0} \,.$$

Blokhintzev *J. Acoust. Soc. Am.* 18(2), 322 (1946)
Pierce *J. Acoust. Soc. Am.* 87(6), 2292 (1990)

High-frequency setting

- High frequencies correspond to $\varepsilon \to 0$ for strongly "ε–oscillatory" initial conditions:

$$\phi(\boldsymbol{x},0) = \phi_0(\boldsymbol{x};\varepsilon)\,, \quad \partial_t\phi(\boldsymbol{x},0) = \psi_0(\boldsymbol{x};\varepsilon)\,,$$

  parameterized by $\varepsilon \equiv \frac{\lambda}{L} \equiv \frac{1}{\omega T} \ll 1$, which quantifies the rate of change of $\boldsymbol{x} \mapsto \phi_0(\boldsymbol{x})$ and $\boldsymbol{x} \mapsto \psi_0(\boldsymbol{x})$ with respect to the typical length/time scales of the ambient flow.
- **Example**: plane waves for some $\boldsymbol{k} \in \mathbb{R}^3 \setminus \{\boldsymbol{0}\}$, $\mathrm{i} = \sqrt{-1}$,

$$\phi_0(\boldsymbol{x};\varepsilon) = \varepsilon A(\boldsymbol{x})\,\mathrm{e}^{\frac{\mathrm{i}}{\varepsilon}\boldsymbol{k}\cdot\boldsymbol{x}}\,, \quad \psi_0(\boldsymbol{x};\varepsilon) = B(\boldsymbol{x})\,\mathrm{e}^{\frac{\mathrm{i}}{\varepsilon}\boldsymbol{k}\cdot\boldsymbol{x}}\,.$$

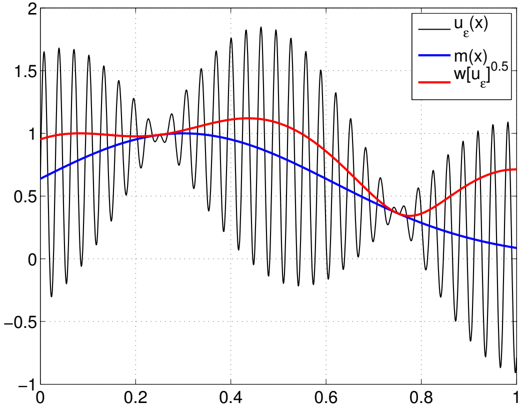
- *For constant ambient quantities $\varrho_0, \boldsymbol{v}_0$ and $c_0 > 0$ the usual parametrix:*

$$\phi_\varepsilon(\boldsymbol{x},t) = \mathcal{F}^{-1}_{\boldsymbol{k}\to\boldsymbol{x}}\left[\cos\left(c_0|\boldsymbol{k}|t\right)\right] \star \phi_0(\boldsymbol{x}-\boldsymbol{v}_0 t;\varepsilon) + \mathcal{F}^{-1}_{\boldsymbol{k}\to\boldsymbol{x}}\left[\frac{\sin\left(c_0|\boldsymbol{k}|t\right)}{c_0|\boldsymbol{k}|}\right] \star \psi_0(\boldsymbol{x}-\boldsymbol{v}_0 t;\varepsilon)$$

  propagates oscillations of wavelength ε which inhibit ϕ_ε from converging strongly in a suitable sense.

Outline

Why quadratic observables?

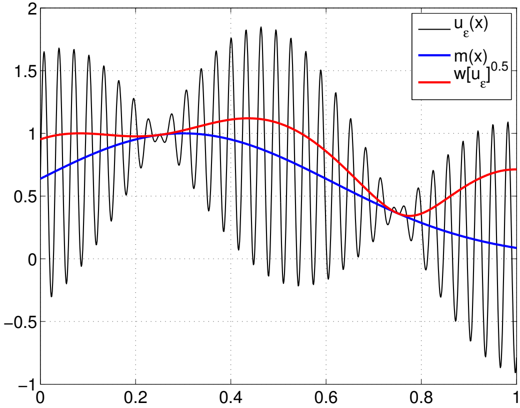


- Let:

$$u_\varepsilon(x) = m(x) + a(x) \sin \frac{x}{\varepsilon} \,,$$

then $(u_\varepsilon) \rightharpoonup m$ weakly in L^2 as $\varepsilon \to 0$, but (u_ε) has no strong limit in any L^p.

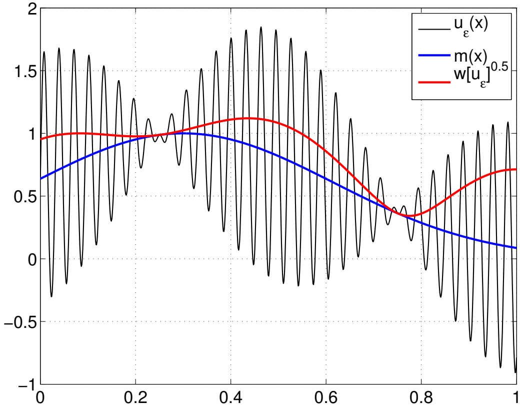
Why quadratic observables?



- Now for any observable $\varphi \in \mathcal{C}_0^\infty(\mathbb{R})$:

$$\lim_{\varepsilon\to 0} (\varphi(x) u_\varepsilon, u_\varepsilon)_{L^2} = \int_{\mathbb{R}} \varphi(x) \left((m(x))^2 + \frac{1}{2}(a(x))^2 \right) \mathrm{d}x \,.$$

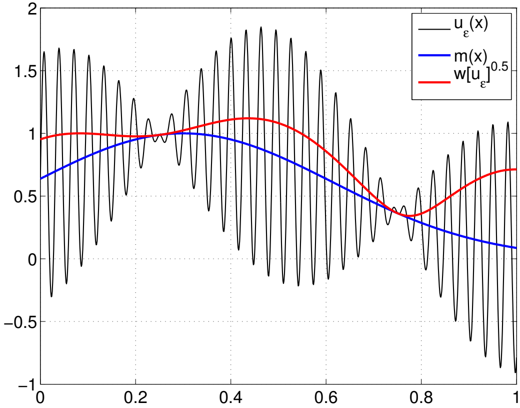
Why quadratic observables?



- Take an observable of the form:

$$\varphi(x,\partial_x)u_\varepsilon(x)=\frac{1}{2\pi}\int_{\mathbb{R}}\mathrm{e}^{\mathrm{i}k\cdot x}\,\varphi(x,\mathrm{i}k)\widehat{u}_\varepsilon(k)\mathrm{d}k\,.$$

Why quadratic observables?



- Take an observable of the form:

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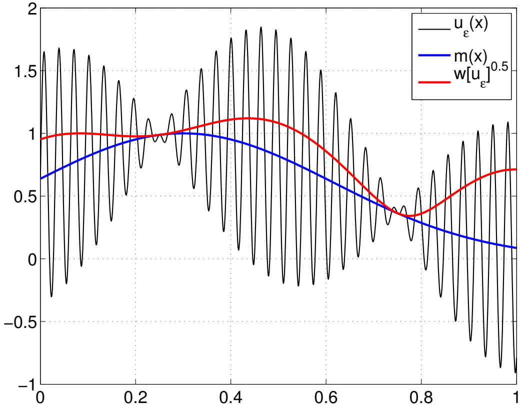
Why quadratic observables?



- Then:

$$\lim_{\varepsilon\to 0} (\varphi(x,\varepsilon\partial_x)u_\varepsilon, u_\varepsilon)_{L^2} = \iint_{\mathbb{R}^2} \varphi(x,\mathrm{i}k) W[u_\varepsilon](\mathrm{d}x,\mathrm{d}k)\,,$$

where $W[u_\varepsilon]$ is called a (positive) Wigner measure of (u_ε).

Wigner measure

- Let $\varphi \in \mathcal{C}_0^\infty(\mathbb{R}^3_{\boldsymbol{x}} \times \mathbb{R}^3_{\boldsymbol{k}})$ (a smooth observable), and for $u \in L^2(\mathbb{R}^3)$:

$$\mathrm{Op}_\varepsilon[\varphi]u(\boldsymbol{x}) = \boldsymbol{\varphi}(\boldsymbol{x}, \varepsilon\boldsymbol{\nabla}_{\boldsymbol{x}})u(\boldsymbol{x}) = \varphi(\boldsymbol{x}, \varepsilon\boldsymbol{\nabla}_{\boldsymbol{x}}) \left(\int \mathrm{e}^{\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{k}}\, \widehat{u}(\boldsymbol{k})\, \mathrm{d}\boldsymbol{k} \right)$$
$$:= \int \mathrm{e}^{\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{k}}\, \varphi(\boldsymbol{x}, \mathrm{i}\varepsilon\boldsymbol{k})\widehat{u}(\boldsymbol{k})\, \mathrm{d}\boldsymbol{k}\,.$$

- Then if (u_ε) is bounded in $L^2(\mathbb{R}^3)$, $\forall\varphi$ a smooth observable:

$$\boxed{\lim_{\varepsilon\to 0}(\varphi(\boldsymbol{x}, \varepsilon\boldsymbol{\nabla}_{\boldsymbol{x}})u_\varepsilon, u_\varepsilon)_{L^2} = \iint \varphi(\boldsymbol{x}, \mathrm{i}\boldsymbol{k})W[u_\varepsilon](\mathrm{d}\boldsymbol{x}, \mathrm{d}\boldsymbol{k})}\,,$$

  and the positive measure $W[u_\varepsilon]$ of the sequence (u_ε) always exists—up to extracting a sub-sequence if need be.

Lions-Paul *Rev. Mat. Iberoamericana* 9(3), 553 (1993)
Gérard-Markowich-Mauser-Poupaud *Commun. Pure Appl. Math.* L(4), 323 (1997)
Martinez *An Introduction to Semiclassical and Microlocal Analysis*, Springer, Berlin (2002)
Zworski *Semiclassical Analysis*, American Mathematical Society, Providence RI (2012)

- The acoustic kinetic and strain energies that would be perceived by an observer moving with the ambient flow are:

$$\lim_{\varepsilon\to 0}\mathcal{T}_\varepsilon(t)=\frac{1}{2}\iint \varrho_0(\boldsymbol{x},t)W[\boldsymbol{\nabla}_{\boldsymbol{x}}\phi_\varepsilon(\cdot,t)](\mathrm{d}\boldsymbol{x},\mathrm{d}\boldsymbol{k}),$$
$$\lim_{\varepsilon\to 0}\mathcal{U}_\varepsilon(t)=\frac{1}{2}\iint \varrho_0(\boldsymbol{x},t)W\left[\frac{\mathrm{d}\phi_\varepsilon}{\mathrm{d}t}(\cdot,t)\right](\mathrm{d}\boldsymbol{x},\mathrm{d}\boldsymbol{k}).$$

- WKB state: $\phi_\varepsilon(\boldsymbol{x})=A(\boldsymbol{x})\,\mathrm{e}^{\frac{\mathrm{i}S(\boldsymbol{x})}{\varepsilon}}$, then $W[\phi_\varepsilon]=|A(\boldsymbol{x})|^2\delta(\boldsymbol{k}-\boldsymbol{\nabla}_{\boldsymbol{x}}S)$.

Ray acoustics

- Computing the space-time Wigner measure of the wave equation one can prove that:

$$W[\phi_\varepsilon] = W_-\delta(\omega-\omega_-) + W_+\delta(\omega-\omega_+)\,,$$

where $\omega_\pm(\boldsymbol{x},t,\boldsymbol{k},\omega) = -\boldsymbol{v}_0(\boldsymbol{x},t)\cdot\boldsymbol{k} \mp c_0(\boldsymbol{x},t)|\boldsymbol{k}|$ (the Doppler-shifted frequencies), and the wave action per unit volume $\mathcal{A}_\pm := \frac{\varrho_0}{c_0}|\boldsymbol{k}|W_\pm$ satisfies:

$$\boxed{\partial_t\mathcal{A}_\pm - \boldsymbol{\nabla}_{\boldsymbol{k}}\omega_\pm\cdot\boldsymbol{\nabla}_{\boldsymbol{x}}\mathcal{A}_\pm + \boldsymbol{\nabla}_{\boldsymbol{x}}\omega_\pm\cdot\boldsymbol{\nabla}_{\boldsymbol{k}}\mathcal{A}_\pm = 0}\,.$$

- Alternatively:

$$\boxed{\frac{d\mathcal{A}}{dt} = 0}$$

along the rays $t \mapsto (\boldsymbol{x}(t),\boldsymbol{k}(t),\omega(t))$ in phase space of the physical space-time with the group velocities $\boldsymbol{v}_g^\pm(\boldsymbol{x},t,\boldsymbol{k}) = \boldsymbol{v}_0(\boldsymbol{x},t) \pm c_0(\boldsymbol{x},t)\frac{\boldsymbol{k}}{|\boldsymbol{k}|}$:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}_g^\pm\,,\quad \frac{d\boldsymbol{k}}{dt} = \boldsymbol{\nabla}_{\boldsymbol{x}}\omega_\pm\,,\quad \frac{d\omega}{dt} = \partial_t\omega_\pm\,,$$

which exhibits the phase shift ($\frac{d\boldsymbol{k}}{dt} \neq \boldsymbol{0}$) and spectral broadening ($\frac{d\omega}{dt} \neq 0$) effects. Also the direction of a sound ray in space is not necessarily along the wavefront normal.

Outline

Randomized ambient flow

- Assume now that $\boldsymbol{v}_0$ and c_0 depend on ε:

$$\frac{1}{c_0^2(\boldsymbol{x},t)} = \frac{1}{C_0^2(\boldsymbol{x},t)}\left[1+\sqrt{\varepsilon}\chi_1\left(\frac{\boldsymbol{x}}{\varepsilon},\frac{t}{\varepsilon}\right)\right],$$

$$\boldsymbol{v}_0(\boldsymbol{x},t) = \boldsymbol{V}_0(\boldsymbol{x},t)+\sqrt{\varepsilon}\boldsymbol{V}_1\left(\frac{\boldsymbol{x}}{\varepsilon},\frac{t}{\varepsilon}\right),$$

  where $\boldsymbol{v}_1 = (\chi_1, \boldsymbol{V}_1)$ is a second order, mean-zero, homogeneous random field.
- The correlation length/time of the random perturbations are the same as the (small) wavelength/period ε, ensuring maximum interactions between waves and the ambient medium.
- Heterogeneities are small though and their size is $\sqrt{\varepsilon}$, which is the relevant scaling that allows them to significantly modify the acoustic energy in the transport regime.

Howe *J. Sound Vib.* 27(4), 455 (1996)
Ryzhik-Papanicolaou-Keller *Wave Motion* 24(4), 327 (1996)
Fannjiang-Ryzhik *SIAM J. Appl. Math.* 61(5), 1545 (2001)
Bal *Wave Motion* 43(2), 132 (2005)
Bal-Komorowski-Ryzhik *Kinet. Relat. Models* 3(4), 529 (2010)

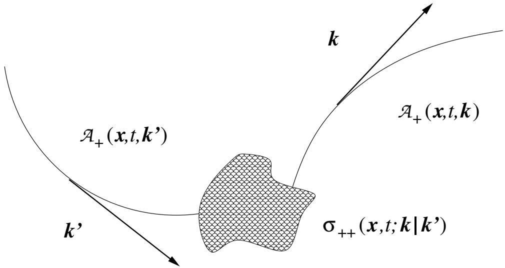
Radiative transfer in random ambient flow

- The radiative transfer (linear Boltzmann) equations for the high-frequency wave action $\mathcal{A}$ describe multiple scattering:

$$\partial_t \mathcal{A}_+(\boldsymbol{k}) - \boldsymbol{\nabla}_{\boldsymbol{k}}\omega_+ \cdot \boldsymbol{\nabla}_{\boldsymbol{x}}\mathcal{A}_+(\boldsymbol{k}) + \boldsymbol{\nabla}_{\boldsymbol{x}}\omega_+ \cdot \boldsymbol{\nabla}_{\boldsymbol{k}}\mathcal{A}_+(\boldsymbol{k}) =$$
$$\int_{\mathbb{R}^3} \sigma_{++}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}')\left(\mathcal{A}_+(\boldsymbol{k}') - \mathcal{A}_+(\boldsymbol{k})\right)\mathrm{d}\boldsymbol{k}' + \int_{\mathbb{R}^3} \sigma_{+-}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}')\left(\mathcal{A}_-(\boldsymbol{k}') - \mathcal{A}_+(\boldsymbol{k})\right)\mathrm{d}\boldsymbol{k}'\,,$$

$$\partial_t \mathcal{A}_-(\boldsymbol{k}) - \boldsymbol{\nabla}_{\boldsymbol{k}}\omega_- \cdot \boldsymbol{\nabla}_{\boldsymbol{x}}\mathcal{A}_-(\boldsymbol{k}) + \boldsymbol{\nabla}_{\boldsymbol{x}}\omega_- \cdot \boldsymbol{\nabla}_{\boldsymbol{k}}\mathcal{A}_-(\boldsymbol{k}) =$$
$$\int_{\mathbb{R}^3} \sigma_{-+}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}')\left(\mathcal{A}_+(\boldsymbol{k}') - \mathcal{A}_-(\boldsymbol{k})\right)\mathrm{d}\boldsymbol{k}' + \int_{\mathbb{R}^3} \sigma_{--}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}')\left(\mathcal{A}_-(\boldsymbol{k}') - \mathcal{A}_-(\boldsymbol{k})\right)\mathrm{d}\boldsymbol{k}'\,.$$

- $\sigma_{\pm\pm}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}')$: the differential scattering cross-sections, which incorporate the macroscopic effects of the small scale heterogeneities.



Scattering cross-sections

- Correlations of the random perturbations of the ambient flow velocity/speed of sound:

$$\mathbb{E}\left\{\widehat{\boldsymbol{v}}_1(\boldsymbol{k},\omega)\otimes\widehat{\boldsymbol{v}}_1(\boldsymbol{k}',\omega')\right\} := (2\pi)^4\delta(\boldsymbol{k}+\boldsymbol{k}')\delta(\omega+\omega')\widehat{\boldsymbol{R}}(\boldsymbol{k},\omega)\,,$$

where the 4×4 correlation tensor is $\boldsymbol{R}(\boldsymbol{x}'-\boldsymbol{x},t'-t) := \mathbb{E}\left\{\boldsymbol{v}_1(\boldsymbol{x},t)\otimes\boldsymbol{v}_1(\boldsymbol{x}',t')\right\}$ and:

$$\widehat{\boldsymbol{R}}(\boldsymbol{k},\omega) := \int_{\mathbb{R}^4}\frac{\mathrm{d}\boldsymbol{k}\mathrm{d}\omega}{(2\pi)^4}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}+\omega t)}\,\boldsymbol{R}(\boldsymbol{x},t) = \begin{bmatrix}\widehat{R}_c(\boldsymbol{k},\omega) & \widehat{\boldsymbol{R}}_{cv}^*(\boldsymbol{k},\omega)\\ \widehat{\boldsymbol{R}}_{cv}(\boldsymbol{k},\omega) & \widehat{\boldsymbol{R}}_v(\boldsymbol{k},\omega)\end{bmatrix}.$$

- The differential scattering cross-sections if one neglects for example the correlation of the perturbations of the speed of sound and the particle velocity ($\widehat{\boldsymbol{R}}_{cv}(\boldsymbol{k}-\boldsymbol{k}',\omega-\omega')=\boldsymbol{0}$):

$$\begin{aligned}\sigma_{++}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}') =& \frac{C_0^2(\boldsymbol{x},t)|\boldsymbol{k}||\boldsymbol{k}'|}{4(2\pi)^3}\widehat{R}_c(\boldsymbol{k}-\boldsymbol{k}',C_0(\boldsymbol{x},t)|\boldsymbol{k}'|-C_0(\boldsymbol{x},t)|\boldsymbol{k}|)\\ &+\frac{(|\boldsymbol{k}|+|\boldsymbol{k}'|)^2}{4(2\pi)^3|\boldsymbol{k}||\boldsymbol{k}'|}\boldsymbol{k}'\cdot\widehat{\boldsymbol{R}}_v(\boldsymbol{k}-\boldsymbol{k}',C_0(\boldsymbol{x},t)|\boldsymbol{k}'|-C_0(\boldsymbol{x},t)|\boldsymbol{k}|)\boldsymbol{k}'\,,\\ \sigma_{+-}(\boldsymbol{x},t;\boldsymbol{k}|\boldsymbol{k}') =& \frac{C_0^2(\boldsymbol{x},t)|\boldsymbol{k}||\boldsymbol{k}'|}{4(2\pi)^3}\widehat{R}_c(\boldsymbol{k}'-\boldsymbol{k},C_0(\boldsymbol{x},t)|\boldsymbol{k}|+C_0(\boldsymbol{x},t)|\boldsymbol{k}'|)\\ &+\frac{(|\boldsymbol{k}|-|\boldsymbol{k}'|)^2}{4(2\pi)^3|\boldsymbol{k}||\boldsymbol{k}'|}\boldsymbol{k}'\cdot\widehat{\boldsymbol{R}}_v(\boldsymbol{k}'-\boldsymbol{k},C_0(\boldsymbol{x},t)|\boldsymbol{k}|+C_0(\boldsymbol{x},t)|\boldsymbol{k}'|)\boldsymbol{k}'\,.\end{aligned}$$

Hot topics

- Diffusion limit(s) $\mathcal{A}(\boldsymbol{x}, t, \boldsymbol{k}) \rightsquigarrow \mathcal{A}(\boldsymbol{x}, t, |\boldsymbol{k}|)$;
- Boundary conditions (very difficult);
- Imaging algorithm: CINT (Gay-Garnier-Peyret-Savin + Bonnet).
- Further reading...

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Prejudice

- Waves in heterogeneous media are far from what they would be in uniform media but their macroscopic features can often be captured by models that do not need the knowledge of the microscopic details.
- The aim is to minimize the efforts required to solve a problem of wave propagation in random media, *i.e.* perform waves coarse-graining.
- The issue is to identify a suitable set of relevant parameters for a coarser target level and express them in terms of the parameters of a finer source level: in other words rely on a few macroscopic parameters to encode microscopic parameters, that do not depend on particular realizations but rather on statistics.
- Coarse-graining consists in rescaling some phenomena into units or cells or models of size close to the uncertainty of measurement, yielding an increase of both entropy and dissipation–hence irreversibility.