# Abstract of forthcoming book

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## 1 Reclaiming the Kantian synthetic a priori:

### 1.1 Frege $\longrightarrow Russell \longrightarrow Wittgenstein \longrightarrow Carnap \longrightarrow Gödel \longrightarrow Tarski \longrightarrow Church \longrightarrow Turing/Kleene \longrightarrow Tait \longrightarrow Girard \longrightarrow Martin{-}Löf \longrightarrow Coquand \longrightarrow Voevodsky'sunivalenceaxiom \longrightarrow Awodey \longrightarrow Altenkirch \longrightarrow HoTT/UF$

I am interested in focusing on Carnap's critiques of Wittgenstein, specifically the deracination of 'meaning' and substitution of 'meaning' with syntax. Analytic philosophy was created at the height of neo-Kantianism in the aftermath of German Idealism. Obscure terms and concepts were much overused at the time (such as Hegel's philosophy). But I am interested in a return to an 'analytic pragmatism' as advocated by Brandom and his readings of Hegel. There was a flood of what Reza Negarestani calls 'metaphysical bloatware' (inflationary metaphysics) at the turn of the 20th century. Russell's pupil Wittgenstein famously said that where we cannot know something, we must remain in silence (Tractatus). Is analyticity only meaningful when it is devoid of all content and meaning (i.e. only positivistic logico-philosophico propositions where all deductions/math are implied from itself deductively like a succession of dominos)? All math is tautology? "5.133 All deductions are made a priori" (Tractatus) And is analyticity only meaningful when philosophy is truly 'tolerant' (in Carnap's sense) for a multitude of interpretative frameworks (metalanguages) for instituting the object language of scientific theory (the language where one has propositions or the arithmetic, e.g. Peano arithmetic) through the supporting "ocean of metalanguages" to use Steve Awodey's term (e.g. Goedel's encoding of the Peano Arithmetic through prime numbers) backing this object language. My plan is to argue for the inferentialist account of meaning (Brandom), that meaning is understood in terms of use and that semantics is inherently an ethical question tied to commitments to discursive norms.

I am interested in surveying the history of early analytic philosophy and then connecting semantic inferentialism to intuitionistic type theory (connecting philosophy to mathematics). I am interested in drawing a connection between the philosophical (semantic anti-realism) and the logical (with the codification of intuitionism in Martin Lof type theory + recent work in HoTT by Awodey and many others). Frege established the analytic demarcation of a priori reasoning which would inform Russell's logicism and Wittgenstein's construction of fact or tautology. Carnap would make the logical framework robust in the notion of analyticity–a radical opposition to the synthetic a priori. But what about the morning star and the evening star? How does one identify the same sense of different denotations? Frege's work on incomplete arithmetic expressions provides a hint of the functional paradigm after the work of Church's lambda calculus, defined by functionals. Tarski posed a great challenge to Carnap with the undefinability of truth theorem wherein every metalanguage necessitates a metalanguage to encode prior object language; the model theoretic paradigm begins. Yet I am interested in an alternative reading of the split between Tarski and Carnap. Through the work of intuitionistic logic, propositions are taken as if the witness matters. This would later culminate in type theories such as Martin-Lof and substructural logics such as Girard's linear logic. The semantic anti-realist position (where the opposition between transcendent truth and constructive truth can be situated between Platonist and intuitionist philosophy of math) harkens back to Carnap's original principle of verification via the witness of intuitionistic logic, yet situated in an inferential network. Through the constructive possibilities of Homotopy Type Theory, a new paradigm for the foundation of mathematics, the identity of types with topology extends the synthetic lineage of the a priori through a mathematical codification of intuitionism (which remain mere mental constructions).

I am interested in the split between Carnap and Tarski over metalogic regarding semantics. Carnap crystallized the bare minimum of structure for an alien civilization to understand our language based on propositions and variables defining such propositions. Carnap argued that we should be tolerant of a multitude of metalanguages for instituting the object language in a logical syntax of language. What are the possibilities of defining a universally-quantified language or is such an endeavor doomed to fail because of Goedel's theorem regarding incompleteness? What about the recent advancements in the foundation of mathematics, i.e. Homotopy Type Theory? There has been a continual casting away of the synthetic a priori by Russell, the positivists and later Quine and Putnam. The central question of the book will be whether to admit ontologically, metaphysical realism or anti-realism and then epistemologically, semantic realism or anti-realism? Per Martin-Löf in his tracts on the philosophy of math accepts the synthetic a priori, which Kant postulated as existent with geometry and arithmetic. I will be arguing for the synthetic a priori in terms of synthetic geometry such as Homotopy Type Theory, which is at the nexus between theoretical computer science and algebraic topology. The computational types can be transported directly into geometric terms through Steve Awodey's interpretation of the Univalence Axiom. One of the main readings of Frege's logicism, which is the school of philosophy of math which believes in the fundamentality of atomic propositions, will be Michael Dummett's work regarding the bivalence of truth statements and the justificatory power of demonstrating a proof to a witness via the assertion. This is what Robert Harper calls "logic as if people matter." The key disagreement between Martin-Löf and Dummett is whether

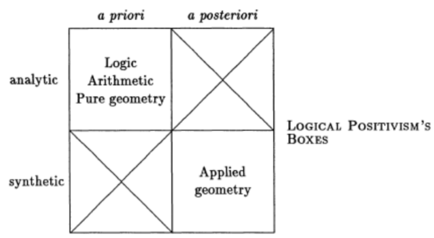


Figure 1: Taken from "The Taming the True"

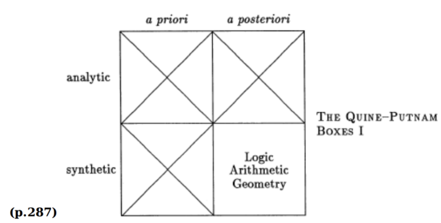


Figure 2: Taken from "The Taming of the True"

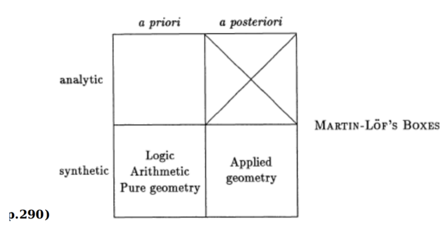


Figure 3: Taken from "The Taming of the True"

proof of a statement is an ontological or epistemological claim with Martin-Löf arguing for the latter and Dummett for the former. Dummett was the key proponent of the semantic anti-realist school and argued that all language can be reduced to its logical basis in the philosophy of math: either one is a Platonist or one is a intuitionist. Dummett was a proponent of verificationism, which Carnap and the logical positivists believed was central to establishing the veracity of empirical phenomenon. David Corfield argues that that inferentialist school of Brandom would be the most faithful way to read the philosophy of Homotopy Type Theory:

*I believe that there are grounds to hope that large portions of Brandom's program can be illuminated by type theory. Since Brandom's inferentialism derives in part from the constructive, proof-theoretic outlook of Gentzen, Prawitz and Dummett, this might not be thought to be such a bold claim. However, his emphasis is generally on material inference, only some aspects of which he takes to be treatable as formal inference. For instance, the argument he gives in Making it Explicit (Brandom 1994, Chap. 6) for why we have substitutable singular terms and directed inferences between predicates, is presented with a minimal formal treatment, and yet this phenomenon makes very good sense in the context of HoTT when understood as arising from the different properties of terms and types. Indeed, the kind of category whose internal structure our type theory describes, an (,1)-topos, presents both aspects—the '1' corresponding to unidirected inference between types (morphisms between objects) and the '' referring to the reversible substitutability of terms (2-morphisms and higher between morphisms). On the other hand, his frequent use of formalisms, even very briefly to category theory (for example, Brandom 2010, p. 14), tells us that he recognizes the organizing power of logical languages. Elsewhere (Brandom 2015, p. 36), he notes that while both Wittgenstein and Sellars emphasized that much of our language is deployed otherwise than for empirical description, whereas Wittgenstein addressed this excess as an assortment, Sellars viewed much of it more systematically as 'broadly metalinguistic locutions'. These additional functions of language pertain to the very framework that allows for our descriptive practices, and are what Brandom looks to make explicit in much of his work. My broader suggestion in this book is that we look to locate these locutions in features of our type theory, especially those modalities we will add in Chap. 4. (Corfield 2020)*

Through inferentialism, truths are not related necessarily to its meaning, but rather meaningfulness in the sense of the meaning in context or meaning-as-use. The book will be situated in the history and philosophy of language and mathematics. I hope to further the research on Homotopy Type Theory being done by Corfield, Andrei Roden and Awodey and extend this mathematical subfield into the larger philosophical lineage of philosophy of language. If time permits, I would like to compare the pragmatist reading of philosophy of language with the rationalist nominalist reading of Jean Cavailles. My book would examine the universality of mathematical necessity through the mathematical philosophy of Cavailles. Through the mathematical Absolute we arrive at both the contingent accidents and mathematical necessity that allows for the development of new theorems, but moreover mathematical experience as a determined concept. Can we found universality (as in Cavailles) on the generality of infinity-groupoids in Homotopy Type Theory, i.e. globally, or is it necessary that metalinguistic locutions be situated in terms of the inferentialism of Homotopy Type Theory, i.e. locally?