THE EMILY HARVEY FOUNDATION

Image source Wikipedia

Hermetic Definition

Curated by Kavita B Schmid

Emily Harvey Foundation
537 Broadway
New York, NY 10012

Saturday, March 24, 2018
6:30pm

Performances by:

Alexander Boland
Lillian Paige Walton
Lauren Burns-Coady
Adrian Rew
Shakespeare the Sadist (Written by Wolfgang Bauer. Starring Cammisa Buerhaus, Tavish Miller, Luke Schumacher, and Dylan Aiello. Directed by Michael Pollard.)

Artwork by Graham Vunderink with a piece by Marlous Borm

Hermetic Definition

If the set of all linearly independent subsets has an upper bound then it has a maximal element that is not smaller than any other element in the set, which is the basis of the vector space. This is proved by Zorn's lemma which is equivalent to the "Axiom of Choice" where a function can choose an element from each set based on index.

"The functioning of the 4G-smartphones depends on the phones ability to quickly carry out certain transformations (DFT/IDFT) in certain (for example) 1024-dimensional subspaces of the space of (periodic) functions." - Jyrki Lahtonen

Proof that every vector space has a basis[edit]

Let **V** be any vector space over some field **F**. Let **X** be the set of all linearly independent subsets of **V**. The set **X** is nonempty since the empty set is an independent subset of **V**, and it is partially ordered by inclusion, which is denoted, as usual, by ⊆.

Let **Y** be a subset of **X** that is totally ordered by ⊆, and let L_Y be the union of all the elements of **Y** (which are themselves certain subsets of **V**).

Since (**Y**, ⊆) is totally ordered, every finite subset of L_Y is a subset of an element of **Y**, which is a linearly independent subset of **V**, and hence every finite subset of $L_Y$is linearly independent. Thus L_Y is linearly independent, so L_Y is an element of **X**. Therefore, L_Y is an upper bound for **Y** in (**X**, ⊆): it is an element of **X**, that contains every element **Y**.

As **X** is nonempty, and every totally ordered subset of (**X**, ⊆) has an upper bound in **X**, Zorn's lemma asserts that **X** has a maximal element. In other words, there exists some element L_{max} of **X** satisfying the condition that whenever $L_{max} \subseteq L$ for some element L of **X**, then $L = L_{max}$.

It remains to prove that L_{max} is a basis of **V**. Since L_{max} belongs to **X**, we already know that L_{max} is a linearly independent subset of **V**.

If L_{max} would not span **V**, there would exist some vector **w** of **V** that cannot be expressed as a linear combination of elements of L_{max} (with coefficients in the field **F**). In particular, **w** cannot be an element of L_{max}. Let $L_w = L_{max} \cup \{\mathbf{w}\}$. This set is an element of **X**, that is, it is a linearly independent subset of **V** (because **w** is not in the span of L_{max}, and L_{max} is independent). As $L_{max} \subseteq L_w$, and $L_{max} \neq L_w$ (because L_w contains the vector **w** that is not contained in L_{max}), this contradicts the maximality of L_{max}. Thus this shows that L_{max} spans **V**.

Hence L_{max} is linearly independent and spans **V**. It is thus a basis of **V**, and this proves that every vector space has a basis.

This proof relies on Zorn's lemma, which is equivalent to the axiom of choice. Conversely, it may be proved that if every vector space has a basis, then the axiom of choice is true; thus the two assertions are equivalent.

(source Wikipedia)

For a more in-depth explanation, proof, examples, theorems and lemmas see:
http://sierra.nmsu.edu/morandi/OldWebPages/Math482Spring2005/Zorn.pdf

More information:
www.emilyharveyfoundation.org