```
(*We begin with the logit demand. We are given the optimal share
 s_i and want to solve for p_i. This is done by solving the insurer's
 profit maximization problem where the optimal p_i is the argmax. *)
(*The equation in the following is the profit function for insurer i. We
 take the first derivative with respect to p_i. I begin with a test
 to ensure that E is the right syntax for the exponential function.*)
D[
 E^x,
 x]
```

Out[∘]= $e^x$

$e^x$

```
D[x^(2 + a), x]
(*We can then proceed.*)
```

Out[∘]= $e^x$

In[∘]:= $D\left[\frac{e^{v-\alpha * p_i}}{1 + e^{v-\alpha * p_i} + e^{v-\alpha * p_j}} * (p_i - \phi_i), p_i\right]$

Out[∘]= $\frac{e^{v-\alpha p_i}}{1 + e^{v-\alpha p_i} + e^{v-\alpha p_j}} + \frac{e^{2 v-2 \alpha p_i} \alpha (p_i - \phi_i)}{\left(1 + e^{v-\alpha p_i} + e^{v-\alpha p_j}\right)^2} - \frac{e^{v-\alpha p_i} \alpha (p_i - \phi_i)}{1 + e^{v-\alpha p_i} + e^{v-\alpha p_j}}$

In[∘]:= (*With this rather awful expression for the FOC in hand we can proceed
 and solve for the optimal p_i by setting this expression equal to zero.*)

$\text{Solve}\left[\frac{e^{v-\alpha p_i}}{1 + e^{v-\alpha p_i} + e^{v-\alpha p_j}} + \frac{e^{2 v-2 \alpha p_i} \alpha (p_i - \phi_i)}{\left(1 + e^{v-\alpha p_i} + e^{v-\alpha p_j}\right)^2} - \frac{e^{v-\alpha p_i} \alpha (p_i - \phi_i)}{1 + e^{v-\alpha p_i} + e^{v-\alpha p_j}} == 0 \,\&\&\, p_i > 0 \,\&\&\, p_j > 0 \,\&\&\, \alpha > 0, p_i\right]$

Out[∘]= $\left\{\left\{p_i \to \boxed{\frac{1 + \text{ProductLog}\left[c_1, \frac{e^{-1+v+\alpha p_j-\alpha \phi_i}}{e^v + e^{\alpha p_j}}\right] + \alpha \phi_i}{\alpha} \text{ if } \textit{condition}}\right\}\right\}$

In[∘]:= (*This isn't very meaningful. However we should be able to make things nicer by
 approaching the simultaneous case and using our symmetry argument. Setting p_i=
 p_j=p in the above FOC and solving we have the following.*)

$\text{Solve}\left[\frac{e^{v-\alpha p}}{1 + e^{v-\alpha p} + e^{v-\alpha p}} + \frac{e^{2 v-2 \alpha p} \alpha (p - \phi_i)}{\left(1 + e^{v-\alpha p} + e^{v-\alpha p}\right)^2} - \frac{e^{v-\alpha p} \alpha (p - \phi_i)}{1 + e^{v-\alpha p} + e^{v-\alpha p}} == 0, p\right]$

Solve: This system cannot be solved with the methods available to Solve.

Out[∘]= $\text{Solve}\left[\frac{e^{v-p \alpha}}{1 + 2 e^{v-p \alpha}} + \frac{e^{2 v-2 p \alpha} \alpha (p - \phi_i)}{\left(1 + 2 e^{v-p \alpha}\right)^2} - \frac{e^{v-p \alpha} \alpha (p - \phi_i)}{1 + 2 e^{v-p \alpha}} == 0, p\right]$

(*This didn't work either. Hmmmmm. *)

In[∘]:= (*With the assumption that v=0 we may reduce things substantially. Suppose that v=0.*)

$$D\left[\frac{e^{-\alpha*p_i}}{1+e^{-\alpha*p_i}+e^{-\alpha*p_j}}*(p_i-\phi_i),\ p_i\right]$$

Out[∘]=
$$\frac{e^{-\alpha\, p_i}}{1+e^{-\alpha\, p_i}+e^{-\alpha\, p_j}}+\frac{e^{-2\,\alpha\, p_i}\,\alpha\,(p_i-\phi_i)}{\left(1+e^{-\alpha\, p_i}+e^{-\alpha\, p_j}\right)^2}-\frac{e^{-\alpha\, p_i}\,\alpha\,(p_i-\phi_i)}{1+e^{-\alpha\, p_i}+e^{-\alpha\, p_j}}$$

In[∘]:=
$$\text{Solve}\left[\frac{e^{-\alpha\, p_i}}{1+e^{-\alpha\, p_i}+e^{-\alpha\, p_j}}+\frac{e^{-2\,\alpha\, p_i}\,\alpha\,(p_i-\phi_i)}{\left(1+e^{-\alpha\, p_i}+e^{-\alpha\, p_j}\right)^2}-\frac{e^{-\alpha\, p_i}\,\alpha\,(p_i-\phi_i)}{1+e^{-\alpha\, p_i}+e^{-\alpha\, p_j}}==0,\ p_i\right]$$

Out[∘]=
$$\left\{\left\{p_i\to\boxed{\frac{1+\text{ProductLog}\left[\mathbb{c}_1,\ \frac{e^{-1+\alpha\, p_j-\alpha\,\phi_i}}{1+e^{\alpha\, p_j}}\right]+\alpha\,\phi_i}{\alpha}\quad \text{if}\quad \mathbb{c}_1\in\mathbb{Z}}\right\}\right\}$$

(*Still the same gnarly expression.*)

Out[∘]= $Aborted

In[∘]:=
$$\text{Solve}\left[e^{v-\alpha*p}+\frac{e^{2\,v-2*\alpha*p}*\alpha*(p-\phi_i)}{1+2*e^{v-\alpha*p}}==e^{v-\alpha*p}*(p-\phi_i)\ \&\&\ p>0,\ p\right]$$

... Solve: This system cannot be solved with the methods available to Solve.

In[∘]:= Clear[v]

In[∘]:= Clear[α]

In[∘]:=
$$\text{Solve}\left[1+2\,e^{v-\alpha*p}+\alpha\,(p-\phi_i)\,e^{v-\alpha*p}==\alpha\,(p-\phi_i)\,\left(1+2\,e^{v-\alpha*p}\right),\ p\right]$$

... Solve: This system cannot be solved with the methods available to Solve.

Out[∘]=
$$\text{Solve}\left[1+2\,e^{v-p\,\alpha}+e^{v-p\,\alpha}\,\alpha\,(p-\phi_i)==\left(1+2\,e^{v-p\,\alpha}\right)\,\alpha\,(p-\phi_i),\ p\right]$$

In[∘]:=
$$\text{Solve}\left[1+2\,e^{(-1/2)*p}+(1/2)\,(p-\phi_i)\,e^{(-1/2)*p}==(1/2)\,(p-\phi_i)\,\left(1+2\,e^{(-1/2)*p}\right),\ p\right]$$

... Solve: This system cannot be solved with the methods available to Solve.

In[∘]:=
$$\text{Solve}\left[1+2\,e^{-p/2}+\frac{1}{2}\,e^{-p/2}\,(p-\phi_i)==\frac{1}{2}\,\left(1+2\,e^{-p/2}\right)\,(p-\phi_i),\ p\right]$$

... Solve: This system cannot be solved with the methods available to Solve.

In[∘]:=
$$\text{Solve}\left[1+2\,e^{-p/2}+\frac{1}{2}\,e^{-p/2}\,(p-\phi_i)==\frac{1}{2}\,\left(1+2\,e^{-p/2}\right)\,(p-\phi_i)\ \&\&\ p>0,\ p,\ \text{Reals}\right]$$

... Solve: This system cannot be solved with the methods available to Solve.

In[∘]:= $\text{Solve}\left[1 + 2\, e^{-p/2} + \frac{1}{2}\, e^{-p/2}\, (p - \phi_i) == \frac{1}{2}\left(1 + 2\, e^{-p/2}\right) (p - \phi_i)\ \&\&\ p > 0,\ p,\ \mathbb{R}\right]$

$\text{ProductLog}\left[1, \frac{e^{-1+\alpha\, p_j - \alpha\, \phi_i}}{1 + e^{\alpha\, p_j}}\right]$

... Solve: This system cannot be solved with the methods available to Solve.

Out[∘]= $\text{Solve}\left[1 + 2\, e^{-p/2} + \frac{1}{2}\, e^{-p/2}\, (p - \phi_i) == \frac{1}{2}\left(1 + 2\, e^{-p/2}\right) (p - \phi_i)\ \&\&\ p > 0,\ p,\ \mathbb{R}\right]$

In[∘]:= $\text{Evaluate}\left[\text{ProductLog}\left[1, \frac{e^{-1+\alpha\, p_j - \alpha\, \phi_i}}{1 + e^{\alpha\, p_j}}\right]\right]$

In[∘]:= $\text{NEvaluate}\left[\text{ProductLog}\left[2, \frac{e^{-1+(1/2)\ (5)-(1/2)\ (3)}}{1 + e^{(1/2)\ (5)}}\right]\right]$

Out[∘]= $\text{NEvaluate}\left[\text{ProductLog}\left[2, \frac{1}{1 + e^{5/2}}\right]\right]$

In[∘]:= `ProductLog[1, 1.5]`

Out[∘]= $-1.12168 + 4.46634\, i$

In[∘]:= $\text{ProductLog}\left[1, \frac{1}{1 + e^{5/2}}\right]$

Out[∘]= $\text{ProductLog}\left[1, \frac{1}{1 + e^{5/2}}\right]$

In[∘]:= $\frac{1}{1 + e^{5/2}}$

Out[∘]= $\frac{1}{1 + e^{5/2}}$

In[∘]:= $N\left[\frac{1}{1 + e^{5/2}}\right]$

Out[∘]= 0.0758582

In[∘]:= `ProductLog[1, %]`

In[∘]:= -4.3391874493414315` + 3.869884106659305` i

$\text{ProductLog}\left[1, N\left[\frac{1}{1 + e^{5/2}}\right]\right]$

Out[∘]= $-4.33919 + 3.86988\, i$

Out[∘]= $-4.33919 + 3.86988\, i$