

$$\text{In}[*]:= \text{Solve}\left[\frac{-2\beta}{3\lambda + \phi_1 - \phi_2} \left(\frac{-\phi_2^2 + \phi_1(3\lambda - \phi_1) + \phi_2(3\lambda + 2\phi_1)}{6\lambda} - \left(\pi_a - \frac{c(3\lambda + \phi_2 - \phi_1)}{6\lambda} \right) \right) + \right. \\ \left. (1 - \beta) \left(\frac{1}{2} + \frac{\phi_1 - \phi_2}{3\lambda} - \pi_b - \frac{c}{6\lambda} \right) = 0, \phi_2 \right]$$

$$\text{Out}[*]= \left\{ \left\{ \phi_2 \rightarrow \frac{1}{4} \left(-c + 3c\beta + 9\lambda - 3\beta\lambda - 6\lambda\pi_b + 6\beta\lambda\pi_b + 4\phi_1 - \sqrt{\left((c - 3c\beta - 9\lambda + 3\beta\lambda + 6\lambda\pi_b - 6\beta\lambda\pi_b - 4\phi_1)^2 - 8(-3c\lambda - 3c\beta\lambda + 9\lambda^2 - 9\beta\lambda^2 + 12\beta\lambda\pi_a - 18\lambda^2\pi_b + 18\beta\lambda^2\pi_b - c\phi_1 + 3c\beta\phi_1 + 9\lambda\phi_1 - 15\beta\lambda\phi_1 - 6\lambda\pi_b\phi_1 + 6\beta\lambda\pi_b\phi_1 + 2\phi_1^2) \right)} \right) \right\}, \right. \\ \left. \left\{ \phi_2 \rightarrow \frac{1}{4} \left(-c + 3c\beta + 9\lambda - 3\beta\lambda - 6\lambda\pi_b + 6\beta\lambda\pi_b + 4\phi_1 + \sqrt{\left((c - 3c\beta - 9\lambda + 3\beta\lambda + 6\lambda\pi_b - 6\beta\lambda\pi_b - 4\phi_1)^2 - 8(-3c\lambda - 3c\beta\lambda + 9\lambda^2 - 9\beta\lambda^2 + 12\beta\lambda\pi_a - 18\lambda^2\pi_b + 18\beta\lambda^2\pi_b - c\phi_1 + 3c\beta\phi_1 + 9\lambda\phi_1 - 15\beta\lambda\phi_1 - 6\lambda\pi_b\phi_1 + 6\beta\lambda\pi_b\phi_1 + 2\phi_1^2) \right)} \right) \right\} \right\}$$

In[*]:=

(*In the above, pi_a is pi_h(Gsetminusminus=2) and pi_b is the partial derivative of it with respect to phi2. Obviously this isn't a very nice expression. A general closed form solution may be difficult. I'll instead proceed by virtue of substituting functional forms for pi_a and pi_b.*)
(*First, I consider the active beliefs setting. The active beliefs setting reduces things significantly, since pi_a becomes (v-\lambda)/2 and pi_b is zero. Further, I substitute \beta=1/2. *)

$$\text{Solve}\left[\frac{1}{2} \left(\frac{-2}{3\lambda + \phi_1 - \phi_2} \right) \left(\frac{1}{6\lambda} (-\phi_2^2 + \phi_1(3\lambda - \phi_1) + \phi_2(3\lambda + 2\phi_1)) - \frac{(v - \lambda)}{2} \right) + \right. \\ \left. \left(\frac{1}{2} \right) \left(\frac{1}{2} + \frac{(\phi_1 - \phi_2)}{3\lambda} - 0 \right) = 0, \phi_2 \right]$$

$$\text{Out}[*]= \left\{ \left\{ \phi_2 \rightarrow \frac{1}{8} \left(15\lambda + 8\phi_1 - \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right) \right\}, \right. \\ \left. \left\{ \phi_2 \rightarrow \frac{1}{8} \left(15\lambda + 8\phi_1 + \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right) \right\} \right\}$$

$$\text{In}[*]:= \left\{ \left\{ \phi_2 \rightarrow \frac{1}{8} \left(15\lambda + 8\phi_1 - \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right) \right\}, \right. \\ \left. \left\{ \phi_2 \rightarrow \frac{1}{8} \left(15\lambda + 8\phi_1 + \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right) \right\} \right\}$$

(*The above then characterize the functional form of Phi_2 in terms of Phi_1 for the active beliefs case. We then want to know how this behaves: namely, what is the derivative of this function with respect to Phi_1. This tells us whether or not there exists a second mover's advantage. The above solution varies significantly from the case Eric described, however. *)

$$D\left[\frac{1}{8} \left(15\lambda + 8\phi_1 - \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right), \phi_1\right]$$

$$\text{Out}[*]= \left\{ \left\{ \phi_2 \rightarrow \frac{1}{8} \left(15\lambda + 8\phi_1 - \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right) \right\}, \right. \\ \left. \left\{ \phi_2 \rightarrow \frac{1}{8} \left(15\lambda + 8\phi_1 + \sqrt{3} \sqrt{-32v\lambda + 59\lambda^2 + 64\lambda\phi_1} \right) \right\} \right\}$$

$$\text{Out}[*]= \frac{1}{8} \left(8 - \frac{32 \sqrt{3} \lambda}{\sqrt{-32 v \lambda + 59 \lambda^2 + 64 \lambda \phi_1}} \right)$$

$$\text{In}[*]:= D \left[\frac{1}{8} \left(15 \lambda + 8 \phi_1 + \sqrt{3} \sqrt{-32 v \lambda + 59 \lambda^2 + 64 \lambda \phi_1} \right), \phi_1 \right]$$

$$\text{In}[*]:= \frac{1}{8} \left(8 + \frac{32 \sqrt{3} \lambda}{\sqrt{-32 v \lambda + 59 \lambda^2 + 64 \lambda \phi_1}} \right)$$

(*Whether or not there is a second-mover's advantage then depends on the behavior of the above expression in response to Lambda and v. *)

$$\frac{1}{8} \left(8 + \frac{32 \sqrt{3} (5)}{\sqrt{-32 (25) (5) + 59 (5)^2 + 64 (5) (19.25)}} \right)$$

$$\text{Out}[*]= \frac{1}{8} \left(8 + \frac{32 \sqrt{3} \lambda}{\sqrt{-32 v \lambda + 59 \lambda^2 + 64 \lambda \phi_1}} \right)$$

$$\text{Out}[*]= 1.57456$$

In[*]:= (*Plugging in numerical values*)

$$\frac{1}{8} \left(15 (5) + 8 (19.375) + \sqrt{3} \sqrt{-32 (25) (5) + 59 (5)^2 + 64 (5) (19.375)} \right)$$

In[*]:= 41.875`

$$\frac{1}{8} \left(15 (5) + 8 (19.375) - \sqrt{3} \sqrt{-32 (25) (5) + 59 (5)^2 + 64 (5) (19.375)} \right)$$

$$\text{Out}[*]= 41.875$$

15.625000000000002`

(*NIIIIIIIIIIIIIIIIIIIIIIICE - the (-) equation works/is consistent with Table 6 for the active beliefs sequential case.*)

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