Homework 7

1.

a) If $\det(A) \neq 0$ then the matrix is invertible, so it has no free variables. Therefore the nullity of A is 0.

b) The nullity is 1 because the matrix is not invertible so it has 1 free variable so the nullity has to be one.

2. Determinant $= 1(-3) - a(4-a) - 2(-3) = a^2 - 4a + 3 = (a-3)(a-1)$

So, setting determinant equal to zero

$(a-3)(a-1) = 0$

$a = 1, 3$ matrix is not invertible

3. $\det((A^{-1})^2 B^4 A^T (5B)) = \frac{1}{9} \cdot 256 \cdot 3 \cdot 15625 = -5333333\frac{1}{3}$

4. $\ker\{T\} \neq \{0\}$ means that the transformation is not 1-1. So, the $\det(A) = 0$

$\det(A) = a^2 + b^2$, Since $a, b \neq 0$ the determinant can never be zero.

So, a & b cannot be found that satisfy the original condition.

5. For there to be a unique solution, the determinant of the matrix $\begin{bmatrix} 2s & 1 \\ 3s & 6s \end{bmatrix} = 12s^2 - 3s \neq 0$

$3s(4s-1) \neq 0$

$s \neq 0, 1/4$

6.

$$A - \lambda I_3 = \begin{bmatrix} 0-\lambda & 1 & 2 \\ 1 & 1-\lambda & 1 \\ 2 & -1 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I_3) = -\lambda(-\lambda + \lambda^2 + 1) - 1(-\lambda - 2) + 2(-1 - 2 + 2\lambda)$$

$$= \lambda^2 - \lambda^3 - \cancel{\lambda} + \cancel{\lambda} + \cancel{2} - \cancel{2} - 4 + 4\lambda$$

$$= -(\lambda^3 - \lambda^2 - 4\lambda + 4) = 0$$

$$= -(\lambda^2(\lambda - 1) - 4(\lambda - 1)) = 0$$

$$(\lambda^2 - 4)(\lambda - 1) = 0$$

$$\lambda = \pm 2, 1$$

7. True. If $\det(A^2) > 0$, Then determinant of $A^2 \cdot A = \det(A^2)\det(A) > 0$.
So, $\det(A^3) > 0$, meaning it is invertible.

True. By the definition of inverse matrices, $\det(A) = \det(C)\det(B)\det(C^{-1})$.
Since the $\det(C)$ and $\det(C^{-1})$ are reciprocals they will cancel leaving
just $\det(A) = \det(B)$ ✓