APMA 3080 Worksheet 2

1. $\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + 5\begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

2.
a) $x_1\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 1 \\ 0 \\ 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 7 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \end{bmatrix}$

Span

1. $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \end{bmatrix} \in \text{span}\left\{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 7 \\ -1 \end{bmatrix}\right\}$

2. a) $\boxed{\text{False}}$ 2 vectors can atmost span R^2, so four vectors with two variables can also span only R^2. So, 2 vectors can span the same region as 4 vectors.

b) Need atleast three vectors to span R^3. So, 2 vectors would not be able to. $\boxed{\text{False}}$

c) $\left[\begin{array}{cc|c} 1 & -1 & 20 \\ -2 & 1 & -27 \\ 3 & 2 & -5 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -13 \\ 0 & 0 & 0 \end{array}\right]$ $\boxed{\text{TRUE}}$

There is a solution for the RREF
$\{ x = 7, y = -13$, so $[20, -27, -5]$ is in the span$\}$

5. $\left[\begin{array}{cc|c} 2 & -4 & 12 \\ 2 & -7 & 18 \\ -1 & 3 & h \end{array}\right] \xRightarrow{\substack{-R_1+R_2 \\ 1/2R_1+R_3}} \left[\begin{array}{cc|c} 2 & -4 & 12 \\ 0 & -3 & 6 \\ 0 & 1 & h+6 \end{array}\right] \xRightarrow{1/3R_2+R_3} \left[\begin{array}{cc|c} 2 & -4 & 12 \\ 0 & -3 & 6 \\ 0 & 0 & h+8 \end{array}\right]$

$h+8=0$ for the system to be consistent

$\boxed{h=-8}$

6.

a) $\begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \end{bmatrix}$ Since every row has a non-zero term the vectors span R^3.

b) $\begin{bmatrix} 2 & 1 & 0 \\ 6 & -3 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1/12 \\ 0 & 1 & 1/6 \end{bmatrix}$

Every row has a pivot, so vectors span all of R^2.

c) $\begin{bmatrix} 2 & 1 & -3 & 5 \\ 1 & 4 & 2 & 6 \\ 0 & 3 & 3 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Since the last row is all rows, the z-coordinates cannot be span

there fore, the four vectors do not span R^3.

d) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$