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4 authors, including:


Azura che soh
Universiti Putra Malaysia


Mohammad Hamiruce Marhaban
Universiti Putra Malaysia


Rubiyah Yusof
Universiti Teknologi Malaysia

Modelling and Optimisation of a Traffic Intersection Based on Queue Theory and Markov Decision Control Methods

Azura Che Soh, Mohammad Hamiruce Marhaban

Department of Electrical & Electronic Engineering,
Universiti Putra Malaysia,
43400, Serdang, Selangor.
E-mail:azura@eng.upm.edu.my

Marzuki Khalid, Rubiyah Yusof

Centre for Artificial Intelligence and Robotics(CAIRO),
Universiti Teknologi Malaysia,
54100 Jalan Semarak, Kuala Lumpur.
E-mail: marzuki.khalid@gmail.com

Abstract

Traffic models play an important role in both today's traffic research and in many traffic applications such as traffic flow prediction, incident detection and traffic control. Modelling traffic dynamics and optimising the control signal are two interrelated problems. Modelling provides fundamental understanding of traffic dynamics and behaviour. In this paper, traffic signal is modelled as a M/M/1 queueing theory. The validation of a simulation model (M/M/1 queue) with different arrival rates is presented. From the result, a traffic light model was developed by applying M/M/1 queue theory for single intersection. In the optimisation strategy, the Markov decision control is applied to minimize queue length and waiting time. Simulation results show the excellent potential of this approach.

1. Introduction

As computer technology has improved, computer models have replaced manual setting and optimisation of signal timing plans. These powerful models use computer simulation to create an optimal signal-timing plan that either minimizes queue length or minimizes total delay. The basic ingredients of these models include (a) a traffic flow model, and (b) an algorithm for optimising a specified performance criterion.

For example, the most widely used computer software package for traffic signal control, TRANSYT-7F (TRAffic Network StudY Tool) [1] relies on historical data and is considered to be an effective off-line control strategy. In SCOOT (Split, Cycle, Offset, Optimisation Technique) [2] and SCATS (Sydney Coordinated Adaptive Traffic System) [3], the control strategy is to match the current traffic conditions obtained from detectors to the best recalculated off-line timing plan.

Generally, these conventional traffic signal control approaches do not hold much promise to achieve fully real-time adaptive control.

Models from queueing theory are now widely recognized as useful aids toward understanding and controlling congestion while maintaining throughput in many production, service, and transportation systems. These include computer systems, voice and data telecommunications, vehicular traffic flow, emergency public service, and industrial customer shops, production lines, and flexible manufacturing systems.

Queueing theory is almost exclusively used to describe traffic behaviour at signalised and unsignalised intersections [4],[5].Queues occur whenever instantaneous demand exceeds the capacity to provide a service. Queueing theory involves the mathematical study of these waiting lines. Using a large number of alternative mathematical models, queueing theory provides various characteristics of the waiting line, like waiting time or length of the queue.

In this paper we will use queueing theory (M/M/1) and standard approach techniques to model single intersection. Setting signals at intersections to minimize the queue length and to minimize vehicle delay time is an important goal in traffic management. The optimisation techniques considered here is Markov decision control, which can be successfully applied to real-time adaptive control strategy for single intersection in the M/M/1 model.

The paper is organized as follows. Section 2 and Section 3 briefly review the queueing theory model and Markov decision control techniques. The Section 4 of the paper focuses on the modelled of the traffic intersection using queueing model and standard approach techniques. In Section 5 shows the optimisation strategy applied to signal timing at intersection in traffic management to reduce the queue length and waiting time in single intersection. Section 6 reports the results from simulations of the validation and optimisation techniques performed under different traffic arrivals rate pattern. Finally, the main conclusions of this paper are summarized in Section 7.

2. Single queueing model

All In the simplest queuing system as shown in Figure 1, customers arrive one at a time. A single server processes the customers one at a time. An arriving customer that finds the idle server enters service immediately; otherwise, it enters a buffer and joins the end of the queue of the customers waiting for service.

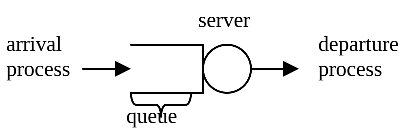

Figure 1. A simple queueing system.

Among others, a queueing model is characterized by[6]:

- The arrival process of customers.
  Usually we assume that the inter-arrival times are independent and have a common distribution. In many practical situations customers arrive according to a Poisson stream (i.e. exponential inter-arrival times). Customers may arrive one by one, or in batches.
- The behaviour of customers.
  Customers may be patient and willing to wait (for a long time). Or customers may be impatient and leave after a while.
- The service times.
  Usually we assume that the service times are independent and identically distributed, and that they are independent of the inter-arrival times.
- The service discipline.
  Customers can be served one by one or in batches. We have many possibilities for the order in which they enter service: FIFO –First In First Out, FCFS – First Come First Serve, priorities, processor sharing.
- The service capacity
  There may be a single server or a group of servers helping the customers.
- The waiting room.
  There can be limitations with respect to the number of customers in the system.

Kendall introduced a shorthand notation to characterize a range of these queueing models. It is a three-part code a=b=c. The first letter specifies the inter-arrival time distribution and the second specifies the service time distribution. In addition, G is used for a general distribution, M is used for an exponential distribution (M stands for Memoryless), and D is used for deterministic times. The third and last letter specifies the number of servers. The notation can be extended with an extra letter to cover other queueing models. For example, a system with exponential inter-arrival and service times, one server and having waiting room only for N customers (including the one in service) is abbreviated by the four letter code M=M=1=N[6].

Relevant performance measures in the queueing models by using the assumptions of Poisson arrivals (with λ = arrival rate) and Exponential service times (with μ = service rate) the following quantitative analysis can be applied for traffic light systems [6].

- Probability that the service facility is idle (probability of 0 units in the system)

$$P_0 = 1 - \lambda/\mu \tag{1}$$

- Probability of n units vehicle in the system (waiting time and service time)

$$P_n = (\lambda/\mu)^n \, P_0 \tag{2}$$

- The average number of vehicle in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \tag{3}$$

- The average number of vehicles in the system

$$L = L_q + (\lambda/\mu) \tag{4}$$

- The average time a vehicle spends waiting in the queue

$$W_q = L_q/\lambda \tag{5}$$

- The average time a vehicle spends in the system

$$W = W_q + 1/\mu \tag{6}$$

3. Markov decision control

A traffic control problem can be formulated as a decision-making problem for a stochastic dynamic system. A Markov model is the mathematical model of stochastic processes. The process is to generate random sequences of outcomes according to certain probabilities.

A discrete time, stationary Markov control model is defined on (X, A, P, R) where X is the state space and every element $x \in X$ is called a state; A is the set of all possible controls (or alternatives); P is a probability measure space, in which an element $P_{i,j}^{k}$ denotes the transition probability from state i to state j under alternative k; and R is a measurable function, also called a one-step reward [7].

From [7], we choose particular alternative results in an immediate reward and a transition probability to the next step. The ultimate objective is to find the supremum (least upper bound) of the total expected discounted reward over an infinite period of time:

$$J = E\left[\sum_{t=0}^{\infty} \beta^{t} r(x_t, a_t)\right] \tag{7}$$

where r is the one-step transition reward, $\beta (0 \le \beta < 1)$ is the discount factor, and a is the policy. The optimal reward v^* is defined as:

$$v^*(x, a^*) = \sup_{a \varepsilon A}[J(x, a)] \tag{8}$$

It can be obtained by solving a Dynamic Programming Equation (DPE):

$$v^* = Tv^* \tag{9}$$

where T is a contraction mapping and:

$$Tv(x) = \max_{a \varepsilon A}\left[r(x, a) + \beta \sum_{j=1}^{N} v(x) p_{i,j}^{a}\right] \tag{10}$$

It has been proven that the optimal solution of the above DPE is unique and can be calculated iteratively by the successive approximation method:

$$v_n(x) = \max_{a \varepsilon A}\left[r(x, a) + \beta \sum_{j=1}^{N} v_{n-1}(x) p_{i,j}^{a}\right]$$

(11)

Therefore, for specific control problem, once the transition matrix and the reward matrix are determined, and then by maximizing the total expected reward, a policy of choosing a certain alternative in each state will be obtained. This represents the optimal strategy, which we will undertake in this problem.

4.Traffic dynamic model for an intersection using queueing theory

A typical four-legged intersection is as shown in Figure 2. There are four approaches in this intersection and each one of them has one through movement and one left turn movement, summing to a total of 8 movements.

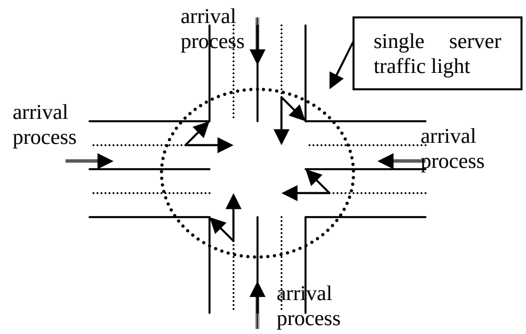

Figure 2. A typical traffic intersection.

The three main concepts in queueing theory are customers, queues, and servers (service mechanisms). In general, in a queueing system, customers for the queueing system are generated by an input source. The customers are generated according to a statistical distribution and the distribution describes their inter-arrival times, in other words, the times between arrivals of customers. The customers join a queue. At various times, customers are selected for service by the server (service mechanism). The basis on which the customers are selected is called the queue discipline.

Traffic signal in this paper is modelled as a M/M/1 queue theory. Here "M" stands for a "memoryless" (that is, exponential) distribution of inter-arrival times. The "M" in the second position stands for a "memoryless" distribution of "i.i.d." service times. The "1" in the third position stands for "one server". The single intersection has a single server, traffic signal, which provides service to a single signal phase at a time. The vehicles queue has the FIFO discipline.

Traffic arrival and service times at a given intersection are considered as independent random variables, with known distributions. Due to the random nature of traffic arrival, the Poisson distribution usually makes a good fit for the memoryless nature of the exponential distribution which has been widely accepted by researchers in fitting randomly distributed service times, such as those at signalised intersections [8].

Vehicles arrive at a single-server facility according to a Poisson process with mean arrival rate λ (vehicles per unit time). Equivalently, the inter-arrival times between vehicles are independent and identically distributed with mean $1/\lambda$. Vehicles, therefore, enter the system according to a Poisson process with arrival rate λ.

Let $q_{in}(t)$ represents the vehicles that enter the system in interval of time $(0,t)$, so that $q_{in}(t)$ is independent. If Δt is small, the probability of one arriving car in $(t, t+\Delta t)$ is independent of t. The probability of two or more arriving cars can be negligible in comparison with the probability of one arriving in a very short interval of time. If these conditions are met, the arrivals of vehicles is considered to be a Poisson process, and the number of arriving vehicles in system per time period obeys the Poisson distribution, which has a PDF as follows [6]:

$$p\{q_{in}(t)=k\}=\frac{(\lambda\Delta t)^k e^{-\lambda\Delta t}}{k!} \qquad (12)$$

where $(\lambda > 0)$ is the arriving rate, that is the mean number of arriving vehicles per time period and k =0,1,2…

Service time is defined as the time used to discharge the individual vehicles from the intersection during the time where traffic light stays green. This should not be confused with the total service time of a given signal phase, which is the effective green time or green phase length. The departure process is the time to cross the intersection (service times) and is arbitrarily and independently distributed.

Assume a continuous traffic flow process that is sampled at every Δt time interval with discrete time index, k. The output of the intersection (i.e., number of vehicles leaving this intersection/departure process) $q_{out}(k)$ can be defined as the following vector:

$$q_{out}(k)=\left[q_{out}^1(k), q_{out}^2(k),..., q_{out}^8(k)\right]^{T_s} \qquad (13)$$

where q^j (j =1,2,...,8) denotes the j-th movement and T_s is the total service time.

In the departure process, the number of vehicles leaving the intersection is dependent on the total time service, T_s for the green phase length of jth movement. So the departure time for vehicles leaving the intersection this can be defined as,

$$T_s = t_{out_1}^j + t_{out_2}^j + t_{out_3}^j + ... + t_{out_{f-1}}^j + t_{out_f}^j \qquad (14)$$

where superscript j (j =1,2,...,8) denotes the j-th movement and t_{outf}^j is departure time for the last vehicles leaving the intersection.

The current queue $q(k)$ will be defined as:

$$q(k)=[q^1(k), q^2(k),..., q^8(k)]^{T_s} \qquad (15)$$

$q_{out}(k)$ can be further expressed as a function of the current control of the intersection, $u(k)$, and $q(k)$:

$$q_{out}(k) = f_{out}(u(k), q(k)) \qquad (16)$$

where $u(k)$ and $f_{out}(k)$ are also vectors and

$$f_{out}(k)=\left[f_{out}^1(k), f_{out}^2(k),..., f_{out}^8(k)\right]^{T_s} \qquad (17)$$

and

$$f_{out}^j(\cdot)=\begin{cases}\min\left[q^j(k); \dfrac{\Delta t}{h_{\min}}\right], when \quad u^j(k)=1\\ 0, when \quad u^j(k)=0\end{cases} \qquad (18)$$

where $j=1,2,..,8$. $h_{\min}$ is the minimum headway, and $u^j(k)$ is the control signal for the jth movement: $u^j(k)=1$ denotes a green signal, $u^j(k)=0$ indicates a red signal.

Headway (h), sometimes called time headway, is a measure of the time interval between the front of one vehicle to the front of a following vehicle, i.e.

$$h = t_{out_f}^j - t_{out_{f-1}}^j \qquad (19)$$

The current queue $q(k)$ can be also be written as:

$$q(k)=q(k-1)+q_{in}(k)-q_{out}(k) \qquad (20)$$

where $q(k-1)$ is the queue at the previous time instant $(k-1)$ and $q_{in}(k)$ is the input (number of vehicles) during time interval $[k-1,k)$.

5. Traffic signal control using markov decision control and queueing theory

To apply Markov decision control to traffic systems, a state space X and a probability measure P must be defined [7]. A threshold (number of vehicles) is chosen for the queue of each movement at an intersection. If the queue length of a specific movement is greater than the threshold value, this movement is defined to be in its congestion mode; otherwise it is in the non-congestion mode. These two modes (congestion and non-congestion) are defined as the two states in the state space X. The signal phasing can be considered as different alternatives in each state.

The state space is discrete, thus the probability measure P is defined as a discrete transition law. An element of this matrix P, i.e., $P_{i,j}^{k}$ denotes the transition probability from state i to state j under alternative k. Let $\pi(k)$ be a row vector of state probabilities (i.e., $\pi_i(k)$ is the probability that the system will occupy state i after k transitions). In the traffic control problem, the probability matrix P is time-varying due to the time-varying traffic flow, therefore:

$$\pi(k+1) = f_{\pi}\left[\pi(k), P(k)\right] \qquad (21)$$

where the probability matrix $P(k)$ is a function of the current queue, the estimated number of arrivals in the next time interval, and the control signal:

$$P(k) = f_p[q(k), \hat{q}_{in}(k+1), u(k), q_g] \qquad (22)$$

The probability matrix can be specified based on different arrival patterns. Under most circumstances, the arrival of vehicles for the external movements follows the Poisson distribution.

Assuming that at a specific time instant, the current queue length of a specific movement i is denoted by q, and there are q_g vehicles passing through the intersection if the signal of this direction is green, then:

$$P_{S_i \to N_i}^{U_i} = p(\hat{q}_{in}^{i} + q^{i} - \delta(u_i) q_g^{i} \le q_{threshold}^{i}) \qquad (23)$$

and

$$P_{S_i \to C_i}^{U_i} = 1 - p_{S_i \to N_i}^{U_i} \qquad (24)$$

where $$\delta(u_i) = \begin{cases} 1, & \text{when } u_i = G_i \\ 0, & \text{otherwise} \end{cases}$$

and $S_i = N_i, C_i$ (N_i for non-congestion and C_i for congestion); $u_i = G_i, R_i$ (G_i for green signal and R_i for red signal).

The reward matrix R has the same dimension and a similar definition to the probability matrix. The control objective herein is to minimize the queue length, so the functions of queue length corresponding to different states are chosen to generate the reward matrix:

$$R_{state1,state2}^{Ui} = f_u(q_0^{i}, q_{threshold}^{i}, u_i) \qquad (25)$$

In real-time traffic control problems, both the probability matrix and reward matrix are time varying. In this research, the sampling time is chosen to be the same as the minimum green extension time, Δt. Every Δt seconds, the P and R matrices are calculated. A decision is then made to choose the control signal for the next time interval based on the current measurement from the detector and our estimation. Once the optimal policy is found, it is implemented only for Δt seconds. At the next time step, the probability matrix and reward matrix are updated and the whole decision-making process is repeated.

6. Simulation results

6.1. Validation of a simulation model

We validate our models by using different stream of arrival rates, λ in running the simulation. The simulation results show that the model was validating correctly. The simulated results and theoretical results of different stream of arrival rates were in accordance.

Table 1. M/M/1 model with arrival rate, λ =3

Iteration	Average Arrival Times (A)	Average Depart Times (D)	Average Waiting Time in Queue (W_q)		Average Waiting Time in System (W)	
			empirical	theory	empirical	theory
1	116.207	116.595	1.212	1.000	0.388	0.333
2	122.123	122.427	0.858	1.000	0.304	0.333
3	122.104	122.443	0.975	1.000	0.338	0.333
4	111.566	111.874	0.922	1.000	0.308	0.333
5	123.487	123.772	0.807	1.000	0.284	0.333
6	119.275	119.629	1.077	1.000	0.354	0.333
7	123.326	123.676	1.054	1.000	0.349	0.333
8	120.785	121.119	1.031	1.000	0.335	0.333
9	119.063	119.383	0.970	1.000	0.319	0.333
10	119.847	120.148	0.866	1.000	0.300	0.333

Table 2. M/M/1 model with arrival rate, λ =6

Iteration	Average Arrival Times (A)	Average Depart Times (D)	Average Waiting Time in Queue (W_q)		Average Waiting Time in System (W)	
			empirical	theory	empirical	theory
1	117.541	117.840	1.787	1.500	0.299	0.250
2	120.565	120.839	1.648	1.500	0.275	0.250
3	117.433	117.692	1.559	1.500	0.259	0.250
4	121.455	121.703	1.483	1.500	0.248	0.250
5	118.908	119.147	1.480	1.500	0.239	0.250
6	117.094	117.295	1.181	1.500	0.201	0.250
7	120.026	120.226	1.187	1.500	0.200	0.250
8	119.351	119.579	1.316	1.500	0.228	0.250
9	118.997	119.245	1.526	1.500	0.249	0.250
10	116.967	117.189	1.350	1.500	0.222	0.250

6.2. Optimisation Strategy of Simulation

The Markov decision control was applied to a single intersection based on M/M/1 queue model (with a Poisson arrival pattern generated as the external input) was performed to evaluate its performance compared to the model without applied Markov decision control in the system.

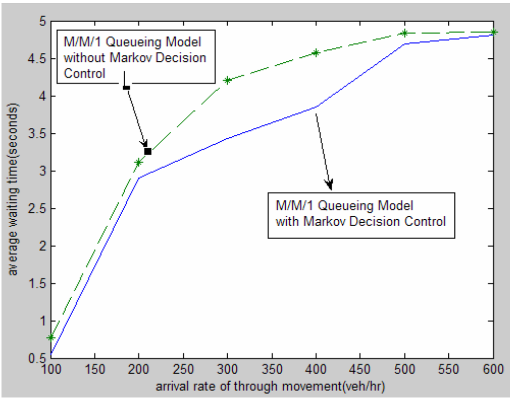

Figure 3. Average of waiting time

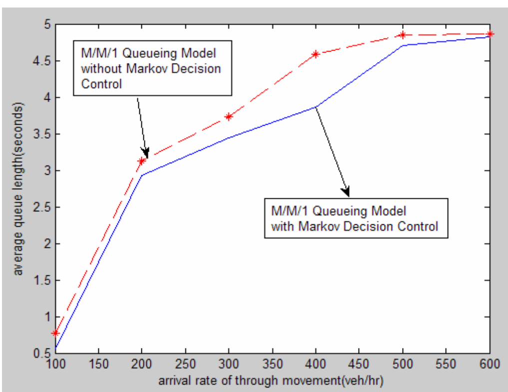

Figure 4. Average of queue length

The algorithms were tested on six different arrival rates, i.e., 100, 200, 300, 400, 500 and 600 vehicles per hour per movement. For each arrival rates, the algorithm was tested. The average of waiting times are plotted in Figure 3 and the average of queue length are also plotted in Figure 4, where the solid line represents the Markov algorithm applied to the M/M/1 queue model and the dotted line represents the model without Markov algorithm. It can be observed that the simulation results indicate that by applying the Markov adaptive control algorithm, the average of waiting time and average of queue length of single intersection can be reduced.

7. Conclusion

In this paper, a single intersection is successfully modelled as M/M/1 queueing theory and standard approach techniques. The proposed model is based on simulation of queues, where service is provided with a certain rate, and that requests for the service come in with a certain rate, usually different, rate. When the request rate is higher than the service rate, a queue for requests will form. The service rate here is generated by a traffic light. The arrival vehicles are assumed to follow Poisson distribution.

Markov Decision Control is applied for optimisation strategy to reduce the queue length and waiting time in single intersection. Computer simulation results for average waiting time and average queue length are also reported. From the simulation, the M/M/1 queueing model with Markov decision control is seen to outperform the M/M/1 queueing model without Markov decision control.

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