```
matN = {{1 - mz^2, (a / r) mr, 1 - mr^2}, {dz, dvz, 0}, {0, dvr, dr}}
```

$$\left\{\left\{1-mz^2,\ \frac{a\,mr}{r},\ 1-mr^2\right\},\ \{dz,\ dvz,\ 0\},\ \{0,\ dvr,\ dr\}\right\}$$

```
Det[matN]
```

$$\frac{-a\,dr\,dz\,mr+dr\,dvz\,r+dvr\,dz\,r-dvr\,dz\,mr^2\,r-dr\,dvz\,mz^2\,r}{r}$$

```
drdzP = (-mz mr + Sqrt[m^2 - 1]) / (1 - mz^2)
```

$$\frac{\sqrt{-1+m^2}-mr\,mz}{1-mz^2}$$

```
drdzN = (-mz mr - Sqrt[m^2 - 1]) / (1 - mz^2)
```

$$\frac{-\sqrt{-1+m^2}-mr\,mz}{1-mz^2}$$

```
dvz = Cos[θ] ⅆv - v Sin[θ] ⅆθ

mr = m Sin[θ] ; mz = m Cos[θ];

dvrdvzS
```

$$\frac{v\,\mathrm{Cos}[\theta]\ \mathrm{d}\theta+\mathrm{d}v\ \mathrm{Sin}[\theta]}{\mathrm{Cos}[\theta]\ \mathrm{d}v-v\ \mathrm{d}\theta\ \mathrm{Sin}[\theta]}$$

```
drP = drdzP dz
```

$$dz\ \mathrm{Tan}\left[\theta-\mathrm{ArcSin}\left[\frac{1}{m}\right]\right]$$

```
drN = drdzN dz
```

$$dz\ \mathrm{Tan}\left[\theta+\mathrm{ArcSin}\left[\frac{1}{m}\right]\right]$$

```
dvr = v Cos[θ] ⅆθ + ⅆv Sin[θ]
```

$$v\,\mathrm{Cos}[\theta]\ \mathrm{d}\theta+\mathrm{d}v\ \mathrm{Sin}[\theta]$$

```
dvz = Cos[θ] ⅆv - v ⅆθ Sin[θ]
```

$$\mathrm{Cos}[\theta]\ \mathrm{d}v-v\ \mathrm{d}\theta\ \mathrm{Sin}[\theta]$$

```
drdzP = Tan[θ + μ]; drdzN = Tan[θ - μ];

μ = ArcSin[1 / m]
```

$$\mathrm{ArcSin}\left[\frac{1}{m}\right]$$

```
(*EQUATIONS
```

$$\frac{-a\,\mathbb{d}r\,dz\,mr + dvr\,dz\,r\left(1 - mr^2\right) + dr\,dvz\,r\left(1 - mz^2\right)}{r} == 0$$

$$-\frac{\mathbb{d}r}{r}\,v\,\mathrm{Sin}[\theta]\,dz + dr\,dvz\left(1 - mz^2\right) + dvr\,dz\left(1 - mr^2\right) == 0$$

$$-\frac{\mathbb{d}r}{r}\,\mathrm{Sin}[\theta] + \frac{dr}{dz}\left(1 - m^2\,\mathrm{Cos}[\theta]^2\right)\left(\frac{\mathbb{d}v}{v}\,\mathrm{Cos}[\theta] - \mathbb{d}\theta\,\mathrm{Sin}[\theta]\right) + \left(1 - m^2\,\mathrm{Sin}[\theta]^2\right)\left(\frac{\mathbb{d}v}{v}\,\mathrm{Sin}[\theta] + \mathbb{d}\theta\,\mathrm{Cos}[\theta]\right) == 0$$

$$-\frac{\mathbb{d}r}{r}\,\mathrm{Sin}[\theta] + \mathrm{Tan}[\theta - \mu]\left(1 - m^2\,\mathrm{Cos}[\theta]^2\right)\left(\frac{\mathbb{d}v}{v}\,\mathrm{Cos}[\theta] - \mathbb{d}\theta\,\mathrm{Sin}[\theta]\right) + \left(1 - m^2\,\mathrm{Sin}[\theta]^2\right)\left(\frac{\mathbb{d}v}{v}\,\mathrm{Sin}[\theta] + \mathbb{d}\theta\,\mathrm{Cos}[\theta]\right) \text{ (* MINUS CHARACTERISTIC}$$

$$-\frac{\mathbb{d}r}{r}\,\mathrm{Sin}[\theta] + \mathrm{Tan}[\theta + \mu]\left(1 - m^2\,\mathrm{Cos}[\theta]^2\right)\left(\frac{\mathbb{d}v}{v}\,\mathrm{Cos}[\theta] - \mathbb{d}\theta\,\mathrm{Sin}[\theta]\right) + \left(1 - m^2\,\mathrm{Sin}[\theta]^2\right)\left(\frac{\mathbb{d}v}{v}\,\mathrm{Sin}[\theta] + \mathbb{d}\theta\,\mathrm{Cos}[\theta]\right) \text{ (* PLUS CHARACTERISTIC}$$

$$-\frac{\mathbb{d}r}{r}\,\mathrm{Sin}[\theta] + \frac{\mathbb{d}v}{v}\left(\mathrm{Cos}[\theta]\left(1 - m^2\,\mathrm{Cos}[\theta]^2\right)\frac{-\mathrm{Cos}[\theta] + \sqrt{1 - \frac{1}{m^2}}\,m\,\mathrm{Sin}[\theta]}{\sqrt{1 - \frac{1}{m^2}}\,m\,\mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]} + \mathrm{Sin}[\theta]\left(1 - m^2\,\mathrm{Sin}[\theta]^2\right)\right) + \mathbb{d}\theta\left(\mathrm{Cos}[\theta]\left(1 - m^2\,\mathrm{Sin}[\theta]^2\right) - \mathrm{Sin}[\theta]\left(1 - m^2\,\mathrm{Cos}[\theta]^2\right)\frac{-\mathrm{Cos}[\theta] + \sqrt{1 - \frac{1}{m^2}}\,m\,\mathrm{Sin}[\theta]}{\sqrt{1 - \frac{1}{m^2}}\,m\,\mathrm{Cos}[\theta] + \mathrm{Sin}[\theta]}\right) \text{ (*MINUS}$$

$$-\frac{dr}{r}\mathrm{Sin}[\theta]+$$

$$\frac{dv}{v}\left(\mathrm{Cos}[\theta]\left(1-m^2\mathrm{Cos}[\theta]^2\right)\frac{\mathrm{Cos}[\theta]+\sqrt{1-\frac{1}{m^2}}\,m\,\mathrm{Sin}[\theta]}{\sqrt{1-\frac{1}{m^2}}\,m\,\mathrm{Cos}[\theta]-\mathrm{Sin}[\theta]}+\mathrm{Sin}[\theta]\left(1-m^2\mathrm{Sin}[\theta]^2\right)\right)+$$

$$d\theta\left(\mathrm{Cos}[\theta]\left(1-m^2\mathrm{Sin}[\theta]^2\right)-\mathrm{Sin}[\theta]\left(1-m^2\mathrm{Cos}[\theta]^2\right)\frac{\mathrm{Cos}[\theta]+\sqrt{1-\frac{1}{m^2}}\,m\,\mathrm{Sin}[\theta]}{\sqrt{1-\frac{1}{m^2}}\,m\,\mathrm{Cos}[\theta]-\mathrm{Sin}[\theta]}\right) \text{ (*PLUS}$$

$$\frac{dr}{r}\mathrm{Sin}[\theta]+\frac{dv}{v}\left(\left(m^2-1\right)\mathrm{Sin}[\theta]-\sqrt{m^2-1}\,\mathrm{Cos}[\theta]\right)-d\theta\left(\mathrm{Cos}[\theta]-\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right) \text{ (*MINUS}$$

$$\frac{dr}{r}\mathrm{Sin}[\theta]+\frac{dv}{v}\left(\left(m^2-1\right)\mathrm{Sin}[\theta]+\sqrt{m^2-1}\,\mathrm{Cos}[\theta]\right)-d\theta\left(\mathrm{Cos}[\theta]+\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right) \text{ (*PLUS}$$

$$\frac{dr}{r}\frac{\mathrm{Sin}[\theta]}{\left(\mathrm{Cos}[\theta]-\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right)}+\frac{dv}{v}\frac{\left(\left(m^2-1\right)\mathrm{Sin}[\theta]-\sqrt{m^2-1}\,\mathrm{Cos}[\theta]\right)}{\left(\mathrm{Cos}[\theta]-\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right)}==d\theta \text{ (*MINUS}$$

$$\frac{dr}{r}\frac{\mathrm{Sin}[\theta]}{\left(\mathrm{Cos}[\theta]+\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right)}+\frac{dv}{v}\frac{\left(\left(m^2-1\right)\mathrm{Sin}[\theta]+\sqrt{m^2-1}\,\mathrm{Cos}[\theta]\right)}{\left(\mathrm{Cos}[\theta]+\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right)}==d\theta \text{ (*PLUS}$$

$$\frac{dr}{r}\frac{\mathrm{Sin}[\theta]}{\left(\mathrm{Cos}[\theta]-\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right)}-\frac{dv}{v}\sqrt{m^2-1}==d\theta$$

$$\frac{dr}{r}\frac{\mathrm{Sin}[\theta]}{\left(\mathrm{Cos}[\theta]+\sqrt{m^2-1}\,\mathrm{Sin}[\theta]\right)}+\frac{dv}{v}\sqrt{m^2-1}==d\theta$$

$$d\theta-\frac{dr}{r}\frac{1}{\left(\mathrm{Cot}[\theta]-\sqrt{m^2-1}\right)}+\frac{dm}{m}\frac{\sqrt{m^2-1}}{1+\frac{\gamma-1}{2}m^2}==0$$

$$d\theta - \frac{dr}{r}\,\frac{1}{\left(\mathrm{Cot}[\theta] + \sqrt{m^2 - 1}\right)} - \frac{dm}{m}\,\frac{\sqrt{m^2 - 1}}{1 + \frac{\gamma - 1}{2}\, m^2} == 0$$

```
dr / dx == Tan[θ - ArcCsc[m]]
dr / dx == Tan[θ + ArcCsc[m]]
```