HW 4

In [1]:
```
# Eric Liu
# Eric Wang
# Austin Du
```

Imports

In [2]:
```
library("ggplot2")
library("gridExtra")
library("dplyr")
library("reshape2")
library("visreg")
```

```
Attaching package: 'dplyr'

The following object is masked from 'package:gridExtra':

    combine

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

Load Data

In [3]:
```
gauge = read.table("./gauge.txt", sep="", header=T)
density = gauge$density
gain = gauge$gain
head(gauge)
```

Out[3]:

A data.frame: 6 × 2

	density	gain
	<dbl>	<dbl>
1	0.686	17.6
2	0.686	17.3
3	0.686	16.9
4	0.686	16.2
5	0.686	17.1
6	0.686	18.5

In [4]:
```
# the unique densities of the blocks
sort(unique(gauge$density))
```

Out[4]:

0.001 · 0.08 · 0.148 · 0.223 · 0.318 · 0.412 · 0.508 · 0.604 · 0.686

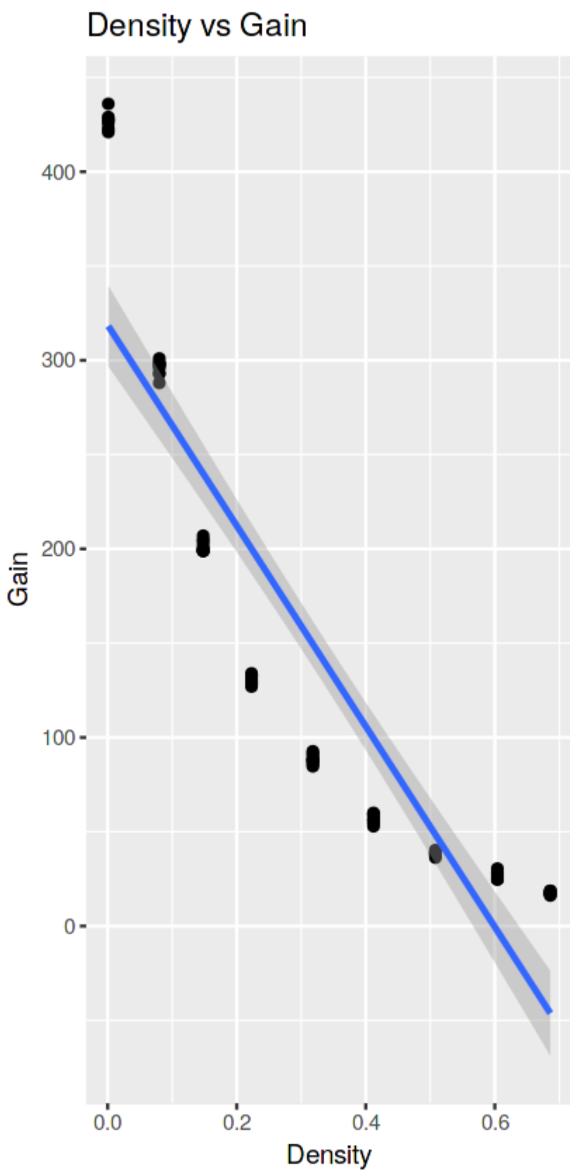
Best Fit Line

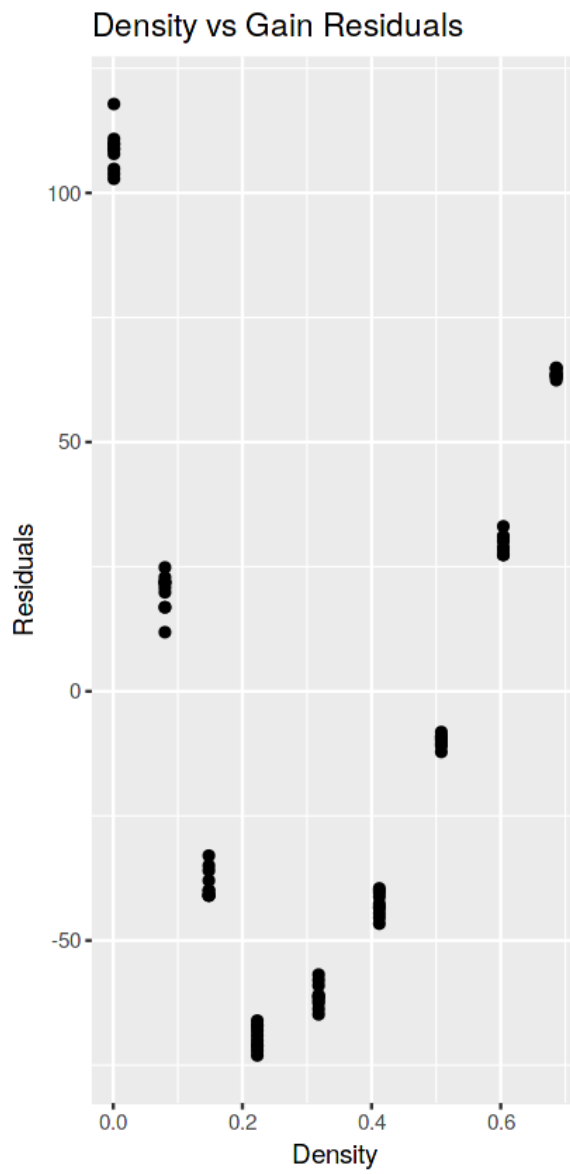
In [5]:
```
# function to print out the equation of the line and the sum of squared r
esiduals
summarize_fit = function(line) {
    cat("intercept:\t\t", line$coefficients[1], "\n")
    cat("slope:\t\t\t", line$coefficients[2], "\n")
    cat("sum squared residuals:\t", sum(resid(line)^2), "\n")
    cat("r squared:\t\t", summary(line)$r.squared)
}
```

In [6]:
```
# without any transformations, fit the data with linear regression and pl
ot the residuals
reg_plot = ggplot(gauge, aes(x=density, y=gain)) +
    geom_point() +
    stat_smooth(method='lm', formula = y~x) +
    labs(title="Density vs Gain", x="Density", y="Gain")

# compute the residuals for the linear regression and plot
lin_reg = lm(gauge$gain ~ gauge$density)
residuals = data.frame(x=gauge$density, resid=resid(lin_reg))
resid_plot = ggplot(residuals, aes(x=x, y=resid)) +
    geom_point() +
    labs(title="Density vs Gain Residuals", x="Density", y="Residuals")

# put both plots side by side
grid.arrange(reg_plot, resid_plot, ncol=2)

# print out the equation of the line and the sum of squared residuals
summarize_fit(lin_reg)
```

```
intercept:                318.7015
slope:                    -531.9507
sum squared residuals:    291335
r squared:                0.8156974
```

Out[6]:



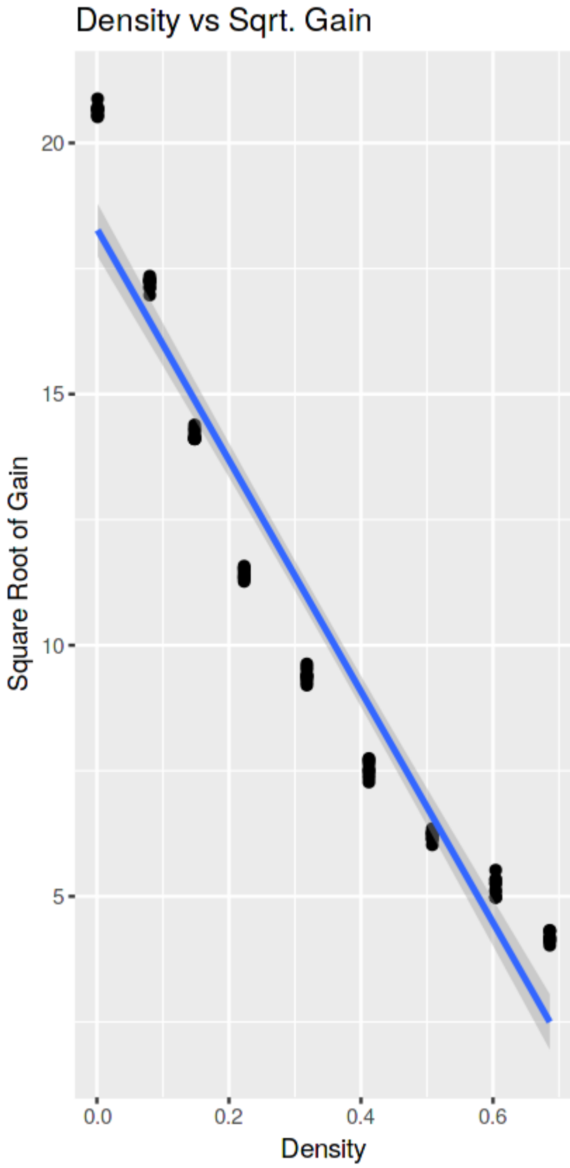


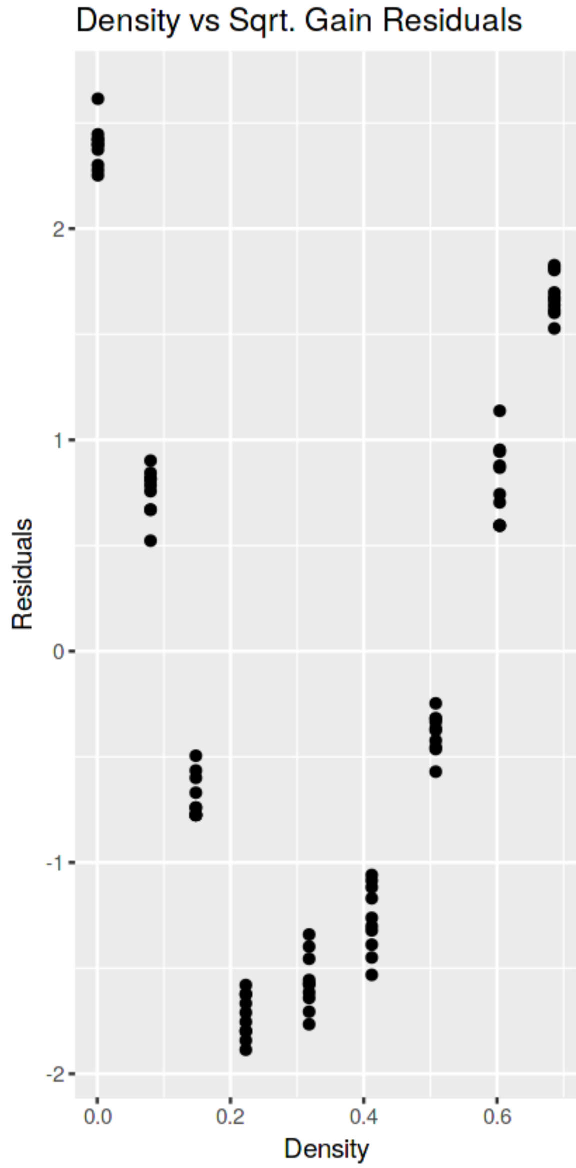
Since the residuals show a polynomial pattern, try doing polynomial regression

In [7]:
```
# transform data (take the sqrt of y axis) and fit to linear regression
gauge$sqrt_gain = sqrt(gauge$gain)
reg_plot = ggplot(gauge, aes(x=density, y=sqrt_gain)) +
    geom_point() +
    stat_smooth(method='lm', formula = y~x) +
    labs(title="Density vs Sqrt. Gain", x="Density", y="Square Root of Ga
in")

# compute the residuals for the linear regression and plot
lin_reg = lm(gauge$sqrt_gain ~ gauge$density)
residuals = data.frame(x=gauge$density, resid=resid(lin_reg))
resid_plot = ggplot(residuals, aes(x=x, y=resid)) +
    geom_point() +
    labs(title="Density vs Sqrt. Gain Residuals", x="Density", y="Residua
ls")

# put both plots side by side
grid.arrange(reg_plot, resid_plot, ncol=2)

# print out the equation of the line and the sum of squared residuals
summarize_fit(lin_reg)
```

```
intercept:               18.28919
slope:                   -23.02002
sum squared residuals:   175.6669
r squared:               0.9321839
```

Out[7]:



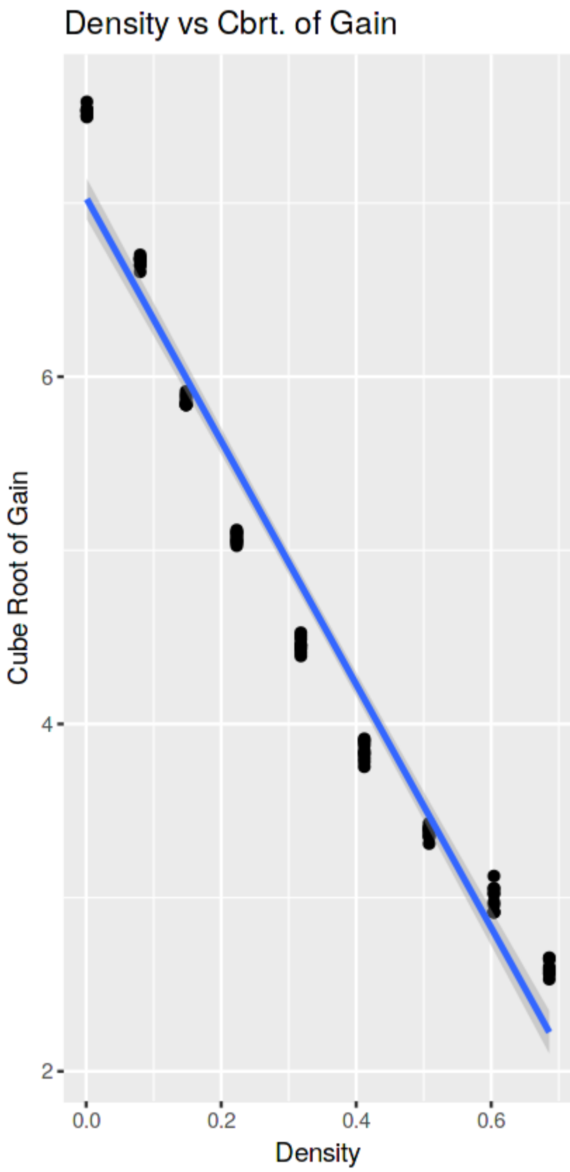


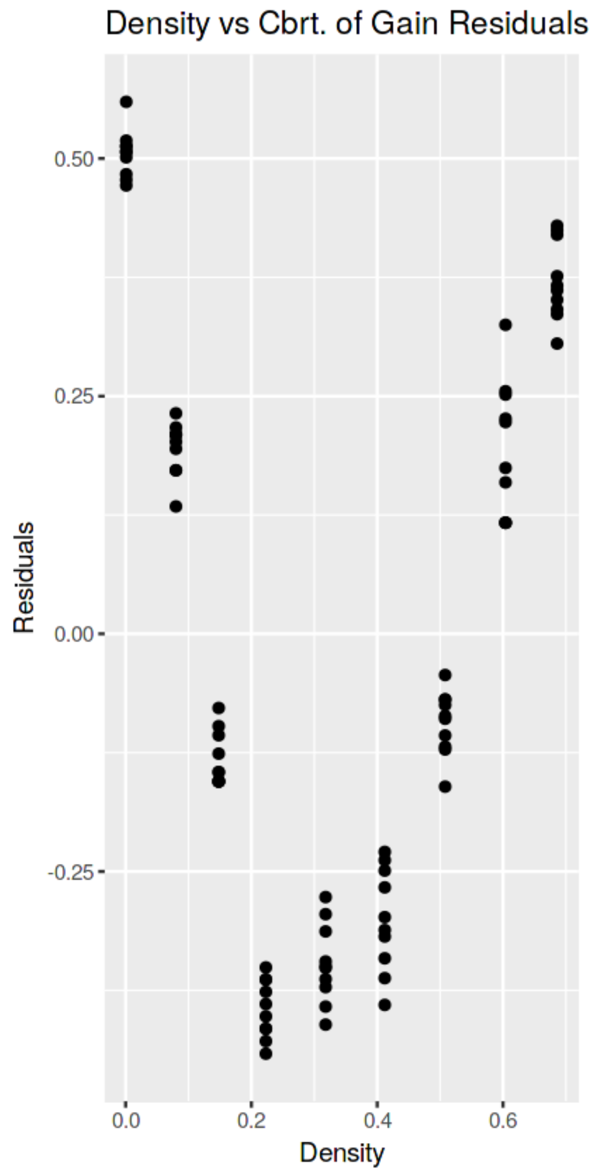
Since the residuals still show a polynomial pattern, try cubic regression

In [8]:
```
# transform data (take the cube root of y axis) and fit to linear regress
ion
gauge$cubert_gain = (gauge$gain)^(1/3)
reg_plot = ggplot(gauge, aes(x=density, y=cubert_gain)) +
    geom_point() +
    stat_smooth(method='lm', formula = y~x) +
    labs(title="Density vs Cbrt. of Gain", x="Density", y="Cube Root of G
ain")

# compute the residuals for the linear regression and plot
lin_reg = lm(gauge$cubert_gain ~ gauge$density)
residuals = data.frame(x=gauge$density, resid=resid(lin_reg))
resid_plot = ggplot(residuals, aes(x=x, y=resid)) +
    geom_point() +
    labs(title="Density vs Cbrt. of Gain Residuals", x="Density", y="Resi
duals")

# put both plots side by side
grid.arrange(reg_plot, resid_plot, ncol=2)

# print out the equation of the line and the sum of squared residuals
summarize_fit(lin_reg)
```

```
intercept:               7.03021
slope:                   -7.004705
sum squared residuals:   8.76708
r squared:               0.9622668
```

Out[8]:



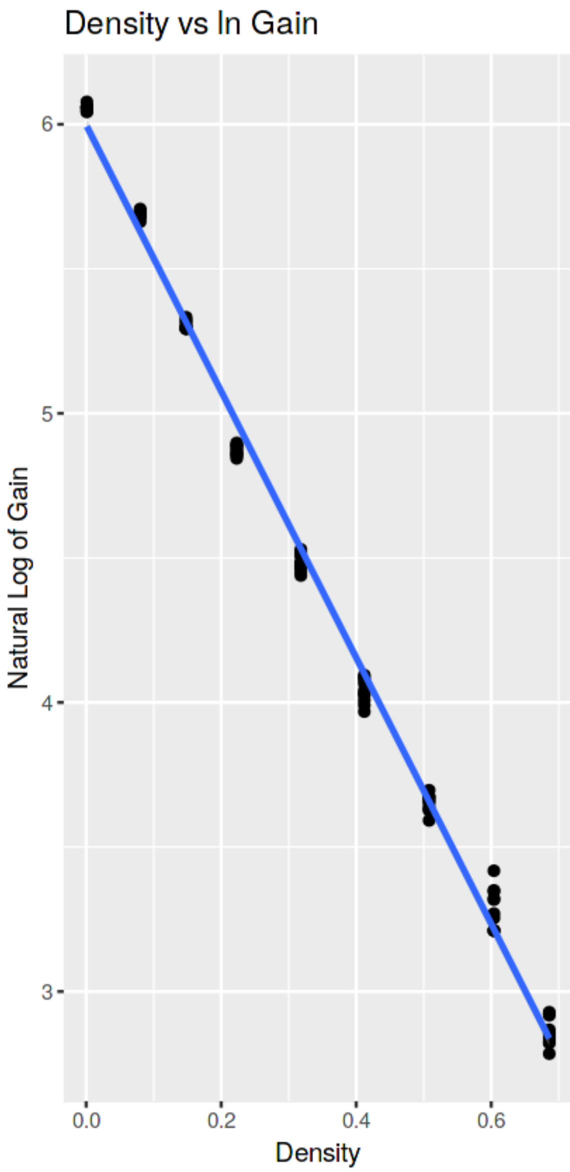


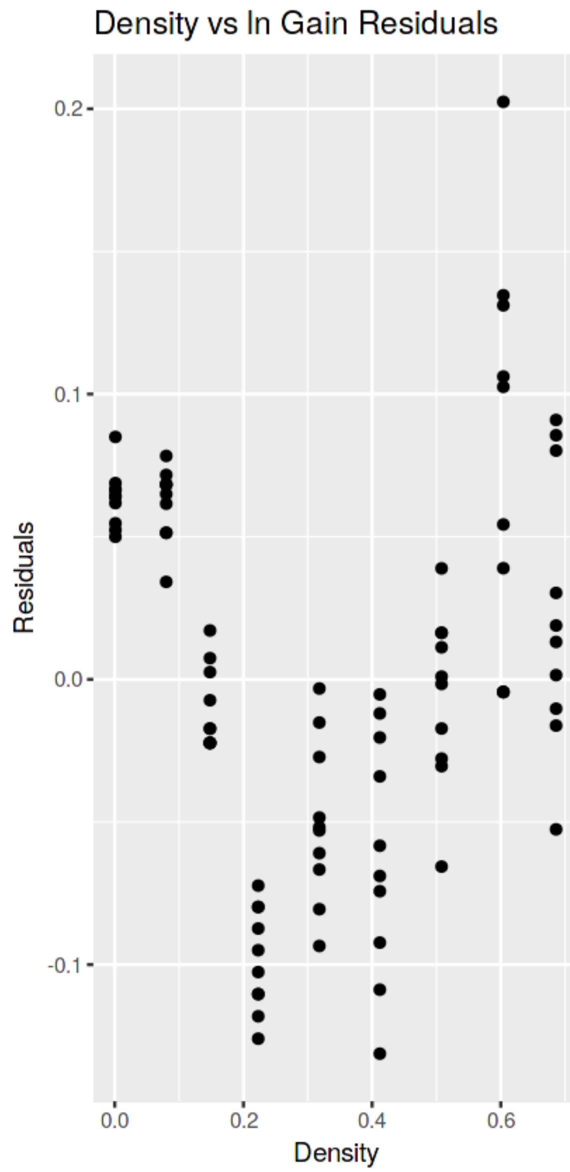
Since the residuals still show a polynomial residuals graph, let's try exponential distribution

In [9]:
```
# transform data (take the ln of y axis) and fit to linear regression
gauge$ln_gain = log(gauge$gain)
reg_plot = ggplot(gauge, aes(x=density, y=ln_gain)) +
    geom_point() +
    stat_smooth(method='lm', formula = y~x) +
    labs(title="Density vs ln Gain", x="Density", y="Natural Log of Gain"
)

# compute the residuals for the linear regression and plot
lin_reg = lm(gauge$ln_gain ~ gauge$density)
residuals = data.frame(x=gauge$density, resid=resid(lin_reg))
resid_plot = ggplot(residuals, aes(x=x, y=resid)) +
    geom_point() +
    labs(title="Density vs ln Gain Residuals", x="Density", y="Residuals"
)

# put both plots side by side
grid.arrange(reg_plot, resid_plot, ncol=2)

# print out the equation of the line and the sum of squared residuals
summarize_fit(lin_reg)
```

```
intercept:               5.997265
slope:                   -4.605937
sum squared residuals:   0.4059345
r squared:               0.9958183
```

Out[9]:



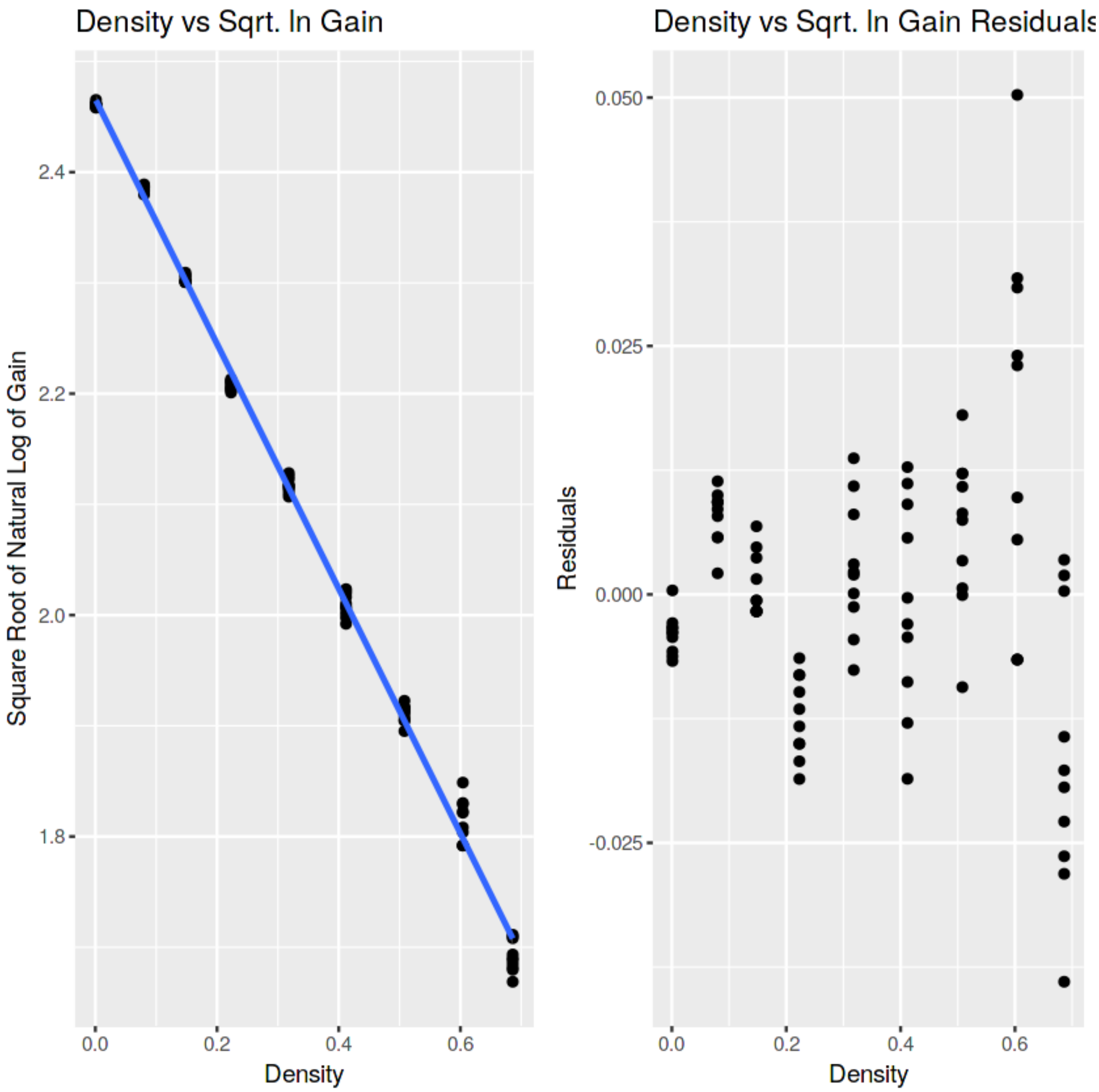


Below is the model we propose

In [10]:
```
# transform data (take the sqrt of the ln of y axis) and fit to linear re
gression
gauge$sqrt_ln_gain = sqrt(log(gauge$gain))
reg_plot = ggplot(gauge, aes(x=density, y=sqrt_ln_gain)) +
    geom_point() +
    stat_smooth(method='lm', formula = y~x) +
    labs(title="Density vs Sqrt. ln Gain", x="Density", y="Square Root of
Natural Log of Gain")

# compute the residuals for the linear regression and plot
lin_reg = lm(gauge$sqrt_ln_gain ~ gauge$density)
residuals = data.frame(x=gauge$density, resid=resid(lin_reg))
resid_plot = ggplot(residuals, aes(x=x, y=resid)) +
    geom_point() +
    labs(title="Density vs Sqrt. ln Gain Residuals", x="Density", y="Resi
duals")

# put both plots side by side
grid.arrange(reg_plot, resid_plot, ncol=2)

# print out the equation of the line and the sum of squared residuals
summarize_fit(lin_reg)
```

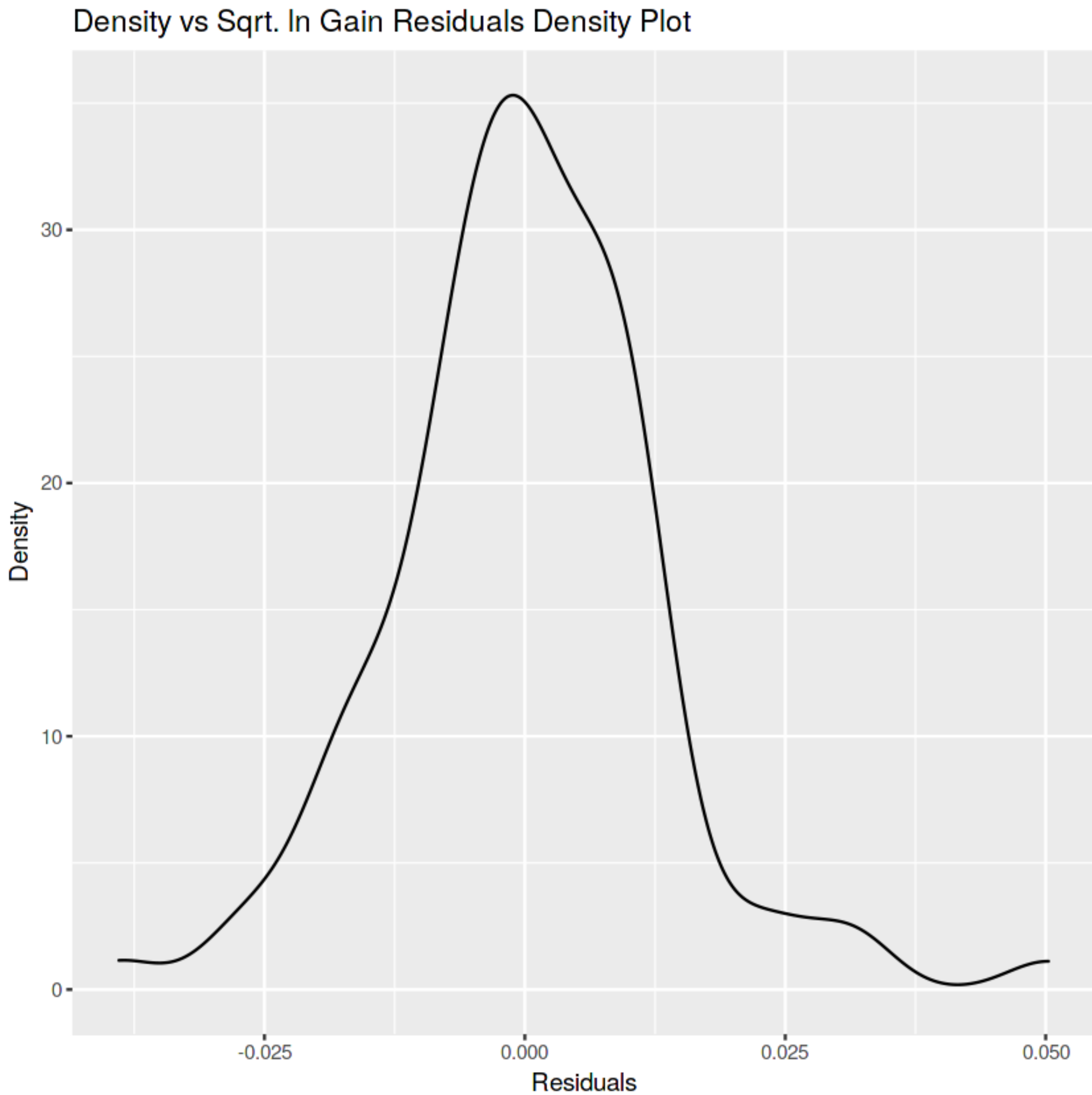
```
intercept:                2.465989
slope:                    -1.105212
sum squared residuals:    0.01539044
r squared:                0.9972425
```

Out[10]:



In [11]:
```
ggplot(residuals, aes(x=resid)) +
    geom_density() +
    labs(title="Density vs Sqrt. ln Gain Residuals Density Plot", x="Resi
duals", y="Density")
```

Out[11]:



In [0]:
```
m = -1.105212
b = 2.465989

f = function(x) exp(1)^((m * x + b)^2)

# plot the original data but with our proposed regression line
ggplot(gauge, aes(x=density, y=gain)) +
    geom_point() +
    stat_function(fun=f, color="blue") +
    labs(title="Density vs Gain", x="Density", y="Gain") +
    annotate(geom="text", x=.35, y=200, label="gain = e^((-1.1052 * densi
ty + 2.4660)^2)", color="blue")
```

What if data was not reported accurately?

My plan is to simulate a large number of times the following procedure:

1. For each density value, vary the value by +- 10% (or some other value in the range with a uniform distribution)
2. Fit the best line again and record the coefficients
3. Plot the distributions of the coefficients and analyze
4. ???
5. Profit

In [0]:

```
# make a copy of the original df to not screw up anything later
# since we will always need to take the log of gain, do so now
original_df = data.frame(select(gauge, sqrt_ln_gain, density))
head(original_df)
```

In [0]:

```
# parameters
N = 5000          # how many times to simulate
V = .15           # range of variation for density (from -V to +V)

# create lists to hold the simulated coefficients
slopes = vector("list", length=N)
intercepts = vector("list", length=N)

# create a function to vary the density column by any amount between -V a
nd +V
vary_density = function(x) {
    vary_amount = runif(n=1, min=-V, max=V) + 1
    return(x * vary_amount)
}

# simulate N times
for(i in 1:N) {
    # first create a copy of the original df so our changes don't carry o
ver between iterations
    copied_df = data.frame(original_df)

    # modify the density column with the random variation
    copied_df$density = unlist(lapply(copied_df$density, vary_density), u
se.names=F)

    # do linear regression as before and record the coefficients
    lin_reg = lm(sqrt_ln_gain ~ density, data=copied_df)
    slopes[i] = lin_reg$coefficients[2]
    intercepts[i] = lin_reg$coefficients[1]
}
```

In [0]:
```
# WARNING: DO NOT UNCOMMENT AND RUN THIS CELL UNLESS YOU WANT TO HAVE TO
# REDO THE CUSTOM TEXT ON THE FOLLOWING GRAPHS

# # combine the lists into a single dataframe and save
# to_save = do.call(rbind, Map(data.frame, slope=slopes, intercept=interc
epts))
# write.table(to_save, "simulated_density_coefficients.txt", sep=" ", ro
w.names=F)
```

In [0]:
```
# load the saved data
sdc = read.table("./simulated_density_coefficients.txt", sep="", header=T
)

# plot the distribution of slopes and intercepts
slope_plot = ggplot(sdc, aes(x=slope)) +
    geom_density() +
    geom_vline(aes(xintercept=mean(slope)), color="blue") +
    annotate("text", label=paste("mean slope at ", round(mean(sdc$slope),
digits=4)), x=mean(sdc$slope)+.005, y=10, angle=90) +
    geom_vline(aes(xintercept=m), color="red") +
    annotate("text", label=paste("actual slope at ", m), x=m+.005, y=5, a
ngle=90) +
    labs(title="Density Plot of Slope", x="Slope", y="Density")
intercept_plot = ggplot(sdc, aes(x=intercept)) +
    geom_density() +
    geom_vline(aes(xintercept=mean(intercept)), color="blue") +
    annotate("text", label=paste("mean intercept at ", round(mean(sdc$int
ercept), digits=4)), x=mean(sdc$intercept)+.001, y=50, angle=90) +
    geom_vline(aes(xintercept=b), color="red") +
    annotate("text", label=paste("actual intercept at ", b), x=b+.001, y=
50, angle=90) +
    labs(title="Density Plot of Intercept", x="Intercept", y="Density")

# put both plots side by side
grid.arrange(slope_plot, intercept_plot, ncol=2)
```

In [0]:
```
# constant for percent variation
C = .01
```

In [0]:
```
# get the mean and standard deviation of simulated slope
slope_mean = mean(sdc$slope)
slope_std = sd(sdc$slope)

# compute the probability of getting within C of the ovserved value
abs(pnorm(m + (m * C), mean=slope_mean, sd=slope_std, lower.tail=T) - pno
rm(m - (m * C), mean=slope_mean, sd=slope_std, lower.tail=T))
```

In [0]:
```
# get the mean and standard deviation of simulated intercept
intercept_mean = mean(sdc$intercept)
intercept_std = sd(sdc$intercept)

# compute the probability of getting within C of the ovserved value
abs(pnorm(b + (b * C), mean=intercept_mean, sd=intercept_std, lower.tail=
T) - pnorm(b - (b * C), mean=intercept_mean, sd=intercept_std, lower.tail
=T))
```

Predicting

In [0]:
```
# create inverse plots for the above models (predicting density with gai
n)

# transform data (take the sqrt(ln(#)) of y axis) and fit to linear regre
ssion
gauge$sqrt_ln_gain = sqrt(log(gauge$gain))
inv_reg_plot = ggplot(gauge, aes(x=sqrt_ln_gain, y=density)) +
    geom_point() +
    stat_smooth(method='lm', formula = y~x) +
    labs(title="ln Gain vs Density", y="Density", x="Natural Log of Gain"
)

# compute the residuals for the linear regression and plot
inv_lin_reg = lm(density ~ sqrt_ln_gain, data = gauge)
inv_residuals = data.frame(x=gauge$sqrt_ln_gain, resid=resid(inv_lin_reg
))
inv_resid_plot = ggplot(inv_residuals, aes(x=x, y=resid)) +
    geom_point() +
    labs(title="ln Gain Residuals vs Density", x="ln(Gain)", y="Residual
s")

# put both plots side by side
grid.arrange(inv_reg_plot, inv_resid_plot, ncol=2)

# print out the equation of the line and the sum of squared residuals
summarize_fit(inv_lin_reg)
```

In [0]:
```
# create a confidence interval for the regression line

# find slope of regression line
slope = inv_lin_reg$coefficients[[2]]

# find standard deviation of slope
mse = sum(inv_lin_reg$residuals^2)/(nrow(gauge)-2)
deviations = var(gauge$sqrt_ln_gain)*(nrow(gauge)-1)
slope_sd = sqrt(mse/deviations)

# create confidence interval for slope
slope_me = qt(.975, nrow(gauge)-2, lower.tail = TRUE) * slope_sd
cat("95% Confidence Interval for slope: (", slope - slope_me, ",", slope
+ slope_me, ")\n")



# find y-int of regression line
yint = inv_lin_reg$coefficients[[1]]

# find standard deviation of y-int
meanx = mean(gauge$sqrt_ln_gain)
yint_sd = sqrt(mse*(1/nrow(gauge) + meanx))

# create confidence interval for y-intervept
yint_me = qt(.975, nrow(gauge)-2, lower.tail = TRUE) * yint_sd
cat("95% Confidence Interval for y-int: (", yint - yint_me, ",", yint + y
int_me, ")")
```

In [0]:
```
# plot prediction interval with confidence bands

# plot data and regression line
newx <- seq(1.6, 2.5, .9/89)
plot(gauge$sqrt_ln_gain, gauge$density, ylim=c(0, .8), xlab="sqrt(log(Gai
n))", ylab="Density", main="Prediction Interval for Density", pch = 20)
abline(inv_lin_reg, col="lightblue")

# get upper and lower bounds for prediction interval of 90 points
conf_interval <- predict(inv_lin_reg, newdata=data.frame(sqrt_ln_gain = n
ewx), interval="prediction",
                         level = 0.95)
# plot points around regression line
lines(newx, conf_interval[,2], col="blue", lty=2)
lines(newx, conf_interval[,3], col="blue", lty=2)
```

In [0]:
```
# create a function that produces a prediction interval for density, give
n a measured gain
pred_interval = function(input){
    transformed = sqrt(log(input))
    output = predict(lm(density ~ sqrt_ln_gain, data = gauge), newdata=da
ta.frame(sqrt_ln_gain = transformed), interval="prediction")
    return (c(output[2], output[3]))
}
```

In [0]:
```
pred_interval(426.7)
```

In [0]:
```
pred_interval(38.6)
```