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DOCTORAL THESIS

# A Language of Polynomials

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*A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy*

*in the*

Research Group Name
Department or School Name

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# Declaration of Authorship

I, Eric UNG, declare that this thesis titled, "A Language of Polynomials" and the work presented in it are my own. I confirm that:

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*“Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism.”*

Dave Barry

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# *Abstract*

Faculty Name
Department or School Name

Doctor of Philosophy

**A Language of Polynomials**

by Eric UNG

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

# *Acknowledgements*

The acknowledgments and the people to thank go here, don't forget to include your project advisor…

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# List of Figures

# List of Tables

# List of Abbreviations

| | |
|---|---|
| **LAH** | **L**ist **A**bbreviations **H**ere |
| **WSF** | **W**hat (it) **S**tands **F**or |

# Physical Constants

Speed of Light $c_0 = 2.997\,924\,58 \times 10^8\ \mathrm{m\,s^{-1}}$ (exact)

# List of Symbols

| | | |
|---|---|---|
| $a$ | distance | m |
| $P$ | power | W ($J\,s^{-1}$) |
| $\omega$ | angular frequency | rad |

*For/Dedicated to/To my…*

## Chapter 1

# A Language of Polynomials

### 1.1 Introduction

This paper is on the re-framing of the one way function to a matrix multiplication problem - that of multiplying two $3 \times 3$ matrices to form a $6 \times 6$ matrix under a locally concatenative property.

### 1.2 Foundations

There exists a language such that it decides each monomial in the polynomial. In other words, there exists a set of deciders for each monomial in the polynomial where it decides if y is in the monomial. A decider in this term is not of the definition found originally in textbooks but one that is redefined in the below definition.

Given a polynomial $p(x) = ax^2 + bx + c$ $p(x) = 3x^2 + 4x + 5$ $p(2) = 3(2)^2 + 4(2) + 5$ $p(2) = 12 + 8 + 5$

Let the decider be defined as the following: Decider is a function $Decider(constant, x, degree) \equiv constant * x^{degree} = y$ such that x1 * x2 * ... * $x$ *degree times* * *constant times* is tested to be equivalent to y and x1 is the start and $x$ *degree times* is the finish then loop around x1 to $x$ *degree times* until it stops

Decider for $ax^2$ is $Decider(3, 2, 2) = 3(2)^2 \equiv 12$ Decider for $bx$ is $Decider(4, 2, 1) = 4(2)^1 \equiv 8$ Decider for $c$ is $Decider(1, 2, 1) = 5(2)^0 \equiv 5$

A <b>Turing Machine</b> can be defined as the following:

1. Q is the set of states 2. Sigma is the input alphabet not containing the blank symbol u 3. Tau is the tape alphabet where u is in tau and sigma is in tau 4. Sigma: Q $\times$ T => Q $\times$ Tau $\times$ $leftbracket$L, R$rightbracket$ is the transition function 5. q0 is in Q is the start state 6. qaccept is in Q is the accept state 7. qreject is in Q is the reject state where qreject != qaccept

Multi-Tape Turing Machine Sigma: Q $\times$ T$k$ => Q $\times$T$k$k $\times$ L,R,S

Rational Expression

K[x] and K[[x]]. Let K[[x]] describe a set of monomial deciders. S is an element of K[[x]] meaning S is a monomial decider.

S = Sigma $leftbracket$ n>=0 $rightbracket$an x$n$

### 1.3 Monomial of One Degree

Given the definition of a decider:

A decider of at least one degree

$Decider(3, x, 4) \equiv 3x^4 = y$ Contains $Decider(3, x, 3) \equiv 3x^3 = y$ Contains $Decider(3, x, 2) \equiv 3x^2 = y$ Contains $Decider(3, x, 1) \equiv 3x^1 = y$ Contains $Decider(3, x, 0) \equiv 3x^0 = y$

Hence it can be generalized to: $Decider(constant, degree, x)$ contains the sequence set $\{Decider(constant, degree, x), Decider(constant, degree - 1, x), ..., Decider(constant, 0, x)\}$ $\{start, ..., finish\}$

## 1.4 Addition

Given the first example: p(x)=3x2+4x+5

p(2)=3(2)2+4(2)+5

p(2)=12+8+5

m2=Decider(3,x,2)=3x2

m1=Decider(4,x,1)=4x

m0=Decider(5,x,0)=5

Generalized to mx where x is the degree Given polynomial functions, p1 and p2, they are commutative p1(x)=ma+...+m0

p2(x)=nb+...+n0

p1(x)+p2(x)=ma+m[x+1]+nb+n[y+1]+...+(mx+ny)+...+(m0+n0) where x=y

Decider(cx,x,dx)+Decider(cy,x,dy)=Decider(cx+cy,x,dx)=Deciders(cx+cy,x,dy)= dx=dy

## 1.5 Product

Given two monomials in the language, a and b, the product of a and b is also in the language.

Given Decider(cx,x,dx) and Decider(cy,x,dy) is in language L

Show that the product Decider(cx+cy,x,dx×dy) is in L

Decider(cx,x,dx)×Decider(cy,x,dy)

=cx×xdx×cy×xdy

=cx×xdx+dy

=(cx+cy)×xdx+dy is in L

=Decider(cx+cy,x,dx+dy)

## 1.6 Problem with Matrices

Given a polynomial p of x, show that the monomial deciders represented in the language can't be contained in a finite matrix after a set number, n, such that x$n$.

axa × bxb = nxn such that a $\neq$ b and a,b<n

## 1.7 Multivariable Polynomials

A monomial with more than one variable can be treated the same way as handling single variables at different degrees.

Decider(c,xyz,d) = c(xyz)d = Decider(c,x,d)×Decider(c,y,d)×Decider(c,z,d) where c is some constant Decider(c,x,d1)×Decider(c,y,d2)×Decider(c,z,d3) = cxd1×yd2×zd3 where c is some constant

Given f(x,y)=3xy2

set x=2,y=3

f(2,3)=3(2)(3)2

f(2,3)=27

## 1.8 Generalized Monomial Deciders

A monomial decider can be represented in a more general graphic

6x6=Decider 6,x,6 =6× xstart,x2,x3,x4,x5,xfinish x6=Decider 1,x,6 =1× xstart,x2,x3,x4,x5,xfinish

In both these examples, xstart is x1 and xfinish is x6.

## 1.9 Concentric Monomial Deciders

Given a polynomial, p x, with a monomial decider represented as ax n in p x and n in N and a = 1 such that p x = x n, there is special property for these monomial deciders that can be illustrated below.

Decider 1,x,4 = x = xi such that i is in start,2,3,finish and xi contains xi minus 1 where i 1 Decider 1,x,2 = x2 = xi such that i is in start,finish and xi contains xi minus 1 where i 1 In both these examples, xi is a state in the monomial decider. Each state in a one constant monomial decider have a property of being concentric. This means that a state can be defined as, x2 = x1,x0 so going from x1 to x0 then going to the next state x3 or looping back to x1.

## 1.10 Constants

Given a constant, c, of p or better described in the example: f(x) = 5. Constants are seen as linear directed acyclic graphs.

f(x) = 5

$Decider(5, x, 0) \equiv 5 \equiv start, 2, 3, 4, finish$

There is no state in the decider where it loops back to the start. In other words, there is no x that represents a monomial in a constant.

## 1.11 Division

Division of monomial deciders

$x^5/x = x^4 \equiv Decider(1, x, 5)/Decider(1, x, 1)$ $Decider(1, x, 4)$ $x^5/x^2 = x^3 \equiv Decider(1, x, 5)/Decider(1, x, 2)$ $Decider(1, x, 3)$ $x^5/x^3 = x^2 \equiv Decider(1, x, 5)/Decider(1, x, 3)$ $Decider(1, x,$ $x^5/x^4 = x^1 \equiv Decider(1, x, 5)/Decider(1, x, 4)$ $Decider(1, x, 1)$ $x^5/x^5 = 1 \equiv Decider(1, x, 5)/Decider(1, x, 5)$

here is represented as a special kind of equivalence that we will get to later.

$Decider(1, x, 5)/Decider(1, x, 1) \equiv sequenceofpermutationsofxi, xjsuchthatthecountofiis4andjis1$
$Decider(1, x, 5)/Decider(1, x, 2) \equiv sequenceofpermutationsofxi, xjsuchthatthecountofiis3andjis2$
$Decider(1, x, 5)/Decider(1, x, 3) \equiv sequenceofpermutationsofxi, xjsuchthatthecountofiis2andjis3$
$Decider(1, x, 5)/Decider(1, x, 4) \equiv sequenceofpermutationsofxi, xjsuchthatthecountofiis1andjis4$
$Decider(1, x, 5)/Decider(1, x, 5) \equiv sequenceofpermutationsofxi, xjsuchthatthecountofiis0andjis5Here, xi \neq xj, meaningxiisofadifferentrepresentationthanxj$

## 1.12 Multiple Divisions

Given multiple operations of division, this forms an interesting space. x5/x2/x=x5/x)/x2

Decider 1,x,5 /Decider 1,x,2 /Decider 1,x,1 =Decider 1,x,5 /Decider 1,x,1 /Decider 1,x,2 iff ignoring order of operations

## 1.13 Equivalence

Decider 1,x,6 /Decider 1,x,1 /Decider 1,x,1 Decider 1,x,6 /Decider 1,x,2 iff the order of operations is next to each other

Determining if y is in f x is easy if we are given any monomial decider in the set of the language of polynomials and their representations has the possibility to give different representations if we consider them as representations of the function f x.

Decider 1,x,6 /Decider 1,x,1 /Decider 1,x,1 =sequence of permutations of xi,xj,xk such that the count of i is 4 and j is 1 and k 1

Decider 1,x,6 /Decider 1,x,2 =sequence of permutations of xi,xj such that the count of i is 4 and i is 2

Theorem of Equivalence Decider 1,x,6 /Decider 1,x,1 /Decider 1,x,1 =Decider 1,x,6 /Decider 1,x,2 such that there is some xsuch that the monomial represented by both deciders exists where f x =y

### 1.13.1 References

The `biblatex` package is used to format the bibliography and inserts references such as this one (**Reference1**). The options used in the `main.tex` file mean that the in-text citations of references are formatted with the author(s) listed with the date of the publication. Multiple references are separated by semicolons (e.g. (**Reference2**; **Reference1**)) and references with more than three authors only show the first author with *et al.* indicating there are more authors (e.g. (**Reference3**)). This is done automatically for you. To see how you use references, have a look at the `Chapter1.tex` source file. Many reference managers allow you to simply drag the reference into the document as you type.

Scientific references should come *before* the punctuation mark if there is one (such as a comma or period). The same goes for footnotes[1]. You can change this but the most important thing is to keep the convention consistent throughout the thesis. Footnotes themselves should be full, descriptive sentences (beginning with a capital letter and ending with a full stop). The APA6 states: “Footnote numbers should be superscripted, [...], following any punctuation mark except a dash.” The Chicago manual of style states: “A note number should be placed at the end of a sentence or clause. The number follows any punctuation mark except the dash, which it precedes. It follows a closing parenthesis.”

The bibliography is typeset with references listed in alphabetical order by the first author’s last name. This is similar to the APA referencing style. To see how LaTeX typesets the bibliography, have a look at the very end of this document (or just click on the reference number links in in-text citations).

#### A Note on bibtex

The bibtex backend used in the template by default does not correctly handle unicode character encoding (i.e. "international" characters). You may see a warning about this in the compilation log and, if your references contain unicode characters, they may not show up correctly or at all. The solution to this is to use the biber backend instead of the outdated bibtex backend. This is done by finding this in `main.tex`: *`backend=bibtex`* and changing it to *`backend=biber`*. You will then need to delete all auxiliary BibTeX files and navigate to the template directory in your terminal (command prompt). Once there, simply type `biber main` and biber will compile your

[1] Such as this footnote, here down at the bottom of the page.

bibliography. You can then compile `main.tex` as normal and your bibliography will be updated. An alternative is to set up your LaTeX editor to compile with biber instead of bibtex, see here for how to do this for various editors.

# Chapter 2

# Chapter Title Here

## 2.1 Main Section 1

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### 2.1.1 Subsection 1

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### 2.1.2 Subsection 2

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## 2.2 Main Section 2

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# Appendix A

# Frequently Asked Questions

## A.1 How do I change the colors of links?

The color of links can be changed to your liking using:
`\hypersetup{urlcolor=red}`, or
`\hypersetup{citecolor=green}`, or
`\hypersetup{allcolor=blue}`.
If you want to completely hide the links, you can use:
`\hypersetup{allcolors=.}`, or even better:
`\hypersetup{hidelinks}`.
If you want to have obvious links in the PDF but not the printed text, use:
`\hypersetup{colorlinks=false}`.