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DOCTORAL THESIS

A Language of Polynomials

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A thesis submitted in fulfillment of the requirements for the degree of Doctor of Philosophy

in the

Research Group Name Department or School Name

May 23, 2024

Declaration of Authorship

I, Eric UNG, declare that this thesis titled, "A Language of Polynomials" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:			
Date:			

"Thanks to my solid academic training, today I can write hundreds of words on virtually any topic without possessing a shred of information, which is how I got a good job in journalism."

Dave Barry

UNIVERSITY

Abstract

Faculty Name Department or School Name

Doctor of Philosophy

A Language of Polynomials

by Eric UNG

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor. . .

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List of Abbreviations

LAH List Abbreviations HereWSF What (it) Stands For

Physical Constants

Speed of Light $c_0 = 2.99792458 \times 10^8 \,\mathrm{m \, s^{-1}}$ (exact)

xxi

List of Symbols

a distance

P power $W(J s^{-1})$

 ω angular frequency rad

xxiii

For/Dedicated to/To my...

Chapter 1

A Language of Polynomials

1.1 Introduction

This paper is on the re-framing of the one way function to a matrix multiplication problem - that of multiplying two 3×3 matrices to form a 6×6 matrix under a locally concatenative property.

1.2 Foundations

There exists a language such that it decides each monomial in the polynomial. In other words, there exists a set of deciders for each monomial in the polynomial where it decides if y is in the monomial. A decider in this term is not of the definition found originally in textbooks but one that is redefined in the below definition.

Given a polynomial
$$p(x) = ax^2 + bx + c$$
 $p(x) = 3x^2 + 4x + 5$ $p(2) = 3(2)^2 + 4(2) + 5$ $p(2) = 12 + 8 + 5$

Let the decider be defined as the following: Decider is a function $Decider(constant, x, degree) \equiv constant * x^{degree} = y$ such that x1 * x2 * ... * x degree times * constant times is tested to be equivalent to y and x1 is the start and x degree times is the finish then loop around x1 to x degree times until it stops

Decider for ax^2 is $Decider(3,2,2) = 3(2)^2 \equiv 12$ Decider for bx is $Decider(4,2,1) = 4(2)^1 \equiv 8$ Decider for c is $Decider(1,2,1) = 5(2)^0 \equiv 5$

A Turing Machine can be defined as the following:

1. Q is the set of states 2. Sigma is the input alphabet not containing the blank symbol u 3. Tau is the tape alphabet where u is in tau and sigma is in tau 4. Sigma: $Q \times T => Q \times Tau \times leftbracketL$, Rrightbracket is the transition function 5. q0 is in Q is the start state 6. qaccept is in Q is the accept state 7. qreject is in Q is the reject state where qreject!= qaccept

Multi-Tape Turing Machine Sigma: $Q \times Tk => Q \times Tkk \times L$, R, S Rational Expression

K[x] and K[[x]]. Let K[[x]] describe a set of monomial deciders. S is an element of K[[x]] meaning S is a monomial decider.

 $S = Sigma\ leftbracket\ n>=0\ rightbracket\ an\ xn$

1.3 Monomial of One Degree

Given the definition of a decider:

A decider of at least one degree

```
Decider(3, x, 4) \equiv 3x^4 = y Contains Decider(3, x, 3) \equiv 3x^3 = y Contains Decider(3, x, 2) \equiv 3x^2 = y Contains Decider(3, x, 1) \equiv 3x^1 = y Contains Decider(3, x, 0) \equiv 3x^0 = y
```

Hence it can be generalized to: Decider(constant, degree, x) contains the sequence set $\{Decider(constant, degree, x), Decider(constant, degree - 1, x), ..., Decider(constant, 0, x)\}$ $\{start, ..., finish\}$

1.4 Addition

```
Given the first example: p(x)=3x2+4x+5

p(2)=3(2)2+4(2)+5

p(2)=12+8+5

m2=Decider(3,x,2)=3x2

m1=Decider(4,x,1)=4x

m0=Decider(5,x,0)=5

Generalized to mx where x is the degree Given polynomial functions, p1 and p2, they are commutative p1(x)=ma+...+m0

p2(x)=nb+...+n0

p1(x)+p2(x)=ma+m[x+1]+nb+n[y+1]+...+(mx+ny)+...+(m0+n0) where x=y

Decider(cx,x,dx)+Decider(cy,x,dy)=Decider(cx+cy,x,dx)=Deciders(cx+cy,x,dy)=dx=dy
```

1.5 Product

Given two monomials in the language, a and b, the product of a and b is also in the language.

```
Given Decider(cx,x,dx) and Decider(cy,x,dy) is in language L
Show that the product Decider(cx+cy,x,dx×dy) is in L
Decider(cx,x,dx)×Decider(cy,x,dy)
=cx×xdx×cy×xdy
=cx×xdx+dy
=(cx+cy)×xdx+dy is in L
=Decider(cx+cy,x,dx+dy)
```

1.6 Problem with Matrices

Given a polynomial p of x, show that the monomial deciders represented in the language can't be contained in a finite matrix after a set number, n, such that xn. $axa \times bxb = nxn$ such that $a \neq b$ and a,b < n

1.6.1 Common LaTeX Math Symbols

There are a multitude of mathematical symbols available for LATEX and it would take a great effort to learn the commands for them all. The most common ones you are likely to use are shown on this page: http://www.sunilpatel.co.uk/latex-type/latex-math-symbols/

You can use this page as a reference or crib sheet, the symbols are rendered as large, high quality images so you can quickly find the LATEX command for the symbol you need.

1.6.2 LATEX on a Mac

The LATEX distribution is available for many systems including Windows, Linux and Mac OS X. The package for OS X is called MacTeX and it contains all the applications you need – bundled together and pre-customized – for a fully working LATEX environment and work flow.

MacTeX includes a custom dedicated LaTeX editor called TeXShop for writing your '.tex' files and BibDesk: a program to manage your references and create your bibliography section just as easily as managing songs and creating playlists in iTunes.

1.7 Getting Started with this Template

If you are familiar with LATEX, then you should explore the directory structure of the template and then proceed to place your own information into the *THESIS INFOR-MATION* block of the main.tex file. You can then modify the rest of this file to your unique specifications based on your degree/university. Section 1.9 on page 5 will help you do this. Make sure you also read section 1.11 about thesis conventions to get the most out of this template.

If you are new to LATEX it is recommended that you carry on reading through the rest of the information in this document.

Before you begin using this template you should ensure that its style complies with the thesis style guidelines imposed by your institution. In most cases this template style and layout will be suitable. If it is not, it may only require a small change to bring the template in line with your institution's recommendations. These modifications will need to be done on the MastersDoctoralThesis.cls file.

1.7.1 About this Template

This LATEX Thesis Template is originally based and created around a LATEX style file created by Steve R. Gunn from the University of Southampton (UK), department of Electronics and Computer Science. You can find his original thesis style file at his site, here: http://www.ecs.soton.ac.uk/~srg/softwaretools/document/templates/

Steve's ecsthesis.cls was then taken by Sunil Patel who modified it by creating a skeleton framework and folder structure to place the thesis files in. The resulting template can be found on Sunil's site here: http://www.sunilpatel.co.uk/thesis-template

Sunil's template was made available through http://www.LaTeXTemplates.com where it was modified many times based on user requests and questions. Version 2.0 and onwards of this template represents a major modification to Sunil's template and is, in fact, hardly recognisable. The work to make version 2.0 possible was carried out by Vel and Johannes Böttcher.

1.8 What this Template Includes

1.8.1 Folders

This template comes as a single zip file that expands out to several files and folders. The folder names are mostly self-explanatory:

Appendices – this is the folder where you put the appendices. Each appendix should go into its own separate .tex file. An example and template are included in the directory.

Chapters – this is the folder where you put the thesis chapters. A thesis usually has about six chapters, though there is no hard rule on this. Each chapter should go in its own separate .tex file and they can be split as:

- Chapter 1: Introduction to the thesis topic
- Chapter 2: Background information and theory
- Chapter 3: (Laboratory) experimental setup
- Chapter 4: Details of experiment 1
- Chapter 5: Details of experiment 2
- Chapter 6: Discussion of the experimental results
- Chapter 7: Conclusion and future directions

This chapter layout is specialised for the experimental sciences, your discipline may be different.

Figures – this folder contains all figures for the thesis. These are the final images that will go into the thesis document.

1.8.2 Files

Included are also several files, most of them are plain text and you can see their contents in a text editor. After initial compilation, you will see that more auxiliary files are created by LaTeX or BibTeX and which you don't need to delete or worry about:

example.bib – this is an important file that contains all the bibliographic information and references that you will be citing in the thesis for use with BibTeX. You can write it manually, but there are reference manager programs available that will create and manage it for you. Bibliographies in LATEX are a large subject and you may need to read about BibTeX before starting with this. Many modern reference managers will allow you to export your references in BibTeX format which greatly eases the amount of work you have to do.

MastersDoctoralThesis.cls – this is an important file. It is the class file that tells LATEX how to format the thesis.

main.pdf – this is your beautifully typeset thesis (in the PDF file format) created by LATEX. It is supplied in the PDF with the template and after you compile the template you should get an identical version.

main.tex – this is an important file. This is the file that you tell LATEX to compile to produce your thesis as a PDF file. It contains the framework and constructs that tell LATEX how to layout the thesis. It is heavily commented so you can read exactly what each line of code does and why it is there. After you put your own information into the THESIS INFORMATION block – you have now started your thesis!

Files that are *not* included, but are created by LATEX as auxiliary files include:

main.aux – this is an auxiliary file generated by LATEX, if it is deleted LATEX simply regenerates it when you run the main .tex file.

main.bbl – this is an auxiliary file generated by BibTeX, if it is deleted, BibTeX simply regenerates it when you run the main.aux file. Whereas the .bib file contains

all the references you have, this .bbl file contains the references you have actually cited in the thesis and is used to build the bibliography section of the thesis.

main.blg – this is an auxiliary file generated by BibTeX, if it is deleted BibTeX simply regenerates it when you run the main .aux file.

main.lof – this is an auxiliary file generated by LATEX, if it is deleted LATEX simply regenerates it when you run the main .tex file. It tells LATEX how to build the *List of Figures* section.

main.log – this is an auxiliary file generated by LATEX, if it is deleted LATEX simply regenerates it when you run the main .tex file. It contains messages from LATEX, if you receive errors and warnings from LATEX, they will be in this .log file.

main.lot – this is an auxiliary file generated by LATEX, if it is deleted LATEX simply regenerates it when you run the main .tex file. It tells LATEX how to build the *List of Tables* section.

main.out – this is an auxiliary file generated by LATEX, if it is deleted LATEX simply regenerates it when you run the main .tex file.

So from this long list, only the files with the .bib, .cls and .tex extensions are the most important ones. The other auxiliary files can be ignored or deleted as LATEX and BibTeX will regenerate them.

1.9 Filling in Your Information in the main.tex File

You will need to personalise the thesis template and make it your own by filling in your own information. This is done by editing the main.tex file in a text editor or your favourite LaTeX environment.

Open the file and scroll down to the third large block titled *THESIS INFORMA-TION* where you can see the entries for *University Name*, *Department Name*, etc...

Fill out the information about yourself, your group and institution. You can also insert web links, if you do, make sure you use the full URL, including the http://for this. If you don't want these to be linked, simply remove the \href{url}{name} and only leave the name.

When you have done this, save the file and recompile main.tex. All the information you filled in should now be in the PDF, complete with web links. You can now begin your thesis proper!

1.10 The main.tex File Explained

The main.tex file contains the structure of the thesis. There are plenty of written comments that explain what pages, sections and formatting the LATEX code is creating. Each major document element is divided into commented blocks with titles in all capitals to make it obvious what the following bit of code is doing. Initially there seems to be a lot of LATEX code, but this is all formatting, and it has all been taken care of so you don't have to do it.

Begin by checking that your information on the title page is correct. For the thesis declaration, your institution may insist on something different than the text given. If this is the case, just replace what you see with what is required in the DECLARATION PAGE block.

Then comes a page which contains a funny quote. You can put your own, or quote your favourite scientist, author, person, and so on. Make sure to put the name of the person who you took the quote from.

Following this is the abstract page which summarises your work in a condensed way and can almost be used as a standalone document to describe what you have done. The text you write will cause the heading to move up so don't worry about running out of space.

Next come the acknowledgements. On this page, write about all the people who you wish to thank (not forgetting parents, partners and your advisor/supervisor).

The contents pages, list of figures and tables are all taken care of for you and do not need to be manually created or edited. The next set of pages are more likely to be optional and can be deleted since they are for a more technical thesis: insert a list of abbreviations you have used in the thesis, then a list of the physical constants and numbers you refer to and finally, a list of mathematical symbols used in any formulae. Making the effort to fill these tables means the reader has a one-stop place to refer to instead of searching the internet and references to try and find out what you meant by certain abbreviations or symbols.

The list of symbols is split into the Roman and Greek alphabets. Whereas the abbreviations and symbols ought to be listed in alphabetical order (and this is *not* done automatically for you) the list of physical constants should be grouped into similar themes.

The next page contains a one line dedication. Who will you dedicate your thesis to?

Finally, there is the block where the chapters are included. Uncomment the lines (delete the % character) as you write the chapters. Each chapter should be written in its own file and put into the *Chapters* folder and named Chapter1, Chapter2, etc...Similarly for the appendices, uncomment the lines as you need them. Each appendix should go into its own file and placed in the *Appendices* folder.

After the preamble, chapters and appendices finally comes the bibliography. The bibliography style (called <code>authoryear</code>) is used for the bibliography and is a fully featured style that will even include links to where the referenced paper can be found online. Do not underestimate how grateful your reader will be to find that a reference to a paper is just a click away. Of course, this relies on you putting the URL information into the BibTeX file in the first place.

1.11 Thesis Features and Conventions

To get the best out of this template, there are a few conventions that you may want to follow.

One of the most important (and most difficult) things to keep track of in such a long document as a thesis is consistency. Using certain conventions and ways of doing things (such as using a Todo list) makes the job easier. Of course, all of these are optional and you can adopt your own method.

1.11.1 Printing Format

This thesis template is designed for double sided printing (i.e. content on the front and back of pages) as most theses are printed and bound this way. Switching to one sided printing is as simple as uncommenting the <code>oneside</code> option of the <code>documentclass</code> command at the top of the <code>main.tex</code> file. You may then wish to adjust the margins to suit specifications from your institution.

The headers for the pages contain the page number on the outer side (so it is easy to flick through to the page you want) and the chapter name on the inner side.

The text is set to 11 point by default with single line spacing, again, you can tune the text size and spacing should you want or need to using the options at the very start of main.tex. The spacing can be changed similarly by replacing the singlespacing with onehalfspacing or doublespacing.

1.11.2 Using US Letter Paper

The paper size used in the template is A4, which is the standard size in Europe. If you are using this thesis template elsewhere and particularly in the United States, then you may have to change the A4 paper size to the US Letter size. This can be done in the margins settings section in main.tex.

Due to the differences in the paper size, the resulting margins may be different to what you like or require (as it is common for institutions to dictate certain margin sizes). If this is the case, then the margin sizes can be tweaked by modifying the values in the same block as where you set the paper size. Now your document should be set up for US Letter paper size with suitable margins.

1.11.3 References

The biblatex package is used to format the bibliography and inserts references such as this one (**Reference1**). The options used in the main.tex file mean that the intext citations of references are formatted with the author(s) listed with the date of the publication. Multiple references are separated by semicolons (e.g. (**Reference2**; **Reference1**)) and references with more than three authors only show the first author with *et al.* indicating there are more authors (e.g. (**Reference3**)). This is done automatically for you. To see how you use references, have a look at the Chapter1.tex source file. Many reference managers allow you to simply drag the reference into the document as you type.

Scientific references should come *before* the punctuation mark if there is one (such as a comma or period). The same goes for footnotes¹. You can change this but the most important thing is to keep the convention consistent throughout the thesis. Footnotes themselves should be full, descriptive sentences (beginning with a capital letter and ending with a full stop). The APA6 states: "Footnote numbers should be superscripted, [...], following any punctuation mark except a dash." The Chicago manual of style states: "A note number should be placed at the end of a sentence or clause. The number follows any punctuation mark except the dash, which it precedes. It follows a closing parenthesis."

The bibliography is typeset with references listed in alphabetical order by the first author's last name. This is similar to the APA referencing style. To see how LATEX typesets the bibliography, have a look at the very end of this document (or just click on the reference number links in in-text citations).

A Note on bibtex

The bibtex backend used in the template by default does not correctly handle unicode character encoding (i.e. "international" characters). You may see a warning about this in the compilation log and, if your references contain unicode characters, they may not show up correctly or at all. The solution to this is to use the biber backend instead of the outdated bibtex backend. This is done by finding this in main.tex: backend=bibtex and changing it to backend=bibter. You will then need to delete all

¹Such as this footnote, here down at the bottom of the page.

Groups	Treatment X	Treatment Y
1	0.2	0.8
2	0.17	0.7
3	0.24	0.75
4	0.68	0.3

TABLE 1.1: The effects of treatments X and Y on the four groups studied.

auxiliary BibTeX files and navigate to the template directory in your terminal (command prompt). Once there, simply type biber main and biber will compile your bibliography. You can then compile main.tex as normal and your bibliography will be updated. An alternative is to set up your LaTeX editor to compile with biber instead of bibtex, see here for how to do this for various editors.

1.11.4 Tables

Tables are an important way of displaying your results, below is an example table which was generated with this code:

```
\begin{table}
\caption{The effects of treatments X and Y on the four groups studied.}
\label{tab:treatments}
\centering
\begin{tabular}{1 1 1}
\toprule
\tabhead{Groups} & \tabhead{Treatment X} & \tabhead{Treatment Y} \\
\midrule
1 & 0.2 & 0.8\\
2 & 0.17 & 0.7\\
3 & 0.24 & 0.75\\
4 & 0.68 & 0.3\\
\bottomrule\\
\end{tabular}
\end{tabular}
\end{table}
```

You can reference tables with \ref{<label>} where the label is defined within the table environment. See Chapter1.tex for an example of the label and citation (e.g. Table 1.1).

1.12 Sectioning and Subsectioning

You should break your thesis up into nice, bite-sized sections and subsections. LATEX automatically builds a table of Contents by looking at all the \chapter{}, \section{} and \subsection{} commands you write in the source.

The Table of Contents should only list the sections to three (3) levels. A chapter{} is level zero (0). A \section{} is level one (1) and so a \subsection{} is level two (2). In your thesis it is likely that you will even use a subsubsection{}, which is level three (3). The depth to which the Table of Contents is formatted is set within MastersDoctoralThesis.cls. If you need this changed, you can do it in main.tex.

1.13. In Closing 9

1.13 In Closing

You have reached the end of this mini-guide. You can now rename or overwrite this pdf file and begin writing your own Chapter1.tex and the rest of your thesis. The easy work of setting up the structure and framework has been taken care of for you. It's now your job to fill it out!

Good luck and have lots of fun!

Guide written by — Sunil Patel: www.sunilpatel.co.uk Vel: LaTeXTemplates.com

Chapter 2

Chapter Title Here

2.1 Main Section 1

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Aliquam ultricies lacinia euismod. Nam tempus risus in dolor rhoncus in interdum enim tincidunt. Donec vel nunc neque. In condimentum ullamcorper quam non consequat. Fusce sagittis tempor feugiat. Fusce magna erat, molestie eu convallis ut, tempus sed arcu. Quisque molestie, ante a tincidunt ullamcorper, sapien enim dignissim lacus, in semper nibh erat lobortis purus. Integer dapibus ligula ac risus convallis pellentesque.

2.1.1 Subsection 1

Nunc posuere quam at lectus tristique eu ultrices augue venenatis. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aliquam erat volutpat. Vivamus sodales tortor eget quam adipiscing in vulputate ante ullamcorper. Sed eros ante, lacinia et sollicitudin et, aliquam sit amet augue. In hac habitasse platea dictumst.

2.1.2 Subsection 2

Morbi rutrum odio eget arcu adipiscing sodales. Aenean et purus a est pulvinar pellentesque. Cras in elit neque, quis varius elit. Phasellus fringilla, nibh eu tempus venenatis, dolor elit posuere quam, quis adipiscing urna leo nec orci. Sed nec nulla auctor odio aliquet consequat. Ut nec nulla in ante ullamcorper aliquam at sed dolor. Phasellus fermentum magna in augue gravida cursus. Cras sed pretium lorem. Pellentesque eget ornare odio. Proin accumsan, massa viverra cursus pharetra, ipsum nisi lobortis velit, a malesuada dolor lorem eu neque.

2.2 Main Section 2

Sed ullamcorper quam eu nisl interdum at interdum enim egestas. Aliquam placerat justo sed lectus lobortis ut porta nisl porttitor. Vestibulum mi dolor, lacinia molestie gravida at, tempus vitae ligula. Donec eget quam sapien, in viverra eros. Donec pellentesque justo a massa fringilla non vestibulum metus vestibulum. Vestibulum in orci quis felis tempor lacinia. Vivamus ornare ultrices facilisis. Ut hendrerit volut-pat vulputate. Morbi condimentum venenatis augue, id porta ipsum vulputate in. Curabitur luctus tempus justo. Vestibulum risus lectus, adipiscing nec condimentum quis, condimentum nec nisl. Aliquam dictum sagittis velit sed iaculis. Morbi tristique augue sit amet nulla pulvinar id facilisis ligula mollis. Nam elit libero, tincidunt ut aliquam at, molestie in quam. Aenean rhoncus vehicula hendrerit.

Appendix A

Frequently Asked Questions

A.1 How do I change the colors of links?

The color of links can be changed to your liking using:

\hypersetup{urlcolor=red}, or

\hypersetup{citecolor=green}, or

\hypersetup{allcolor=blue}.

If you want to completely hide the links, you can use:

\hypersetup{allcolors=.}, or even better:

\hypersetup{hidelinks}.

If you want to have obvious links in the PDF but not the printed text, use:

\hypersetup{colorlinks=false}.