

第一題

| t | Exact | Euler | Euler Error | Taylor | Taylor Error |
|-----|----------|----------|-------------|----------|--------------|
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.1 | 0.105160 | 0.100000 | 0.005160 | 0.105000 | 0.000160 |
| 1.2 | 0.221243 | 0.209917 | 0.011325 | 0.220919 | 0.000324 |
| 1.3 | 0.349121 | 0.330471 | 0.018651 | 0.348612 | 0.000509 |
| 1.4 | 0.489682 | 0.462354 | 0.027328 | 0.488954 | 0.000728 |
| 1.5 | 0.643875 | 0.606285 | 0.037590 | 0.642883 | 0.000993 |
| 1.6 | 0.812753 | 0.763041 | 0.049711 | 0.811438 | 0.001315 |
| 1.7 | 0.997494 | 0.933475 | 0.064019 | 0.995787 | 0.001707 |
| 1.8 | 1.199439 | 1.118537 | 0.080902 | 1.197252 | 0.002187 |
| 1.9 | 1.420116 | 1.319293 | 0.100823 | 1.417344 | 0.002772 |
| 2.0 | 1.661282 | 1.536943 | 0.124338 | 1.657795 | 0.003487 |

第二題

步長 $h = 0.05$:

$t = 0.0$:

數值解: $u_1 = 1.333333$, $u_2 = -0.666667$

精確解: $u_1 = 1.333333$, $u_2 = 0.666667$

誤差: $u_1 = 0.000000$, $u_2 = 1.333333$

$t = 0.5$:

數值解: $u_1 = 0.630794$, $u_2 = -0.449730$

精確解: $u_1 = 0.738788$, $u_2 = -0.515658$

誤差: $u_1 = 0.107994$, $u_2 = 0.065928$

$t = 1.0$:

數值解: $u_1 = 0.307225$, $u_2 = -0.237293$

精確解: $u_1 = 0.279675$, $u_2 = -0.229888$

誤差: $u_1 = 0.027550$, $u_2 = 0.007405$

步長 $h = 0.1$:

$t = 0.0$:

數值解: $u_1 = 1.333333$, $u_2 = -0.666667$

精確解: $u_1 = 1.333333$, $u_2 = 0.666667$

誤差: $u_1 = 0.000000$, $u_2 = 1.333333$

$t = 0.5$:

數值解: $u_1 = -346.402403$, $u_2 = 712.725642$

精確解: $u_1 = 0.738788$, $u_2 = -0.515658$

誤差: $u_1 = 347.141191$, $u_2 = 713.241300$

$t = 1.0$:

數值解: $u_1 = -1300265.083415$, $u_2 = 2672128.044580$

精確解: $u_1 = 0.279675$, $u_2 = -0.229888$

誤差: $u_1 = 1300265.363090$, $u_2 = 2672128.274467$