9.

$H_3(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

$H_3'(x) = 3a_3x^2 + 2a_2x + a_1$

$f(-1) = 1 \Rightarrow -a_3 + a_2 - a_1 + a_0 = 1$ ①

$f'(-1) = 1 \Rightarrow 3a_3 - 2a_2 + a_1 = 1$ ②

$f'(1) = 2 \Rightarrow 3a_3 + 2a_2 + a_1 = 2$ ③

$f(2) = 1 \Rightarrow 8a_3 + 4a_2 + 2a_1 + a_0 = 1$ ④

③ − ②: $4a_2 = 1 \Rightarrow a_2 = \frac{1}{4}$

② $\Rightarrow 3a_3 + a_1 = \frac{3}{2}$

$\Rightarrow a_1 = \frac{3}{2} - 3a_3$

① $\Rightarrow -a_3 + \frac{1}{4} - \left(\frac{3}{2} - 3a_3\right) + a_0 = 1$

$\Rightarrow 2a_3 + a_0 = \frac{9}{4}$

$\Rightarrow a_0 = \frac{9}{4} - 2a_3$

④ $\Rightarrow 8a_3 + 4 \cdot \frac{1}{4} + 2\left(\frac{3}{2} - 3a_3\right) + \left(\frac{9}{4} - 2a_3\right) = 1$

$\Rightarrow \frac{25}{4} = 1$ $\times$

Hence such H_3 doesn't exist.

12

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

Note that $\frac{1}{1+b_2x^2} = 1 - b_2x^2 + b_2^2x^4 - b_2^3x^6 + \cdots$

$$r(x) = (a_0 + a_2x^2 + a_4x^4)(1 - b_2x^2 + b_2^2x^4 - b_2^3x^6 + \cdots)$$

$$= a_0(1 - b_2x^2 + b_2^2x^4 - b_2^3x^6) + a_2x^2(1 - b_2x^2 + b_2^2x^4) + a_4x^4(1 - b_2x^2) + O(x^8)$$

$$= a_0 + (-a_0b_2 + a_2)x^2 + (a_0b_2^2 - a_2b_2 + a_4)x^4 + (-a_0b_2^3 + a_2b_2^2 - a_4b_2)x^6 + O(x^8)$$

$$\begin{cases} a_0 = 1 \\ -a_0b_2 + a_2 = -\frac{1}{2} \\ a_0b_2^2 - a_2b_2 + a_4 = \frac{1}{24} \\ -a_0b_2^3 + a_2b_2^2 - a_4b_2 = -\frac{1}{720} \end{cases} \xRightarrow{a_0=1} \begin{cases} a_0 = 1 \\ -b_2 + a_2 = -\frac{1}{2} \\ \left.\begin{array}{l} b_2^2 - a_2b_2 + a_4 = \frac{1}{24} \\ -b_2^3 + a_2b_2 - a_4b_2 = \frac{-1}{720} \end{array}\right\} b_2 = \frac{1}{30} \end{cases}$$

$$\xRightarrow{b_2=\frac{1}{30}} \begin{cases} a_0 = 1 \\ \frac{-1}{30} + a_2 = -\frac{1}{2} \Rightarrow a_2 = -\frac{7}{15} \\ \frac{1}{900} - \frac{a_2}{30} + a_4 = \frac{1}{24} \\ b_2 = \frac{1}{30} \end{cases} \Rightarrow \begin{cases} a_0 = 1 \\ a_2 = -\frac{7}{15} \\ \frac{1}{900} + \frac{7}{450} + a_4 = \frac{1}{24} \Rightarrow a_4 = \frac{1}{40} \\ b_2 = \frac{1}{30} \end{cases}$$

$$\Rightarrow a_0 = 1,\ a_2 = -\frac{7}{15},\ a_4 = \frac{1}{40},\ b_2 = \frac{1}{30} \quad \#$$