

Poincaré inequality. For every interval $[a,b]$, $\exists C_p > 0$ s.t.

$$\|v\|_{L^2(a,b)} \leq C_p \|v'()\|_{L^2(a,b)}$$

for all $v \in C^1([a,b])$ s.t. $v(a) = v(b) = 0$ and where $\|\cdot\|_{L^2(a,b)}$ is the norm in $L^2(a,b)$.

We use Poincaré inequality to get the energy estimate,

but $u(x,t) = 1$ doesn't satisfy the assumption of the boundary condition.

So the energy estimate doesn't hold.