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Poincaré inequality. For every interval $[a,b]$, $\exists C_p > 0$ s.t.

$$\|v\|_{L^2(a,b)} \leq C_p \|v'\|_{L^2(a,b)}$$

for all $v \in C^1([a,b])$ s.t. $v(a)=v(b)=0$ and where $\|\cdot\|_{L^2(a,b)}$ is the norm in $L^2(a,b)$.

We use Poincaré inequality to get the energy estimate,

but $u(x,t)=1$ doesn't satisfy the assumption of the boundary condition.

So the energy estimate doesn't hold.