ST 501 Final Project Q3 Write Up

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Contents

Here we are going to let X_1 and X_2 be random variables with $E[X_i] = \mu_i$ and $\text{Var}(X_i) = \sigma_i^2$ for i = 1, 2. We are going to assume that $0 < \sigma_1^2 \leq \sigma_2^2$. We are also going to define our weighted random variable X(w), which will end up being our portfolio between two stocks. We will say that $X(w) = wX_1 + (1-w)X_2$ for $w \in [0,1]$ and ρ is the correlation between X_1 and X_2.

For any $w \in [0,1]$, find c_0, c_1, a, b, and c such that $E(X(w)) = \mu(w) = c_0 + c_1 w$ and $\text{Var}(X(w)) = \sigma^2(w) = aw^2 - 2bw + c$.

Here we can first find the $E(X(w)) = E(wX_1 + (1-w)X_2) = E(wX_1) + E((1-w)X_2) = wE(X_1) + (1-w)E(X_2) = w\mu_1 + (1-w)\mu_2 = w\mu_1 + \mu_2 - w\mu_2 = mu_2 + w(\mu_1 - \mu_2)$. Therefore, $c_0 = \mu_2$ and $c_1 = \mu_1 - \mu_2$. Now we can find $\text{Var}(X(w)) = Var(wX_1 + (1-w)X_2) = w^2 Var(X_1) + (1-w)^2 Var(X_2) + 2Cov(xX_1, (1-w)X_2) = w^2\sigma_1^2 + (w^2 - 2w + 1)\sigma_2^2 + 2w(1-w)Cov(X_1, X_2) = w^2\sigma_1^2 + (w^2 - 2w + 1)\sigma_2^2 + (2w - 2w^2)(\rho\sigma_1\sigma_2) = w^2\sigma_1^2 + w^2\sigma_2^2 - 2w\sigma_2^2 + \sigma_2^2 + 2w\rho\sigma_1\sigma_2 - 2w^2\rho\sigma_1\sigma_2 = w^2(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2) - 2w(\sigma_2^2 - \rho\sigma_1\sigma_2) + \sigma_2^2$. Also if we say that $\rho\sigma_1\sigma_2 = Cov(X_1, X_2)$ then our final answer of $w^2(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2) - 2w(\sigma_2^2 - \rho\sigma_1\sigma_2) + \sigma_2^2 = w^2(\sigma_1^2 + \sigma_2^2 - 2Cov(X_1, X_2)) - 2w(\sigma_2^2 - Cov(X_1, X_2)) + \sigma_2^2$. Therefore, we can clearly see that $a = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 = \sigma_1^2 + \sigma_2^2 - 2Cov(X_1, X_2), b = \sigma_2^2 - \rho\sigma_1\sigma_2 = \sigma_2^2 - Cov(X_1, X_2)$, and $c = \sigma_2^2$.