ST 518 Homework 6

Eric Warren

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1 Problem 1

A completely randomized experiment investigates the effects of increasing nitrogen (N) and copper (Cu) in the diet of chickens. Feed conversion ratio (FCR) is observed on $n = 4$ chickens for each of four treatment combinations(diets), with output below. Data are available online as "fcr.dat" so you can check your answers, but you should be able to complete these problems without software.

1.1 Part A

Write a factorial effects model for the 16 observed FCR measurements which assumes that, for a given diet, FCR is normally distributed, with variance σ^2 that is constant across diets.

We can say that this factorial effects model is $Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + E_{ijk}$ where for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ and E_{ijk} are i.i.d $N(0, \sigma^2)$ errors.

1.2 Part B

Estimate the simple effect of increasing copper when $N = 25$.

We can estimate the simple effect by only looking at the output for when $N = 25$. This occurs when $Cu = 10$ and gives us a mean value of 133 and when $Cu = 100$ and gives us a mean value of 146. Therefore, the simple effect of increasing copper when $n = 25$ is 146 - 133 = 13. For future purposes we will call this answer $\hat{\theta_1} = \bar{y_3} - \bar{y_1} = 13$ where $\bar{y_3} = 146$ and $\bar{y_1} = 133$.

1.3 Part C

Estimate the simple effect of increasing copper when $N = 45$.

We can estimate the simple effect by only looking at the output for when $N = 45$. This occurs when $Cu = 10$ and gives us a mean value of 130 and when $Cu = 100$ and gives us a mean value of 127. Therefore, the simple effect of increasing copper when $n = 45$ is subtracting these two values of 127 - 130 = -3. For future purposes we will call this answer $\hat{\theta_2} = \bar{y_4} - \bar{y_2} = -3$ where $\bar{y_4} = 127$ and $\bar{y_2} = 130$.

1.4 Part D

Estimate the difference in the simple effects of increasing copper across levels of Nitrogen.

This is just finding the difference between **Part B** and **Part C**. For purposes of this problem, we will call this estimate $\hat{theta_3} = \hat{\theta_1} - \hat{\theta_2} = \bar{y_3} - \bar{y_1} - (\bar{y_4} - \bar{y_2}) = \bar{y_3} - \bar{y_1} - \bar{y_4} + \bar{y_2} = -13 - (-3) = 16$. Therefore, this difference shown by $\hat{\theta_3} = 16$.

1.5 Part E

Using significance level $\alpha = .05$, test the hypothesis that the simple effects of copper are constant across levels of nitrogen.

Here we are going to do a F-test to see if this simple effects of copper are consistent across the different levels of nitrogen. Our null hypothesis is that the effects of copper are consistent across all levels of nitrogen. The alternative hypothesis is that they are not. We are going to use our result from **Part D** to help us get our appropriate F-value. In this case, we have shown that $\hat{\theta_3} = 16$. Now let us note the contrast which is $(-1, 1, 1, -1)'$ which will get us our c_i values later. To get our F-value, we must use the formula $F = \frac{SS(\hat{\theta_3})/(a-1)(b-1)}{MS(E)}$ where $a = 2$ because there are only 2 nitrogen levels, $b = 2$ because there are only 2 copper levels, $SS(\hat{\theta_3}) = \frac{(\hat{\theta_3})^2}{\sum_{i=1}^{4} \frac{c_i^2}{n_i}} = \frac{16^2}{\frac{-1^2}{4} + \frac{1^2}{4} + \frac{1^2}{4} + \frac{-1^2}{4}} = \frac{256}{4 * \frac{1}{4}} = \frac{256}{1} = 256$, $MS(E) = \frac{1}{4} * \sum_{i=1}^{4} s_i^2 = 36 + 16 + 58.6666667 + 57.3333333 = \frac{1}{4} * 168 = 42$. Now that we have found all the values for our F-test we can get our F-value to be $F = \frac{SS(\hat{\theta_3})/(a-1)(b-1)}{MS(E)} = \frac{256/(2-1)(2-1)}{42} = \frac{256}{42} = 6.095238$ on degrees of freedom $df = (a-1) * (b-1), n(ab-1) = (2-1)(2-1), 4(2*2-1) = 1*1, 4*3 = 1, 12$. We can get our F-critical value (or F^*) by using the `qf()` function in R. Our F^* value is `qf(1-alpha, df1, df2)` = `qf(1-.05, 1, 12)` = 4.7472253. Since the F-value we obtained is greater the F-critical value ($6.095238 > 4.7472253$) then we reject our null hypothesis and say that the simple effects of copper are **not** consistent across the different levels of nitrogen.

1.6 Part F

Report the smallest level of significance at which the difference between simple copper effects across levels of nitrogen may be declared significant.

I am assuming for this problem that we are essentially finding the p-value we obtain from our F-value and use that as the smallest significance level we can use for the difference between copper effects still be declared significant. Note in **Part E** we showed that the F-value we obtained was 6.095238. Now we can use the `pf()` function to get the smallest level of significance at which we can still declare significance. To do this, we will put into the function `pf(F-value, df1, df2, lower.tail = F)` = `pf(6.095238, 1, 12, lower.tail = F)` = 0.0295555. Therefore, the smallest level of significance at which the difference between simple copper effects across levels of nitrogen may be declared significant is 0.0295555.