

# ST 518 Homework 5

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## 1 Problem 1

In a completely randomized design,  $N = 80$  preschoolers are randomized to four instruction groups to study possible effect on development. The means and variances of improvements in a spatio-temporal reasoning score are given in the table below.

$i$	Group	$n_i$	$\bar{y}_i$	$s_i^2$
1	Instructor	20	7.3	14.2
2	Computer	20	4.5	9.3
3	Control	20	0.3	11.0
4	Piano	20	3.7	7.5

## 1.1 Part A

Use Tukey's procedure to carry out all pairwise comparisons while controlling the familywise error rate at level  $\alpha = 0.05$ . Which of the observed differences may be declared significant. Convey this info by producing a recorded table of means that uses letters to enable the reader to know which differences are significant.

First we need to remember how a Tukey test works. The formula we should use is we reject the  $H_0$  if  $|\hat{\theta}| > q(\alpha, t, N - t) \sqrt{\frac{MS(E)}{n}}$  where  $\hat{\theta} = \bar{y}_i - \bar{y}_j$  and  $MS(E) = \frac{1}{N-t} \sum_{i=1}^t (n_i - 1)s_i^2$ . Below we are going to get the MSE and do some tests.

```
# Put in the means
mean_1 <- 7.3; mean_2 <- 4.5; mean_3 <- 0.3; mean_4 <- 3.7

# Put in all the si2 values
s2_1 <- 14.2; s2_2 <- 9.3; s2_3 <- 11.0; s2_4 <- 7.5

# Put in the N, t, and ni values
N = 80; n1 <- 20; n2 <- 20; n3 <- 20; n4 <- 20; t <- 4

# Get the MSE; show output for case of problem
mse_problem_1 <- (1 / (N-1)) * ((n1-1) * s2_1 + (n2-1) * s2_2 + (n3-1) * s2_3 + (n4-1) * s2_4)
mse_problem_1

## [1] 10.10127

# Get the Tukey 95% HSD (use qtkey(1-alpha, t, N-t) * sqrt(mse / n)) where n = any ni value; show output
tukey_critical_value <- qtkey(.95, t, N-t) * sqrt(mse_problem_1 / n1)
tukey_critical_value

## [1] 2.640063

# Compare means from groups 1 and 2; show for case in problem
tukey_test_1_against_2 <- abs(mean_1 - mean_2)
tukey_test_1_against_2

## [1] 2.8

tukey_test_1_against_2 > tukey_critical_value # Do we reject?

## [1] TRUE
```

```
# Compare means from groups 1 and 3; show for case in problem
tukey_test_1_against_3 <- abs(mean_1 - mean_3)
tukey_test_1_against_3
```

```
## [1] 7
```

```
tukey_test_1_against_3 > tukey_critical_value # Do we reject?
```

```
## [1] TRUE
```

```
# Compare means from groups 1 and 4; show for case in problem
tukey_test_1_against_4 <- abs(mean_1 - mean_4)
tukey_test_1_against_4
```

```
## [1] 3.6
```

```
tukey_test_1_against_4 > tukey_critical_value # Do we reject?
```

```
## [1] TRUE
```

```
# Compare means from groups 2 and 3; show for case in problem
tukey_test_2_against_3 <- abs(mean_2 - mean_3)
tukey_test_2_against_3
```

```
## [1] 4.2
```

```
tukey_test_2_against_3 > tukey_critical_value # Do we reject?
```

```
## [1] TRUE
```

```
# Compare means from groups 2 and 4; show for case in problem
tukey_test_2_against_4 <- abs(mean_2 - mean_4)
tukey_test_2_against_4
```

```
## [1] 0.8
```

```
tukey_test_2_against_4 > tukey_critical_value # Do we reject?
```

```
## [1] FALSE
```

```
# Compare means from groups 3 and 4; show for case in problem
tukey_test_3_against_4 <- abs(mean_3 - mean_4)
tukey_test_3_against_4
```

```
## [1] 3.4
```

```
tukey_test_3_against_4 > tukey_critical_value # Do we reject?
```

```
## [1] TRUE
```

As we can see by doing our Tukey test, the only test that results in **FALSE** for “Do we reject?” is the comparison between the second group (computer) and the fourth group (piano). We are going to show a table below to make this easier to follow.

Statistic	Output
Alpha	0.05
Error Degrees of Freedom	76
Error Mean Square	10.1012658
Critical Value of Test (HSD)	2.6400635

In this table, we will show the means and means with the same letter are **NOT** significantly different when doing the pairwise Tukey Test.

Tukey Grouping	Group	n	Mean
A	Instructor	20	7.3
	Computer	20	4.5
	Control	20	0.3
A	Piano	20	3.7

## 1.2 Part B

Use Scheffe’s procedure to obtain simultaneous 95% confidence intervals for all the pairwise differences considered in *part (a)* in addition to the contrast of the control with the average of the three non-control treatment groups.

First we are going to find the pairwise comparisons using Scheffe’s procedure. To find the intervals we use the confidence interval formula:

$$\bar{y}_i - \bar{y}_j \pm \sqrt{(t-1)F(1-\alpha, t-1, N-t) * MS(E) * \sum_{i=1}^t \frac{c_i^2}{n_i}}$$

Please note, that the margin of error in the confidence interval for all pairwise comparisons are the same since there are just two values (and contrasts have a value of 1 and -1 while all the  $n_i$  values are the same). We know that  $\sum_{i=1}^t \frac{c_i^2}{n_i} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} = \frac{1}{10}$ . We are going to obtain the margin of error first.

```
alpha <- 0.05 # 95% confidence interval
```

```
scheffe_moe <- sqrt((t-1) * qf(1-alpha, t-1, N-t) * mse_problem_1 * .1)
scheffe_moe # Show for purpose of the problem
```

```
## [1] 2.873607
```

Now we are going to get our grouping confidence intervals and make a table below to make it easy to follow.

```

# Compare group 1 and 2
lower_CI_groups_1_vs_2 <- (mean_1 - mean_2) - scheffe_moe
upper_CI_groups_1_vs_2 <- (mean_1 - mean_2) + scheffe_moe

# Compare group 1 and 3
lower_CI_groups_1_vs_3 <- (mean_1 - mean_3) - scheffe_moe
upper_CI_groups_1_vs_3 <- (mean_1 - mean_3) + scheffe_moe

# Compare group 1 and 4
lower_CI_groups_1_vs_4 <- (mean_1 - mean_4) - scheffe_moe
upper_CI_groups_1_vs_4 <- (mean_1 - mean_4) + scheffe_moe

# Compare group 2 and 3
lower_CI_groups_2_vs_3 <- (mean_2 - mean_3) - scheffe_moe
upper_CI_groups_2_vs_3 <- (mean_2 - mean_3) + scheffe_moe

# Compare group 2 and 4
lower_CI_groups_2_vs_4 <- (mean_2 - mean_4) - scheffe_moe
upper_CI_groups_2_vs_4 <- (mean_2 - mean_4) + scheffe_moe

# Compare group 3 and 4
lower_CI_groups_3_vs_4 <- (mean_3 - mean_4) - scheffe_moe
upper_CI_groups_3_vs_4 <- (mean_3 - mean_4) + scheffe_moe

```

Here we will show a table of the groups with their corresponding Scheffe Confidence Interval. For note of the table, we are subtracting the first mean value minus the second mean value. So a negative value means the second group in the comparison has a larger average value and a positive value means the first group's average value is larger.

Comparison of Groups	Lower Confidence Interval	Upper Confidence Interval
Instructor and Computer	-0.0736066	5.6736066
Instructor and Control	4.1263934	9.8736066
Instructor and Piano	0.7263934	6.4736066
Computer and Control	1.3263934	7.0736066
Computer and Piano	-2.0736066	3.6736066
Control and Piano	-6.2736066	-0.5263934

Now we are going to show the contrast of the control with the average of the three non-control treatment groups. The formula we will use is

$$\frac{\bar{y}_1. + \bar{y}_2. + \bar{y}_4.}{3} - \bar{y}_3. \pm \sqrt{(t-1)F(1-\alpha, t-1, N-t) * MS(E) * \sum_{i=1}^t \frac{c_i^2}{n_i}}$$

where  $t = 4$  and  $\sum_{i=1}^t \frac{c_i^2}{n_i} = \frac{-1^2}{20} + \frac{3 * \frac{1}{3}^2}{20} = \frac{1}{20} + \frac{13}{20} = \frac{1}{20} + \frac{1}{60} = \frac{4}{60} = \frac{1}{15}$ . Now we are going to get the confidence interval for all the groups compared to the control.

```

# Get margin of error
scheffe_moe_all <- sqrt((t-1) * qf(1-alpha, t-1, N-t) * mse_problem_1 * (1/15))
scheffe_moe_all # Show for purpose of the problem

```

```
## [1] 2.34629
```

```
# Get bounds for confidence interval
lower_CI_control_vs_all <- (((mean_1 + mean_2 + mean_4) / 3) - mean_3) - scheffe_moe_all
upper_CI_control_vs_all <- (((mean_1 + mean_2 + mean_4) / 3) - mean_3) + scheffe_moe_all

# Show bounds for case of problem
lower_CI_control_vs_all; upper_CI_control_vs_all
```

```
## [1] 2.520377
```

```
## [1] 7.212957
```

As we can see the lower bound for our 95% confidence interval is 2.5203767 and our upper bound is 7.2129566 which is saying that we are 95% confident that the average mean reasoning score when using a program is between 2.5203767 and 7.2129566 points higher than doing nothing (which is the control group).

### 1.3 Part C

Would Bonferroni (with  $k = 7$ ) have been more efficient than Scheffe here? To answer this question, compare the widths, or relevant component multipliers of the intervals.

Since we are trying to see what is more efficient we need to compare the margin of error on the intervals. Let us look at the margin of errors for both Scheffe and Bonferroni. For Scheffe the  $MOE_{scheffe} = \sqrt{(t-1)F(1-\alpha, t-1, N-t) * MS(E) * \sum_{i=1}^t \frac{c_i^2}{n_i}}$  and for Bonferroni the  $MOE_{bonferroni} = t(\frac{\alpha/k}{2}, \nu) * \sqrt{MSE * \sum_{i=1}^t \frac{c_i^2}{n_i}}$ . As we can see both margin of errors have  $\sqrt{MSE * \sum_{i=1}^t \frac{c_i^2}{n_i}}$  so we are really comparing which is bigger between  $\sqrt{(t-1)F(1-\alpha, t-1, N-t)}$  for Scheffe or  $t(\frac{\alpha/k}{2}, \nu)$  for bonferroni. Let us examine this below. Note,  $\alpha = 0.05$  since we are looking at 95% confidence intervals.

```
# Get Scheffe width
scheffe_width <- sqrt((t-1) * qf(1-alpha, t-1, N-t))

# Get Bonferroni width; k = 7
k <- 7
bonferroni_width <- qt(1 - ((alpha / k) / 2), N-t)

# Compare both widths
scheffe_width; bonferroni_width
```

```
## [1] 2.859166
```

```
## [1] 2.764879
```

As we can see, the Bonferroni procedure would have been better than Scheffe because the multipliers of the Bonferroni procedure is less; moreover, the width of the Bonferroni procedure is less. Therefore, Bonferroni would have been more efficient than Scheffe here. Please note, when the degrees of freedom is than two, Bonferroni will have a smaller width than Scheffe, just like we have shown here.

## 2 Problem 2

Cheese is produced by bacterial fermentation of milk. Consider designing an experiment to test for the possible effect of adding two strains (A and B) of nonstarter bacteria on total free amino acids ( $y$ ) in cheese. There will be  $t = 4$  conditions: neither strain added, A added, B added, both added. Suppose the mean total free amino acids under the first condition is believed to be  $\mu_1 = 4.4$ . The smallest effect the experiment needs to be able to find is  $H_1 : \mu_1 = 4, \mu_A = 4.4, \mu_B = 4.4, \mu_{AB} = 4.8$ . (The effects under this hypothesis are said to be “additive.”)

Suppose that, for the first condition, we might expect to produce cheese that could have amino acid content as high as 4.3 or as low as 3.7. Use the range to roughly approximate the error variance.

For the part above we know that  $\sigma = \frac{Range}{4}$ . We know the  $Range = 4.3 - 3.7$  so  $\sigma = \frac{Range}{4} = \frac{4.3-3.7}{4} = \frac{0.6}{4} = 0.15$ . So  $\sigma = 0.15$  which means our error variance is  $\sigma^2 = 0.15^2 = 0.0225$ .

$r = 5$  cheeses for will be produced under each condition (after suitable randomization of the  $t = 4$  treatments to conditions) for a total of  $N = rt = 20$  cheeses. Total free amino acid content will be measured and the F-test will be used to test the null hypothesis  $H_0 : \mu_1 = \mu_A = \mu_B = \mu_{AB}$

### 2.1 Part A

Give the critical value of the F-distribution and the decision rule for the F-ratio for a test of  $H_0$  at level  $\alpha = 0.05$ .

We can find this below by using our `qf(1 - alpha, t-1, N-t)` where we are told that  $t = 4$  and  $N = rt = 20$ .  $\alpha = 0.05$ .

```
# Put in needed terms
alpha <- 0.05; r <- 5; t <- 4; N <- r*t

# Get critical value
problem_2_critical_value <- qf(1 - alpha, t-1, N-t)
```

Our critical value when  $\alpha = 0.05$  for a test on  $H_0$  is 3.2388715. The decision rule for this F-ratio test is that if the corresponding F-statistic value we get when performing our test is less than or equal to 3.2388715 we fail to reject  $H_0$  but if our F-statistic value is greater than 3.2388715 we reject  $H_0$  and proceed with accepting the  $H_A$ .

### 2.2 Part B

Report the power to reject  $H_0$  when  $H_1$  is true.

We are going to report the power or  $P(\text{Reject } H_0 \mid H_A \text{ is true})$ . In this case it is  $P(\text{Reject } H_0 \mid H_1 \text{ is true})$ . We can show this in our code below. Remember  $\sigma^2 = 0.15^2 = 0.0225$  and  $H_1 : \mu_1 = 4, \mu_A = 4.4, \mu_B = 4.4, \mu_{AB} = 4.8$ .

```
alpha <- 0.05 # Get alpha
sigma2 <- 0.0225 # Get sigma^2

# Get the power through this chain
my_ncp <- r * (t-1) * var(c(4, 4.4, 4.4, 4.8)) / sigma2
power <- 1 - pf(qf(1-alpha, t-1, N-t), t-1, N-t, my_ncp); power # Show the power for case of problem

## [1] 0.9999991
```

As we can see the power to reject  $H_0$  when  $H_1$  is true is 0.9999991.

## 2.3 Part C

How would the power change if we used an  $\alpha = 0.01$  critical value?

We know that as  $\alpha$  decreases, so does the power but we will show that in our example here.

```
alpha2 <- 0.01
power2 <- 1 - pf(qf(1-alpha2, t-1, N-t), t-1, N-t, my_ncp); power2 # Show the power for case of problem

## [1] 0.9999095
```

As we can see when  $\alpha = 0.01$ , the power is now 0.9999095 as opposed to when the power was 0.9999991 when  $\alpha = 0.05$ . Therefore we can see that the power decreases when  $\alpha$  decreases.

## 2.4 Part D

How would the power change if we used  $\alpha = 0.05$  but  $\sigma$  was really larger than our approximation?

We know from our lecture that when the  $\sigma^2$  increases, the power decreases. Thus if  $\sigma$  was really larger than our approximation then  $\sigma^2$  would be larger than our approximation and our power would decrease. This is because we are increasing our denominator which means the overall value (of power) should be decreasing. We will show that in our example here where we will say our  $\sigma$  value is really 0.5 so  $\sigma^2 = 0.25$  now.

```
alpha <- 0.05
sigma2_d <- 0.25 # Get sigma^2

# Get the power through this chain
my_ncp2 <- r * (t-1) * var(c(4, 4.4, 4.4, 4.8)) / sigma2_d
power3 <- 1 - pf(qf(1-alpha, t-1, N-t), t-1, N-t, my_ncp2); power3 # Show the power for case of problem

## [1] 0.4433369
```

As we can see when  $\sigma = 0.5$  (which means  $\sigma$  is increasing), the power is now 0.4433369 as opposed to when the power was 0.9999991 when  $\sigma = 0.15$  (which was when  $\sigma$  was lower). Therefore we can see that the power decreases when  $\sigma$  increases.

## 2.5 Part E

How would the type II error rate,  $\beta$  change if we produced  $r + 1$  cheeses per treatment rather than  $r$ ?

Note in our past calculations,  $r = \frac{N}{t}$  so  $r + 1 = \frac{N}{t} + 1$ . When we will do the example we will change  $\frac{N}{t} = 5$  to  $\frac{N}{t} + 1 = r + 1 = 6$ . Now we know from the lecture that by the number in each treatment (in this  $r$  increasing to  $r + 1$ ) the power increases. Since  $\beta = 1 - \text{power}$ , we know that if power increases,  $\beta$  must decrease. Our example will show this.

```
alpha <- 0.05 # Get alpha
sigma2 <- 0.0225 # Get sigma^2
new_r <- 6
new_N <- new_r * t

# Get the power through this chain
my_ncp3 <- new_r * (t-1) * var(c(4, 4.4, 4.4, 4.8)) / sigma2
power4 <- 1 - pf(qf(1-alpha, t-1, new_N-t), t-1, new_N-t, my_ncp3); power4 # Show the power for case of
```



```
## [1] 1
```

As we can see in this example, the power is increasing (in this example we can see the power is 1 as before which was 0.9999991). This means that the  $\beta$  is decreasing since it has an opposite relationship of power (if power goes up  $\beta$  goes down and vice versa). Therefore, we have shown from this example and what we know from before that if we produced  $r + 1$  cheeses per treatment rather than  $r$ ,  $\beta$  (or Type II Error) will decrease.

## 2.6 Part F

Find the smallest  $r$  so that the power exceeds 0.9.

Here we are going to look at different values of  $r$  that will allow us to see what is the smallest value that will allow us to still get a power exceeding 0.9.

```
# Set up loop
library(tidyverse)
results <- list()
results <- vector("list", length = 4)

# Make for loop to go through r = 2, 3, 4, 5 (we can stop at 5 since we know the power > 0.9 at r = 5)
for (i in 2:5){
  alpha <- 0.05; r <- i; t <- 4; N <- r*t; sigma2 <- 0.0225
  my_ncp_value <- r * (t-1) * var(c(4, 4.4, 4.4, 4.8)) / sigma2
  power_value <- 1 - pf(qf(1-alpha, t-1, N-t), t-1, N-t, my_ncp_value)
  results_data <- data.frame(r, ncp = my_ncp_value, power = power_value)
  results[[i]] <- results_data
}

# Show the results
final_results <- as_tibble(do.call(rbind, results)); final_results # show output
```

```
## # A tibble: 4 x 3
##       r    ncp power
##   <int> <dbl> <dbl>
## 1     2  28.4  0.775
## 2     3  42.7  0.993
## 3     4  56.9  1.00
## 4     5  71.1  1.00
```

As we can see in our table, the  $r = 3$  is when our power first gets above 0.9 so the smallest  $r$  we need is when  $r = 3$ .

## 3 Problem 3

Recall the battery experiment from Homework #4. The four battery types are actually combinations of two factors, Duty and Brand:

Battery Type	Duty	Brand
1	Alkaline	Name

Battery Type	Duty	Brand
2	Alkaline	Store
3	Heavy	Name
4	Heavy	Store

First we are going to read in the `battery` data and will show what it looks like.

```
library(tidyverse)
battery <- read_table("battery.txt")
battery$TypeBat <- factor(battery$TypeBat)
battery
```

```
## # A tibble: 16 x 5
##   TypeBat Life UCost LPUC Order
##   <fct>   <dbl> <dbl> <dbl> <dbl>
## 1 1      602 0.985 611     1
## 2 2      863 0.935 923     2
## 3 1      529 0.985 537     3
## 4 4      235 0.495 476     4
## 5 1      534 0.985 542     5
## 6 1      585 0.985 593     6
## 7 2      743 0.935 794     7
## 8 3      232 0.52  445     8
## 9 4      282 0.495 569     9
## 10 2      773 0.935 827    10
## 11 2      840 0.935 898    11
## 12 3      255 0.52  490    12
## 13 4      238 0.495 480    13
## 14 3      200 0.52  384    14
## 15 4      228 0.495 460    15
## 16 3      215 0.52  413    16
```

Now we are going to show the mean of each battery type.

```
mean_output <- battery %>%
  group_by(TypeBat) %>%
  summarize(mean = mean(LPUC))
mean_output # show table
```

```
## # A tibble: 4 x 2
##   TypeBat mean
##   <fct>   <dbl>
## 1 1      571.
## 2 2      860.
## 3 3      433
## 4 4      496.
```

Therefore we can turn this table below:

Battery Type	Duty	Brand
1	Alkaline	Name
2	Alkaline	Store
3	Heavy	Name
4	Heavy	Store

into a 2x2 table with the means

	Duty = Alkaline	Duty = Heavy	Marginal Mean
Brand = Name	570.75	433	501.875
Brand = Store	860.5	496.25	678.375
Marginal Mean	715.625	464.625	

### 3.1 Part A

Estimate each contrast listed below:

#### 3.1.1 Part i

The simple effect of Duty for Name Brand batteries.

To find this we need to look at the mean difference of battery type with the fixed variable being name batteries. Thus, we are going to look at the difference of means between battery type 1 and 3. Thus,

$$\theta_1 = (1, 0, -1, 0)' \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \mu_1 - \mu_3 \text{ so to find the estimate we are going to say that } \hat{\theta}_1 = \hat{\mu}_1 - \hat{\mu}_3 = \bar{y}_1 - \bar{y}_3.$$

We know  $\bar{y}_1 = 570.75$  and  $\bar{y}_3 = 433$ .  $\hat{\theta}_1 = \bar{y}_1 - \bar{y}_3 = 137.75$ . Thus, the simple effect of Duty for Name Brand batteries is 137.75.

#### 3.1.2 Part ii

The simple effect of Duty for Store Brand batteries.

To find this we need to look at the mean difference of battery type with the fixed variable being store batteries. Thus, we are going to look at the difference of means between battery type 2 and 4. Thus,

$$\theta_2 = (0, 1, 0, -1)' \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \mu_2 - \mu_4 \text{ so to find the estimate we are going to say that } \hat{\theta}_2 = \hat{\mu}_2 - \hat{\mu}_4 = \bar{y}_2 - \bar{y}_4.$$

We know  $\bar{y}_2 = 860.5$  and  $\bar{y}_4 = 496.25$ .  $\hat{\theta}_2 = \bar{y}_2 - \bar{y}_4 = 364.25$ . Thus, the simple effect of Duty for Store Brand batteries is 364.25.

#### 3.1.3 Part iii

The main effect of Duty.

To find this we need to look at the mean difference of batteries with the different duties with no fixed variable. Thus, we are going to look at the difference of means between battery types 1 and 2 and then compare to

types 3 and 4. Thus,  $\theta_3 = \frac{1}{2}(1, 1, -1, -1)' \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \frac{1}{2}(\mu_1 + \mu_2 - \mu_3 - \mu_4)$  so to find the estimate we are going

to say that  $\hat{\theta}_3 = \frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2 - \hat{\mu}_3 - \hat{\mu}_4) = \frac{1}{2}(\bar{y}_1 + \bar{y}_2 - \bar{y}_3 - \bar{y}_4)$ . We know  $\bar{y}_1 = 570.75$ ,  $\bar{y}_2 = 860.5$ ,  $\bar{y}_3 = 433$ , and  $\bar{y}_4 = 496.25$ .  $\hat{\theta}_3 = \frac{1}{2}(\bar{y}_1 + \bar{y}_2 - \bar{y}_3 - \bar{y}_4) = 251$ . Thus, the main effect of Duty batteries is 251.

### 3.1.4 Part iv

The interaction effect.

To find this we need to look at the mean difference of batteries for the interaction effect (which means we want to look at the opposite battery characteristics to compare this). Thus, we are going to look at the difference of means between battery types 1 and 4 and then compare to types 2 and 3 (since we want to

compare with the opposites to find the interaction). Thus,  $\theta_4 = (1, -1, -1, 1)' \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \mu_1 - \mu_2 - \mu_3 + \mu_4$

so to find the estimate we are going to say that  $\hat{\theta}_4 = \hat{\mu}_1 - \hat{\mu}_2 - \hat{\mu}_3 + \hat{\mu}_4 = \bar{y}_1 - \bar{y}_2 - \bar{y}_3 + \bar{y}_4$ . We know  $\bar{y}_1 = 570.75$ ,  $\bar{y}_2 = 860.5$ ,  $\bar{y}_3 = 433$ , and  $\bar{y}_4 = 496.25$ .  $\hat{\theta}_4 = \bar{y}_1 - \bar{y}_2 - \bar{y}_3 + \bar{y}_4 = -226.5$ . Thus, the interaction effect of batteries is -226.5. We can also see this is the same as saying  $\hat{\theta}_4 = \hat{\theta}_1 - \hat{\theta}_2$  as said in the lectures, and with  $\hat{\theta}_1 = 137.75$  and  $\hat{\theta}_2 = 364.25$  then  $\hat{\theta}_4 = \hat{\theta}_1 - \hat{\theta}_2 = 137.75 - 364.25 = -226.5$ .

### 3.1.5 Part v

The main effect of Brand.

To find this we need to look at the mean difference of batteries with the different brands with no fixed variable. Thus, we are going to look at the difference of means between battery types 1 and 3 and then

compare to types 2 and 4. Thus,  $\theta_5 = \frac{1}{2}(1, -1, 1, -1)' \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix} = \frac{1}{2}(\mu_1 - \mu_2 + \mu_3 - \mu_4)$  so to find the estimate

we are going to say that  $\hat{\theta}_5 = \frac{1}{2}(\hat{\mu}_1 - \hat{\mu}_2 + \hat{\mu}_3 - \hat{\mu}_4) = \frac{1}{2}(\bar{y}_1 - \bar{y}_2 + \bar{y}_3 - \bar{y}_4)$ . We know  $\bar{y}_1 = 570.75$ ,  $\bar{y}_2 = 860.5$ ,  $\bar{y}_3 = 433$ , and  $\bar{y}_4 = 496.25$ .  $\hat{\theta}_5 = \frac{1}{2}(\bar{y}_1 - \bar{y}_2 + \bar{y}_3 - \bar{y}_4) = -176.5$ . Thus, the main effect of brand batteries is -176.5.

## 3.2 Part B

If you were to compute the contrast sum of squares for the last three listed contrasts, what would they add up to?

We know that by adding the sum of squares of the main effects and the interaction effect it will equal the sum of squares for treatment. That is we know that  $SS(\hat{\theta}_3) + SS(\hat{\theta}_4) + SS(\hat{\theta}_5) = SS(Trt)$ . From homework 4, we know that this value is 427915.25, so adding up the last three listed contrasts' sum of squares ( $SS(\hat{\theta}_3) + SS(\hat{\theta}_4) + SS(\hat{\theta}_5)$ ) would equal the  $SS(Trt)$  of 427915.25.