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Econ436 Project 2: Stationarity of Relative Stock Prices

Introduction

The goal of this project is to determine whether a country's stock index is mean reverting, relative to the United States or United Kingdom's index. Mean reversion of a country's stock index relative to a reference country means that if the stock prices of the two countries diverge, they will eventually converge back to their average difference. This implies that excessive divergences are temporary; the "winning" country will eventually fall and the "losing" country will eventually rise and regain its previous losses, much like a stretched spring will undergo harmonic motion around its original position.

If this pattern holds true, a contrarian investment strategy that sells the country on a winning streak and buys the country on the losing streak will outperform a momentum strategy (buy-and-hold) because mean reversion implies that the streaks are temporary. Although the momentum strategy may generate positive returns, the contrarian strategy will take advantage of these market patterns and generate higher returns.

A way to test this hypothesis is to test the stationarity of the differences between a country's index and the US or UK's index. This project completed that analysis and found that, indeed, some countries exhibited stationarity over the long term with the USA or the UK. An analysis of effects of the financial crisis on index mean reversion followed after. The stationarity of relative stock prices was tested before and after the respective financial crises in America and Europe, and it was found that the financial crisis affected the stationarity results significantly.

Data Description

The stock indices of 23 countries were downloaded from Morgan Stanley Capital International (MSCI), which is a major compiler of stock index data and portfolio analysis tools. MSCI has stock indices for a large number of countries around the globe, and tracks the returns on each index. The countries are stratified by market maturity (developed markets, emerging markets, etc.) and the index level can be observed as the price return, total (gross) return, or net return. The performance data for country stock indexes can be found under "end of day index data search" on MSCI's website.

The countries analyzed included the United States and the United Kingdom as base countries, and the 21 other countries available under "developed markets" (17 European countries, Japan, Hong Kong, Singapore, and Israel). The data contained monthly entries of gross returns on stock indexes, starting in 1970. For some countries, such as Israel and Finland, the index started in 1993 and 1988, respectively.

Standard Statistics for Stock Prices Relative to the USA:

	mean	stdDev	skewness	kurtosis
AUSTRALIA	-0.41952	0.321248	-0.73307	3.575503
AUSTRIA	0.055716	0.593214	-0.35868	2.086107
BELGIUM	0.678832	0.320941	-0.28092	2.437738
CANADA	-0.01788	0.351276	-0.36668	2.354096
DENMARK	0.803235	0.387758	-0.16633	2.786313
FINLAND	-1.91797	0.410601	-0.3658	3.161824
FRANCE	0.159574	0.229352	-0.06142	2.056525

GERMANY	0.113508	0.236073	0.210468	2.460398
HONG.KONG	1.642944	0.462387	-0.88128	3.777897
IRELAND	-2.28858	0.62308	-0.50777	1.711712
ISRAEL	-2.90436	0.323202	0.136624	2.428147
ITALY	-1.10708	0.53812	-0.09445	2.748987
JAPAN	0.648919	0.744749	0.344911	2.025353
NETHERLANDS	0.738436	0.357194	-0.94179	2.971636
NEW.ZEALAND	-2.56748	0.382311	-0.46291	3.71549
NORWAY	0.465404	0.382504	-0.2874	2.753983
PORTUGAL	-2.95077	0.545023	-0.50922	3.069396
SINGAPORE	0.731357	0.506753	0.01677	2.285162
SPAIN	-0.31086	0.427063	0.448129	3.255153
SWEDEN	0.900631	0.500367	-0.39097	2.244524
SWITZERLAND	0.428868	0.241637	-0.21337	2.387442
UNITED.KINGDOM	0.236838	0.242338	-0.61376	3.012474
USA	NA	NA	NA	NA

Standard Statistics for Stock Prices Relative to the UK:

	mean	stdDev	skewness	kurtosis
AUSTRALIA	-0.65636	0.350456	-0.22698	2.312903
AUSTRIA	-0.18112	0.535352	0.420988	2.759788
BELGIUM	0.441994	0.278201	-0.82564	3.400219
CANADA	-0.25472	0.392975	0.054292	2.546326
DENMARK	0.566397	0.453694	0.468562	2.361905
FINLAND	-2.18979	0.487979	-0.69477	3.258071
FRANCE	-0.07726	0.189687	-0.25965	3.371235
GERMANY	-0.12333	0.236466	1.336278	4.896282
HONG.KONG	1.406106	0.42926	-0.46392	2.847132
IRELAND	-2.5604	0.474215	-0.70268	1.862102
ISRAEL	-3.12022	0.234387	0.094184	2.724392
ITALY	-1.34392	0.509033	1.044748	3.806142
JAPAN	0.412081	0.633418	0.219677	1.640878
NETHERLANDS	0.501598	0.318807	-0.48671	2.59357
NEW.ZEALAND	-2.83929	0.310704	-0.65092	2.98096
NORWAY	0.228566	0.374504	0.485968	4.20914
PORTUGAL	-3.22259	0.340819	-0.29265	3.209213
SINGAPORE	0.494519	0.467231	0.199835	2.447907
SPAIN	-0.5477	0.506426	0.971277	4.433767
SWEDEN	0.663793	0.506714	-0.0879	2.064143
SWITZERLAND	0.19203	0.304605	-0.01987	2.055752
UNITED.KINGDOM	NA	NA	NA	NA
USA	-0.23684	0.242338	0.613755	3.012474

Methodology (econometric models)

In order to test the connection between the various OECD stock markets to the USA and UK, we test the stationarity between these countries are those base countries. This is done using two unit root tests: the Augmented Dickey-Fuller (linear) and ESTAR (nonlinear) unit root tests. Both are evaluated as first order equations (with only one lag variable). These two models are fitted to the relative price data, and we test the estimated parameters of these models for proof of stationarity.

First, the natural log of the stock prices were calculated in order to correct for the exponential time trend. Second, the relative stock price for each country was created by contrasting it with the USA and UK's returns. Therefore, the relative stock prices for each country can be stated as:

$$rp_{USA} = \ln(p_{country}) - \ln(p_{USA})$$
$$rp_{UK} = \ln(p_{country}) - \ln(p_{UK})$$

Once these relative prices were obtained, the linear and nonlinear root tests can be applied to check stationarity. In these tests, a specific ARIMA model is fitted to the data, and a hypothesis test is conducted on the estimated parameter of the lag variable, rp_{t-1}. Specifically, the t-value of the parameter from the regression results is compared to a chart appropriate for the model used.

In applying the first order, intercept-only (no trend term) Dickey-Fuller (ADF) test, this linear model was fitted to the data:

$$rp_t = \alpha + \rho(rp_{t-1}) + \beta_1(\Delta rp_{t-1}) + \eta_t$$

(Where Δr is the first difference of the relative price data). The t-value on the coefficient $\hat{\rho}$ is compared to the Linear ADF Critical Values given by Balvers et al. If this t-value is less than the critical value (this is a one-sided test), the null hypothesis is rejected, implying that the data exhibits stationarity.

The same is done for the nonlinear model in the approximate form:

$$\Delta rp_t = \delta(rp_{t-1}^3) + \beta_1(\Delta rp_{t-1}) + \epsilon_t$$

The first difference of the relative prices are regressed onto the previous price, cubed, and the previous differenced price. The t-value on the coefficient $\hat{\delta}$ is compared to the appropriate critical t-values for the nonlinear (ESTAR) unit root test, given by Kapetanios et al.

These tests were translated into R code so that they could be run in a loop to evaluate multiple countries. Some of the R code for the above tests is given here.

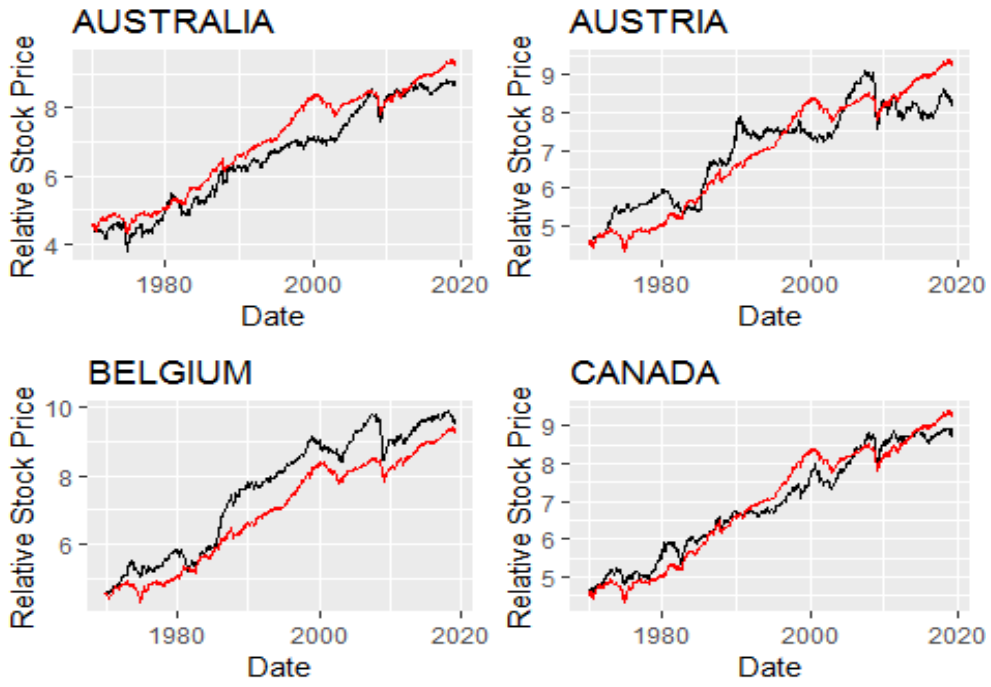
```
#linear regression, intercept only, no trend
get_tval_linregress_intOnly <- function(rp) {
  if (is.na(rp[1])) return(NA)

  n <- length(rp)
  d.rp <- rp[2:n] - rp[1:n-1]
  mod <- lm(d.rp[2:n] ~ rp[1:n-1] + d.rp[1:n-1])

  sum <- summary(mod)$coefficients
  t_val <- sum[8]
  return(t_val)
}

#nonlinear regression, intercept only
```

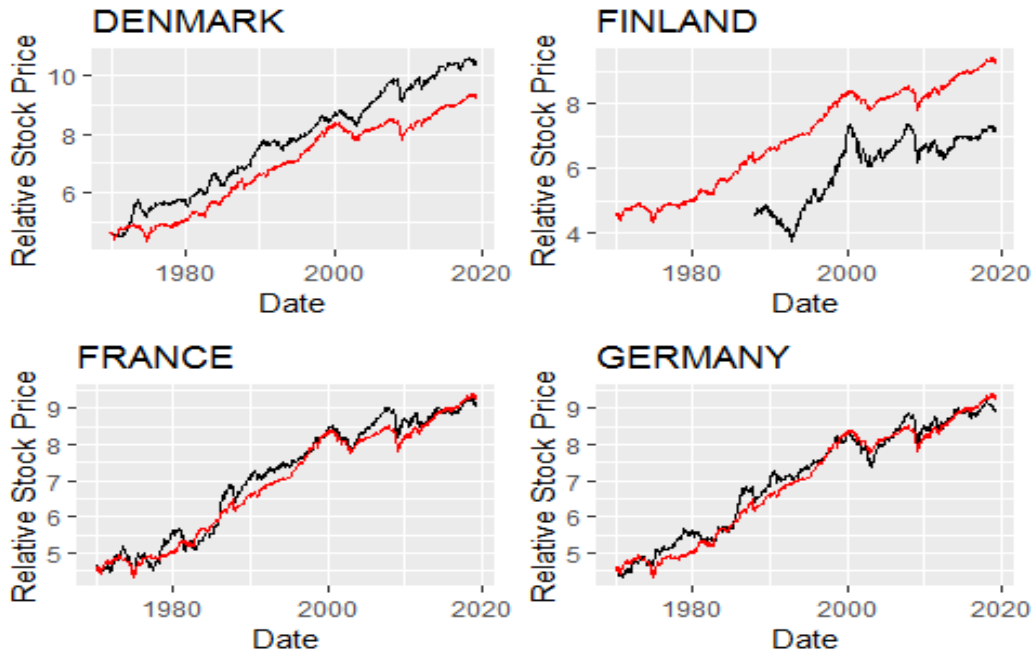
```
get_tval_nonlin_regress_intOnly <- function(rp) {
  if (is.na(rp[1])) return(NA)

  n <- length(rp)
  d.rp <- rp[2:n] - rp[1:n-1]
  rpcubed <- (rp^3)
  mod <- lm(d.rp[2:n] ~ rpcubed[1:n-1] + d.rp[1:n-1])

  sum <- summary(mod)$coefficients
  t_val <- sum[8]
  return(t_val)
}
```

Two tables of the test results were generated in R: one for stationarity with the USA stock index, and the other with the UK's. These tables show the level at which the null hypothesis is rejected, if at all.

Analysis of the test results

An illustration of the stock prices for a sample of countries compared to the US market.





Stationarity with the USA Market Results, Table 1:

	USA ADF	ADF Result	USA Nonlinear	Nonlinear Result
AUSTRALIA	-2.20	...	-1.06	...
AUSTRIA	-0.80	...	-1.51	...
BELGIUM	-2.36	...	-2.07	...
CANADA	-0.94	...	-1.76	...
DENMARK	-1.99	...	-1.82	...
FINLAND	-1.92	...	-1.69	...
FRANCE	-2.64	< 10%	-2.30	...
GERMANY	-2.11	...	-2.53	...
HONG.KONG	-3.53	< 1%	-3.33	< 5%
IRELAND	0.04	...	0.05	...
ISRAEL	-1.42	...	-1.41	...
ITALY	-1.55	...	-0.65	...
JAPAN	-0.45	...	-0.97	...
NETHERLANDS	-2.11	...	-2.27	...
NEW.ZEALAND	-2.37	...	-2.07	...
NORWAY	-2.48	...	-3.24	< 5%
PORTUGAL	-1.47	...	-0.97	...
SINGAPORE	-1.73	...	-1.64	...
SPAIN	-1.57	...	-1.69	...
SWEDEN	-1.83	...	-1.85	...
SWITZERLAND	-2.43	...	-1.87	...
UNITED.KINGDOM	-1.94	...	-2.83	< 10%

According to the Augmented Dickey Fuller test conducted on the relative stock prices of multiple countries, the USA is only stationary with France (weakly) and Hong Kong (strongly). The nonlinear root test (ESTAR) also shows stationarity between the US and Hong Kong, and also Norway and the United Kingdom. Based on these results, the USA is only reliably stationary with Hong Kong.

Stationarity with the UK Market Results, Table 2:

	UK ADF	ADF Result	UK Nonlinear	Nonlinear Result
AUSTRALIA	-2.29	...	-1.44	...
AUSTRIA	-1.57	...	-2.82	< 10%
BELGIUM	-2.84	< 10%	-4.57	< 1%
CANADA	-1.74	...	-2.00	...
DENMARK	-1.25	...	-1.16	...
FINLAND	-1.49	...	-1.14	...
FRANCE	-3.57	< 1%	-4.11	< 1%
GERMANY	-3.09	< 5%	-5.44	< 1%
HONG.KONG	-3.30	< 5%	-2.91	< 10%
IRELAND	-0.60	...	-0.68	...
ISRAEL	-2.56	...	-2.60	...
ITALY	-2.57	...	-2.06	...
JAPAN	-1.15	...	-1.79	...
NETHERLANDS	-1.71	...	-1.16	...
NEW.ZEALAND	-1.60	...	-1.49	...
NORWAY	-2.63	< 10%	-3.00	< 5%
PORTUGAL	-2.45	...	-2.07	...
SINGAPORE	-1.99	...	-2.14	...
SPAIN	-1.86	...	-2.41	...
SWEDEN	-1.50	...	-1.14	...
SWITZERLAND	-1.70	...	-2.86	< 10%
UNITED.KINGDOM	NA	NA	NA	NA
USA	-1.94	...	-2.83	< 10%

According to the ADF test, the UK is stationary with Belgium and Norway at the 10% level, Germany and Hong Kong at the 5% level, and France at the 1% level. The ESTAR test shows the UK is stationary with Austria, Hong Kong, Switzerland, and the USA at the 10% level. Additionally, it found stationarity with New Zealand at the 5% level, and Belgium, France, and Germany at the 1% level.

Based on these results, the UK is reliable stationary with France, Germany, Hong Kong, Belgium, and possibly Norway.

<u>Sub-sample Analysis: Before and After the 2008-2009 Crises</u>

Observing Changes in Stationarity Results with the USA: Before and After the Financial Crisis

Compared to USA	ADF Result Full Sample	ADF Sub Result Before	ADF Sub Result After	Nonlin Result Full Sample	Nonlin Sub Result Before	Nonlin Sub Result After
AUSTRALIA	...	...	...	...	...	...
AUSTRIA	...	...	...	...	...	...
BELGIUM	...	...	...	...	...	...
CANADA	...	...	...	...	...	...
DENMARK	...	...	...	...	...	...
FINLAND	...	...	...	...	...	...
FRANCE	< 10%	...	...	...	...	...
GERMANY	...	...	...	...	...	...
HONG.KONG	< 1%	< 5%	...	< 5%	...	< 5%
IRELAND	...	...	...	...	...	...
ISRAEL	...	...	...	...	...	...
ITALY	...	< 10%	...	...	...	...
JAPAN	...	...	...	...	...	...
NETHERLANDS	...	...	...	...	...	...
NEW.ZEALAND	...	...	...	...	...	...
NORWAY	...	...	...	< 5%	...	...
PORTUGAL	...	< 1%	...	...	...	< 5%
SINGAPORE	...	...	...	...	...	...
SPAIN	...	...	...	...	...	...
SWEDEN	...	...	...	...	...	...
SWITZERLAND	...	...	...	...	...	...
UNITED.KINGDOM	...	...	...	< 10%	...	< 5%
USA	NA	NA	NA	NA	NA	NA

The ADF test results show that stationarity with some countries was higher before the financial crisis. After the financial crisis, stationarity with the USA dropped. Conversely, the nonlinear test results show that stationarity existed after the financial crisis, but was not present before the financial crisis.

Observing Changes in Stationarity Results with the UK: Before and After the Eurozone Crisis

Compared to the UK	ADF Result Full Sample	ADF Sub Result Before	ADF Sub Result After	Nonlin Result Full Sample	Nonlin Sub Result Before	Nonlin Sub Result After
AUSTRALIA	...	...	...	...	...	...
AUSTRIA	...	...	...	< 10%	...	< 10%
BELGIUM	< 10%	< 10%	...	< 1%	...	< 1%
CANADA	...	...	...	...	...	...
DENMARK	...	...	...	...	...	< 5%
FINLAND	...	...	...	...	...	...

FRANCE	< 1%	< 5%	...	< 1%	...	< 1%
GERMANY	< 5%	< 10%	...	< 1%	...	< 1%
HONG.KONG	< 5%	< 1%	...	< 10%	...	< 1%
IRELAND	...	...	...	...	...	...
ISRAEL	...	...	...	...	...	...
ITALY	...	< 10%	< 5%	...	< 5%	< 10%
JAPAN	...	...	...	...	...	...
NETHERLANDS	...	...	...	...	...	...
NEW.ZEALAND	...	...	...	...	...	...
NORWAY	< 10%	...	...	< 5%	...	< 10%
PORTUGAL	...	< 1%	...	...	...	< 1%
SINGAPORE	...	...	...	...	...	...
SPAIN	...	...	< 5%	...	< 5%	...
SWEDEN	...	...	< 10%	...	< 10%	...
SWITZERLAND	...	...	...	< 10%	...	< 1%
UK	NA	NA	NA	NA	NA	NA
USA	...	< 10%	...	< 10%	...	< 5%

This table compares the test results between three sets: the full sample, sub sample before, and sub sample after. For the ADF test, the stationarity results of the full historical time period somewhat matchup with the results from the era before the Eurozone crisis. Stationarity with the UK drops after the Eurozone crisis, although stationarity with Spain and Sweden seem to increase.

The nonlinear test shows the opposite direction: stationarity results after the crisis more closely matches the long-term stationarity results than before the crisis. Curiously, the results of the ADF test and nonlinear test mirror each other: the nonlinear results of the "before" era match the ADF results of the "after" era, and so on.

Conclusion

The tests conducted have shown that some OECD countries do exhibit long-term stationarity with the USA or UK. Specifically, it seems that Hong Kong is reliably stationary with the USA and UK, and France, Germany, Belgium, and Norway are also stationary with the UK. This implies that these markets are connected, and that a rise or fall in one of these markets will be reflected in the others. Additionally, the financial crises of 2008-2009 had a significant effect on the stationarity results of both the USA and UK. This implies that financial crashes may not always affect foreign markets in predictable ways.

These methods of determining stationarity could be applied to a wide variety of data, including studying exchange rates, unemployment rates, and other metrics across countries, in order to determine the connectedness of these markets.

References

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