- Least Squares Estimation

$$RSS = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad (1)$$

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = -2 \sum_{i=1}^{n} x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0 \quad (2)$$

$$(1) \quad \sum_{i=1}^{n} y_i - n\hat{\beta}_0 - \hat{\beta}_1 \sum_{i=1}^{n} x_i = 0$$

$$\Rightarrow n\hat{\beta}_0 = \sum_{i=1}^{n} y_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i \quad \Rightarrow \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

(2) Solve using substitution

$$\sum_{i=1}^{n} x_i y_i - \hat{\beta}_0 \sum_{i=1}^{n} x_i - \hat{\beta}_1 \sum_{i=1}^{n} x_i^2 = 0$$

$$\sum_{i=1}^{n} x_i y_i = \hat{\beta}_0 \sum_{i=1}^{n} x_i + \hat{\beta}_1 \sum_{i=1}^{n} x_i^2$$

$$= (\bar{y} - \hat{\beta}_1 \bar{x})(n\bar{x}) + \hat{\beta}_1 \sum_{i=1}^{n} x_i^2$$

$$= n\bar{x}\bar{y} - \hat{\beta}_1 n\bar{x}^2 + \hat{\beta}_1 \sum_{i=1}^{n} x_i^2$$

$$\cancel{= n\bar{x}\bar{y} - \hat{\beta}_1 (n\bar{x}^2)}$$

$$= n\bar{x}\bar{y} + \hat{\beta}_1 \left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2 \right)$$

$$\therefore \hat{\beta}_1 = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$