Lecture 11:
Inference for Two Means
STAT 310, Fall 2020

Difference Between Two Means

- In this lecture we discuss how to construct confidence intervals and perform hypothesis tests for the difference between two populations means $\mu_1 - \mu_2$, where the data come from two independent samples.
- Just as with a single sample, we need to check whether certain conditions are satisfied for the confidence interval or hypothesis test to be valid.
- An important question we address is whether the difference between the two population means is significantly different than 0.

Confidence Interval

Confidence interval for the difference between two population means $\mu_1 - \mu_2$:

SE $\Rightarrow$ Standard error

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

point estimate $\pm\, t^*$ SE

- The degrees of freedom for the critical value t^* can be calculated with the formula $df = \min(n_1 - 1, n_2 - 1)$
- The formula for the degrees of freedom computed using software (`t.test()` function in R) is more complex.[1]

[1] https://en.wikipedia.org/wiki/Welch%27s_t-test

Hypothesis Test

$H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 \neq 0$

Hypothesis test for the difference between two population means:

$H_0 : \mu_1 = \mu_2$ (the two means are the same)

$H_A : \mu_1 \neq \mu_2$ (the two means are different)

Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad ; \quad df = \min(n_1 - 1, n_2 - 1)$$

- The degrees of freedom are the same as the confidence interval.
- Can also do a one-sided test (e.g., $H_A : \mu_1 > \mu_2$), but we will focus on two-sided tests when comparing two means.

Conditions

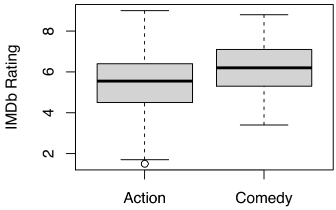
Conditions for a confidence interval or hypothesis test for the difference between two population means:

- The data in each group comes from a random sample, or randomized experiment. Additionally, the two groups are independent of each other (the cases in the first group are not related to the cases in the second group).
- The sample sizes are large ($n_1 \geq 30$ and $n_2 \geq 30$). Otherwise, if the samples sizes are small, the data in each group should be approximately normal.
- There should be no extreme outliers.

Example

Are action or comedy movies rated higher on IMDb? Below are some summary statistics for a random sample of 50 action movies and 50 comedy movies rated on IMDb. Use a hypothesis test to determine whether there is a statistically significant difference between the two means.

	IMDb Rating	
	Action	Comedy
Mean	5.46	6.18
SD	1.55	1.24
n	50	50



μ_A: Pop mean for action movies
μ_C: " " for comedy movies

(a) Write the null and alternative hypotheses.

$H_0: \mu_A = \mu_C \qquad H_A: \mu_A \neq \mu_C$

(b) Check the conditions for the test.

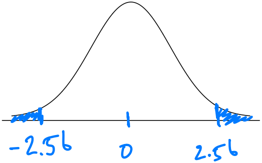
- Data come from two independent random samples ✓
- Large samples sizes ($n_A \geq 30$ and $n_C \geq 30$), and no extreme outliers ✓

(c) Calculate the test statistic.

$$t = \frac{\bar{x}_A - \bar{x}_C}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_C^2}{n_C}}} = \frac{5.46 - 6.18}{\sqrt{\frac{1.55^2}{50} + \frac{1.24^2}{50}}} = -2.56$$

$t = -2.56 \qquad df = \min(50-1, 50-1) = 49$

(d) Calculate the *p*-value, and and make a decision using $\alpha = 0.05$ significance level.



$$\text{p-value} = 2 * pt(-2.56, df = 49)$$

$$= 0.0136$$

Since p-value < 0.05, we reject H_0

(e) What is the conclusion of the test in the context of the data?

There is a statistically significant difference between the mean rating of action and comedy movies on IMDb. Sample means suggest comedy movies are rated higher.

Example

$df = \min(50-1, 50-1) = 49$

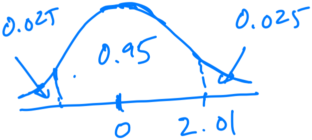
Calculate and interpret a 95% confidence interval for the difference between the mean rating of action and comedy movies on IMDb.

$$\bar{X}_A - \bar{X}_C \pm t^* \sqrt{\frac{S_A^2}{n_A} + \frac{S_C^2}{n_C}}$$

$$5.46 - 6.18 \pm 2.01 \sqrt{\frac{1.55^2}{50} + \frac{1.24^2}{50}}$$

$$-0.72 \pm 2.01(0.2807)$$

$$\boxed{(-1.28, -0.16)}$$



$t^* = qt(0.975, df = 49)$

$= 2.01$

Example

Calculate and interpret a 95% confidence interval for the difference between the mean rating of action and comedy movies on IMDb.

95% CI for $\mu_A - \mu_c$ is $(-1.28, -0.16)$

- We are 95% confident that $\mu_A - \mu_c$ is between -1.28 and -0.16
- We are 95% confident that the average rating for action movies is between 0.16 and 1.28 less than the average rating for comedy movies.