Lecture 10:
Hypothesis Test for One Mean
STAT 310, Spring 2021

Hypothesis Test for One Mean

Key components:

- Null hypothesis:
  $H_0 : \mu = \mu_0$

- Alternative hypothesis (use one of these):
  $H_A : \mu > \mu_0$ (one-sided, upper-tail)
  $H_A : \mu < \mu_0$ (one-sided, lower-tail)
  $H_A : \mu \neq \mu_0$ (two-sided)

- Test statistic:

$$t = \frac{\text{observed value} - \text{null value}}{\text{SE}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- A rule to either reject or not reject H_0 (based on p-value)

p-value (review)

Decision rule using the p-value:

- If p-value $< \alpha$, then reject H_0.
- If p-value $> \alpha$, then do not reject H_0.

α is called the **signficance level**. Common values for $\alpha = 0.05, 0.01$

p-value (review)

- When the p-value $< \alpha$ (we reject H_0) the result is said to be **statistically significant**.
- In other words, a result is statistically significant when it is unlikely to of occurred by random chance, assuming that the null hypothesis is true.
- The smaller the p-value, the stronger the data favor H_A over H_0.

Computing p-values

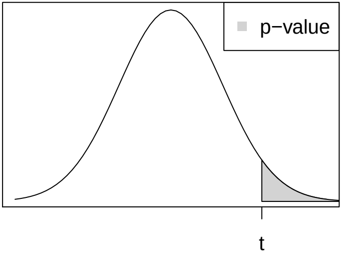
One-sided test (upper-tail):
$H_0 : \mu = \mu_0$
$H_A : \mu > \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

p-value $=$ `1 - pt(t, df = n-1)`

Reject H_0 if p-value $< \alpha$



Computing p-values

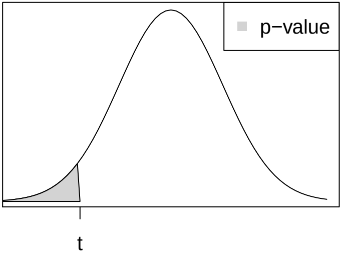
One-sided test (lower-tail):
$H_0 : \mu = \mu_0$
$H_A : \mu < \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

p-value $=$ `pt(t, df = n-1)`

Reject H_0 if p-value $< \alpha$



Computing p-values

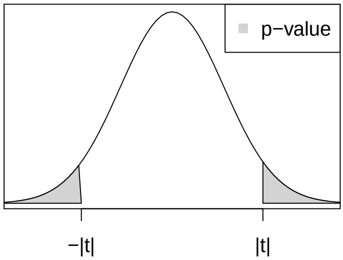
Two-sided test:
$H_0 : \mu = \mu_0$
$H_A : \mu \neq \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

p-value $=$ `2*pt(-abs(t), df = n-1)`

Reject H_0 if p-value $< \alpha$



Conditions

The hypothesis test is valid if the following conditions are satisfied:

- The data come from a random sample. (This is called the **independence condition** in the textbook.)
- The sample size n is large ($n \geq 30$). Otherwise, if the sample size is small ($n < 30$), the data should have an approximate normal distribution. (This is called the **normality condition** in the textbook.)
- Additionally, the data should not contain any extreme outliers.

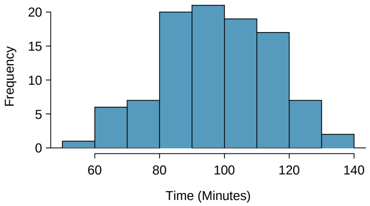
Graphical methods (box plots, histograms) can be used to check if the data have an approximate normal distribution when the sample size n is small.

Note that these are exactly the same conditions we check for a confidence interval.

Example 1 (from *Open Intro*, Chapter 7.1)

- Is the typical US runner getting faster or slower over time? We consider this question in the context of the Cherry Blossom Race, which is a 10-mile race in Washington, DC each spring.
- The average time for all runners who finished the Cherry Blossom Race in 2006 was 93.29 minutes. Using data from a random sample of 100 participants in the 2017 Cherry Blossom Race, we want to determine whether runners in this race are getting faster or slower, versus the other possibility that there has been no significant change.
- Summary statistics and a histogram of the race times (in minutes) for the 2017 Cherry Blossom Race are shown below.

n	$\bar{x}$	s
100	97.32	16.98



Example 1

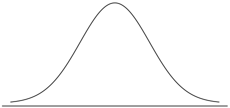
(a) Write the null and alternative hypothesis for a two-sided test.

(b) Check the conditions for the hypothesis test.

(c) Calculate the test statistic.

Example 1

(d) Calculate the *p*-value and make a decision using $\alpha = 0.05$ significance level.

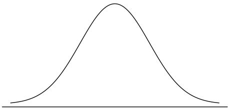


(e) What is the conclusion of the test in the context of the data?

Example 2

Find the p-value for the given t-test statistic and sample size. Also determine if the null hypothesis would be rejected at $\alpha = 0.05$. Assume all the conditions for the hypothesis test are satisfied.

(a) $H_A : \mu > \mu_0,\ n = 9,\ t = 1.7$



(b) $H_A : \mu \neq \mu_0,\ n = 40,\ t = -3.1$