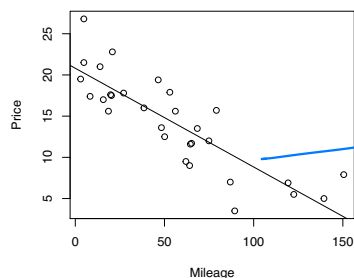


Exam 3 Practice Problems
STAT 310, Spring 2021

Exercise 1. The following scatterplot shows the association between price (in \$1,000's) and mileage (number of miles driven in 1,000's) for a sample of 30 used Honda Accords in 2017. Also provided below is the output from fitting a simple linear regression model in R.



$$\hat{y} = 20.8 - 0.12x$$

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	20.8096	0.9529	21.84	< 2e-16 ***
Mileage	-0.1198	0.0141	-8.50	3.06e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

- (a) Do the data provide strong evidence of a linear association between the price of the used cars and the number of miles driven, i.e., is the slope of the regression line significantly different than 0? State the null and alternative hypothesis, report the test statistic and p -value, and state your conclusion.

$$H_0: \beta_1 = 0 \quad t = -8.5, \quad p\text{-value} = 3.06e-09 = 3.06 \times 10^{-9}$$

$H_A: \beta_1 \neq 0$ Since $p\text{-value} < 0.05$, we reject H_0
The data provide strong evidence of a linear association between price and mileage.

- (b) Calculate a 95% confidence interval for the slope parameter β_1 .

$$t^* = qt(0.975, df=28) = 2.048$$

$$\hat{\beta}_1 \pm t^* SE$$

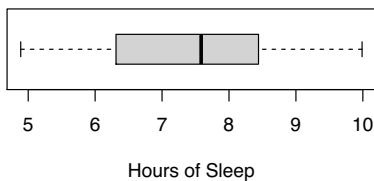
$$-0.1198 \pm 2.048 (0.0141)$$

$$(-0.1487, -0.0909)$$

- (c) Do the results from the hypothesis test and confidence interval agree? Explain.

Yes, since 0 is not contained in the interval.

Exercise 2. A social worker at a local high school is interested in testing whether the average amount of sleep students get is significantly different than 8 hours, which is considered healthy. A random sample of $n = 25$ students are interviewed. The sample mean $\bar{x} = 7.6$ and standard deviation $s = 1.4$. A boxplot of the data is also shown below.



(a) Which of the following is the correct null and alternative hypothesis for a two-sided test?

- (i) $H_0 : \bar{x} = 8, H_A : \bar{x} \neq 8$
- (ii) $H_0 : \mu = 8, H_A : \mu \neq 8$ \rightarrow hypotheses are a statement about population mean μ
- (iii) $H_0 : \bar{x} = 8.2, H_A : \bar{x} \neq 8.2$

(b) Check the conditions for the hypothesis test.

- Data come from a random sample ✓
 - Data has approximate normal distribution ✓
(need to check since $n = 25 < 30$)
- Yes, the conditions are satisfied.

(c) Calculate the test statistic.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{7.6 - 8}{1.4/\sqrt{25}} = -1.43$$

(d) Calculate the p -value and make a decision using $\alpha = 0.05$.



$$p\text{-value} = 2 * p(t(-1.43, df = 24)) = 0.1656$$

Since $p\text{-value} > 0.05$, we do not reject H_0

(e) What is the conclusion of the test in the context of the data?

The average amount of sleep students get is not significantly different than 8 hours.