

Lecture 8: Hypothesis Testing for a Proportion  
Practice Problems  
STAT 310, Spring 2021

**Exercise 1.** It is believed that nearsightedness affects about 8% of all children. In a random sample of 194 children, 21 are nearsighted. Conduct a hypothesis test for the following question: do these data provide evidence that the 8% value is inaccurate?

- (a) Write the null and alternative hypothesis for a two-sided test.

$$H_0: p = 0.08$$

$$H_A: p \neq 0.08$$

$$\hat{p} = \frac{21}{194} = 0.108$$

$$n = 194$$

$$p_0 = 0.08$$

- (b) Check the conditions for the hypothesis test.

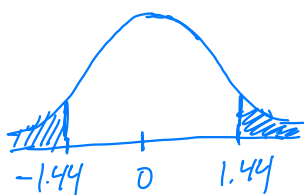
— Data come from random sample ✓

$$— np_0 = 194(0.08) = 15.5 \geq 10 \text{ and } n(1-p_0) = 194(0.92) = 178.5 \geq 10$$

- (c) Calculate the test statistic.

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.108 - 0.08}{\sqrt{\frac{0.08(0.92)}{194}}} = 1.44$$

- (d) Calculate the  $p$ -value and make a decision using  $\alpha = 0.05$  significance level.



$$p\text{-value} = 2 \cdot P(Z < -1.44)$$

$$= 2 \times \text{pnorm}(-1.44) = \boxed{0.15}$$

Since  $p\text{-value} > 0.05$  we do not reject  $H_0$

- (e) What is the conclusion of the test in the context of the data?

The data do not provide convincing evidence that the percentage of children that are nearsighted is different than 8%.

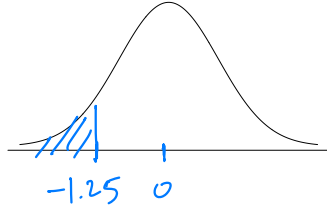
- (f) What type of testing error might we have made?

	Do not reject $H_0$	reject $H_0$
$H_0$ true	✓	type 1 error
$H_A$ true	type 2 error	✓

type 2 error

**Exercise 2.** Find the  $p$ -value for the given  $z$ -test statistic for a hypothesis test for a proportion. Also determine if the null hypothesis would be rejected at  $\alpha = 0.05$ . Assume all the conditions for the hypothesis test are satisfied.

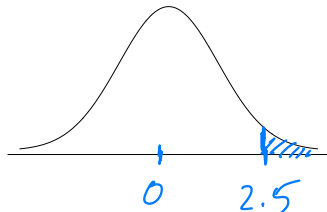
(a)  $H_A : p < p_0; z = -1.25$



$$\begin{aligned} p\text{-value} &= \text{pnorm}(-1.25) \\ &= 0.106 \end{aligned}$$

Since  $p\text{-value} > 0.05$  we do not reject  $H_0$

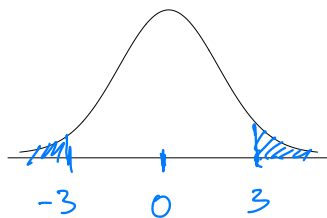
(b)  $H_A : p > p_0; z = 2.5$



$$\begin{aligned} p\text{-value} &= 1 - \text{pnorm}(2.5) \\ &= 0.0062 \end{aligned}$$

Since  $p\text{-value} < 0.05$  we reject  $H_0$

(c)  $H_A : p \neq p_0; z = -3$



$$\begin{aligned} p\text{-value} &= 2 \times \text{pnorm}(-3) \\ &= 0.0027 \end{aligned}$$

Since  $p\text{-value} < 0.05$  we reject  $H_0$