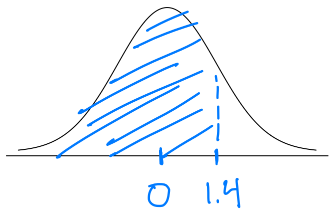
Lecture 5: Normal Distribution
Practice Problems
STAT 310, Spring 2021

Exercise 1. Suppose $Z \sim N(\mu = 0, \sigma = 1)$ is a random variable following a standard normal distribution. Use the R function `pnorm()` to compute the following probabilities:

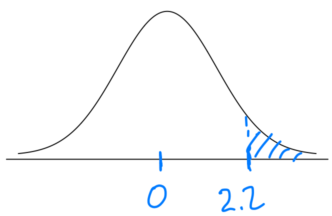
(a) $P(Z < 1.4)$ $= \texttt{pnorm}(1.4) = \boxed{0.919}$



(b) $P(Z > 2.2)$ $= 1 - P(Z < 2.2)$

$= 1 - \texttt{pnorm}(2.2)$

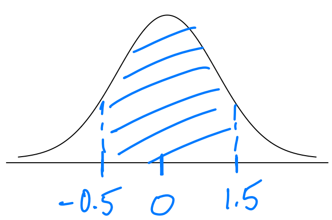
$= \boxed{0.0139}$



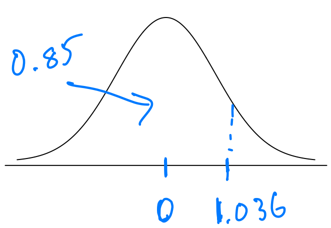
(c) $P(-0.5 < Z < 1.5)$ $= P(Z < 1.5) - P(Z < -0.5)$

$= \texttt{pnorm}(1.5) - \texttt{pnorm}(-0.5)$

$= \boxed{0.625}$



Exercise 2. Use the R function `qnorm()` to find 85^{th} percentile of the standard normal distribution $N(\mu = 0, \sigma = 1)$.

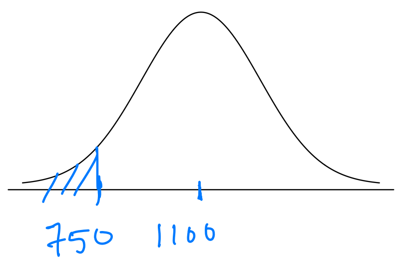


$z = \texttt{qnorm}(0.85) = \boxed{1.036}$

$$Z = \frac{X - \mu}{\sigma}$$

Exercise 3. The SAT score X closely follows a normal distribution with mean $\mu = 1100$ and standard deviation $\sigma = 200$. That is, $X \sim N(\mu = 1100, \sigma = 200)$

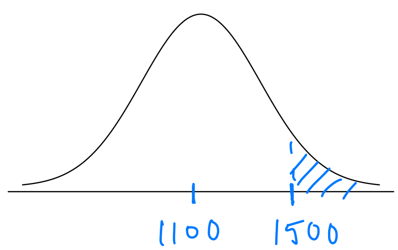
(a) About what percent of test takers score below a 750?



$$Z = \frac{750 - 1100}{200} = -1.75$$

$$P(X < 750) = P(Z < -1.75)$$
$$= \text{pnorm}(-1.75)$$
$$= \boxed{0.04}$$

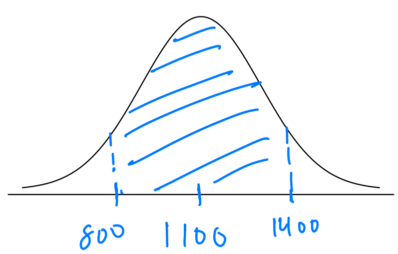
(b) About what percent of test takers score above a 1500?



$$Z = \frac{1500 - 1100}{200} = 2$$

$$P(X > 1500) = 1 - P(X < 1500)$$
$$= 1 - P(Z < 2)$$
$$= 1 - \text{pnorm}(2) = \boxed{0.023}$$

(c) About what percent of test takes score between 800 and 1400?

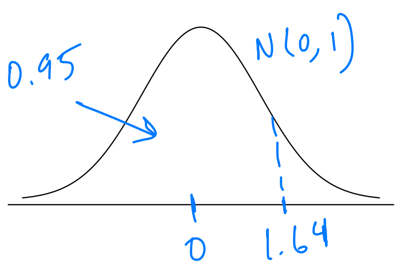


$$Z = \frac{800 - 1100}{200} = -1.5$$

$$Z = \frac{1400 - 1100}{200} = 1.5$$

$$P(800 < X < 1400) = P(X < 1400) - P(X < 800)$$
$$= P(Z < 1.5) - P(Z < -1.5)$$
$$= \text{pnorm}(1.5) - \text{pnorm}(-1.5)$$
$$= \boxed{0.866}$$

(d) What is the 95^{th} percentile for SAT scores?



$$Z = \text{qnorm}(0.95) = 1.64$$

Solve for x

$$Z = \frac{X - \mu}{\sigma}$$

$$1.64 = \frac{X - 1100}{200}$$

$$1.64(200) = X - 1100$$

$$X = 1100 + 1.64(200) = \boxed{1428}$$