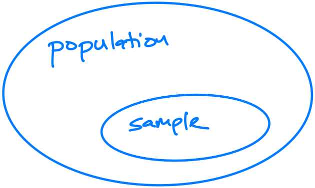
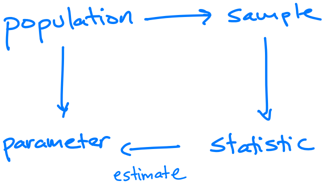
Lecture 6:
Sampling Distributions
STAT 310, Spring 2023

- A **parameter** is a numerical characteristic of the population (fixed number that is usually unknown).
- A **statistic** is a numerical characteristic of the sample (varies depending on sample).
- The statistic is also referred to as a **point estimate**, since it is our best guess at the value of a population parameter.





Notation for the proportion:

- p: population proportion
- $\hat{p}$: sample proportion

Notation for the mean:

- μ: population mean
- $\bar{x}$: sample mean

The sample size is denoted as n

Example

A recent Gallup poll found that 63% of Americans are dissatisfied with US gun laws. The survey results were based on a random sample of 1,011 adults.[1]

(a) What is the sample proportion?

$\hat{p} = 0.63$

(b) What is the sample size?

$n = 1{,}011$

(c) Describe the population proportion?

p: proportion of all American adults who are dissatisfied with gun laws.

(d) Suppose another poll is conducted with a different random sample of adults. Would you expect the sample proportion to be the same, or slightly different?

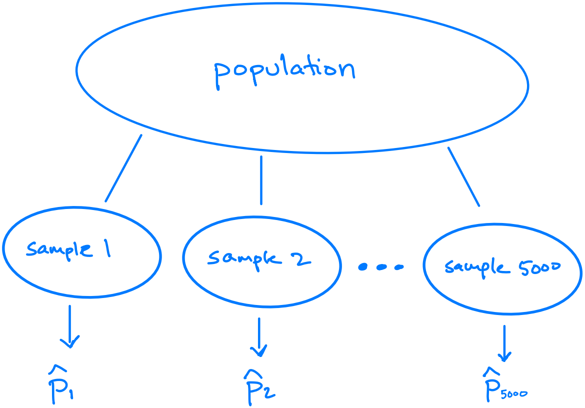
slightly different

[1] https://news.gallup.com/poll/470588/dissatisfaction-gun-laws-hits-new-high.aspx

- **Sampling Error** refers to how much a statistic, such as the sample proportion, will vary from one random sample to the next.

- For example, the Gallup poll reported a sampling error of ± 4 percentage points. This means that the population proportion of Americans that are dissatisfied with US gun laws is likely between 59% and 67%. $63\% \pm 4\%$

A **sampling distribution** is the distribution of a statistic when repeatedly taking random samples from a population.

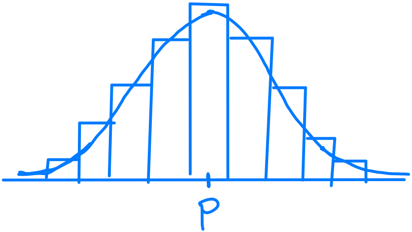


$p \Rightarrow$ population proportion

$n \Rightarrow$ sample size (same for each sample)

$5000 \Rightarrow$ number of samples

We can visualize the sampling distribution by making a histogram of the 5000 sample proportions.



- When n is large, histogram looks normal and centered around population proportion p.
- The standard deviation of the 5000 sample proportions is called the <u>standard error</u> (SE). It measures the variability, or spread, of the estimates from sample to sample.

- In real-world applications, we never actually observe the sampling distribution, since we usually take a single random sample.
- However, it is useful to always think of a statistic, such as the sample proportion, as coming from such a hypothetical distribution.
- The concept of a sampling distribution is very important when trying to quantify sampling error.

Central Limit Theorem (CLT)

The sampling distribution for $\hat{p}$ follows an approximate normal distribution centered around the population proportion p, and with standard error $\sqrt{p(1-p)/n}$.

$$\hat{p} \sim N\Big(\underbrace{p}_{\text{mean}}, \underbrace{\sqrt{\frac{p(1-p)}{n}}}_{\text{standard error (SE)}}\Big)$$

The CLT can be verified using computer simulations.

Conditions for CLT

The following conditions should be met to apply the CLT:

- The data come from a simple random sample. This is called the **independence condition** since it implies that the individuals or cases in the data are unrelated.
- $np \geq 10$ and $n(1-p) \geq 10$. This is sometimes called the **success-failure condition** since np can be interpreted as the expected number of successes and $n(1-p)$ the expected number of failures.

Example

Suppose that the population proportion of Americans who support the expansion of solar energy is $p = 0.88$, and $n = 1000$ Americans are randomly sampled.

(a) What is the mean, or center, of the sampling distribution for $\hat{p}$?

$$\text{mean} = p = 0.88$$

(b) What is the standard error of the sampling distribution for $\hat{p}$?

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.88(0.12)}{1000}} = 0.01$$

(c) What distribution does $\hat{p}$ follow?

$$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N(0.88, 0.01)$$

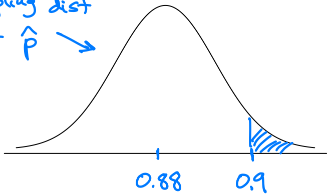
(d) Are the conditions for the CLT satisfied? yes

- random sample ✓
- $np = 1000(0.88) = 880 \geq 10$ ✓ $\quad n(1-p) = 1000(0.12) = 120 \geq 10$ ✓

Example

Suppose that population proportion of Americans who support the expansion of solar energy is $p = 0.88$, and $n = 1000$ Americans are randomly sampled. What is the probability that the sample proportion $\hat{p}$ will be greater than 0.9?

sampling dist for $\hat{p}$



By CLT, $\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$

$= N(0.88, 0.01)$

$\Downarrow$ mean $\quad \Downarrow$ SE

$$Z = \frac{\hat{p} - p}{SE} = \frac{0.9 - 0.88}{0.1} = 2$$

$$P(Z > 2) = 1 - P(Z < 2)$$

$$= 1 - \text{pnorm}(2)$$

$$= \boxed{0.023}$$