Lecture 10:
Hypothesis Testing for One Mean
STAT 310, Spring 2023

# Hypothesis Test for One Mean

**Key components:**

- Null hypothesis:
  $H_0 : \mu = \mu_0$
- Alternative hypothesis (use one of these):
  $H_A : \mu > \mu_0$ (one-sided, upper-tail)
  $H_A : \mu < \mu_0$ (one-sided, lower-tail)
  $H_A : \mu \neq \mu_0$ (two-sided)
- Test statistic:

$$t = \frac{\text{observed value} - \text{null value}}{\text{SE}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- A rule to either reject or not reject $H_0$ (based on $p$-value)

# $p$-value (review)

Decision rule using the $p$-value:

- If $p$-value $< \alpha$, then reject $H_0$.
- If $p$-value $> \alpha$, then do not reject $H_0$.

$\alpha$ is called the **signficance level**. The most common value for $\alpha = 0.05$, but other values are also sometimes used such as $\alpha = 0.01$.

# $p$-value (review)

- When the $p$-value $< \alpha$ (we reject $H_0$) the result is said to be **statistically significant**.
- In other words, a result is statistically significant when it is unlikely to of occurred by random chance (i.e., natural sampling variability), assuming that the null hypothesis is true.
- The smaller the $p$-value, the stronger the data favor $H_A$ over $H_0$.

# Computing $p$-values

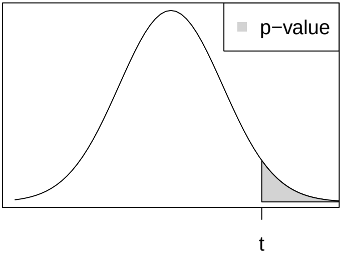
One-sided test (upper-tail):
$H_0 : \mu = \mu_0$
$H_A : \mu > \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$p$-value $=$ `1 - pt(t, df = n-1)`

Reject $H_0$ if $p$-value $< \alpha$



# Computing $p$-values

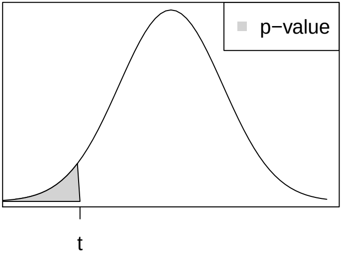
One-sided test (lower-tail):
$H_0 : \mu = \mu_0$
$H_A : \mu < \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$p$-value $=$ `pt(t, df = n-1)`

Reject $H_0$ if $p$-value $< \alpha$



# Computing $p$-values

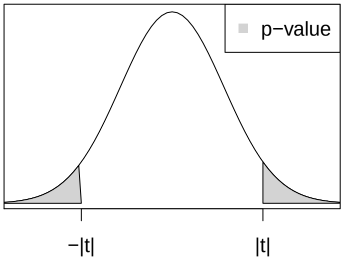
Two-sided test:
$H_0 : \mu = \mu_0$
$H_A : \mu \neq \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$p$-value $=$ `2*pt(-abs(t), df = n-1)`

Reject $H_0$ if $p$-value $< \alpha$



# Conditions

A hypothesis test for a mean is valid if the following conditions are satisfied:

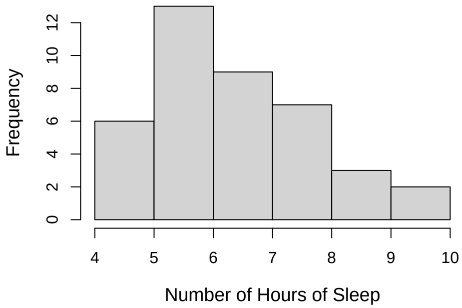
- The data come from a random sample. (This is called the **independence condition** in the textbook.)
- The sample size $n$ is large ($n \geq 30$). Otherwise, if the sample size is small ($n < 30$), the data should have an approximate normal distribution. (This is called the **normality condition** in the textbook.)

Note that these are the same conditions we check for a confidence interval. For small sample sizes ($n < 30$), look at a histogram of the data to check normality.

# Example

As part of a survey conducted by the US National Center for Health Statistics, a random sample of $n = 40$ Americans were asked how many hours of sleep they get on a typical weekday. Some summary statistics and a histogram are shown below. Do these data provide evidence that the average amount of sleep Americans get on a typical weekday is significantly less than 8 hours?

| n | $\bar{x}$ | s | min | max |
|---|---|---|---|---|
| 40 | 6.8 | 1.4 | 4 | 10 |



(a) Write the null and alternative hypothesis for a one-sided test.
$H_0 : \mu = 8$
$H_A : \mu < 8$

(b) Check the conditions for the hypothesis test.

- Data come from a random sample.
- Large sample size: $n = 40 \geq 30$

The conditions are satisfied.

(c) Calculate the test statistic.

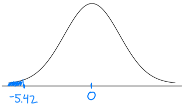
$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{6.8 - 8}{1.4/\sqrt{40}} = \frac{-1.2}{1.4/\sqrt{40}} = -5.42$$

R command:

```
> -1.2 / (1.4 / sqrt(40))
[1] -5.421047
```

(d) Calculate the $p$-value and make a decision using $\alpha = 0.05$ significance level.

$p$-value $=$ `pt(-5.42, df = 39)` $= 0.0000017$
Since $p$-value $< 0.05$, we reject $H_0$.[1]




(e) What is the conclusion of the test in the context of the data?

The data provide evidence that on average Americans get significantly less than 8 hours of sleep on a typical weekday.

[1] Scientific notation: `1.7e-06` $= 1.7 \cdot 10^{-6} = 0.0000017$