Lecture 10:
Hypothesis Testing for One Mean
STAT 310, Spring 2023

Hypothesis Test for One Mean

Key components:

- Null hypothesis:
  $H_0 : \mu = \mu_0$
- Alternative hypothesis (use one of these):
  $H_A : \mu > \mu_0$ (one-sided, upper-tail)
  $H_A : \mu < \mu_0$ (one-sided, lower-tail)
  $H_A : \mu \neq \mu_0$ (two-sided)
- Test statistic:

$$t = \frac{\text{observed value} - \text{null value}}{\text{SE}} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- A rule to either reject or not reject H_0 (based on p-value)

p-value (review)

Decision rule using the p-value:

- If p-value $< \alpha$, then reject H_0.
- If p-value $> \alpha$, then do not reject H_0.

α is called the **signficance level**. The most common value for $\alpha = 0.05$, but other values are also sometimes used such as $\alpha = 0.01$.

p-value (review)

- When the p-value $< \alpha$ (we reject H_0) the result is said to be **statistically significant**.
- In other words, a result is statistically significant when it is unlikely to of occurred by random chance (i.e., natural sampling variability), assuming that the null hypothesis is true.
- The smaller the p-value, the stronger the data favor H_A over H_0.

Computing p-values

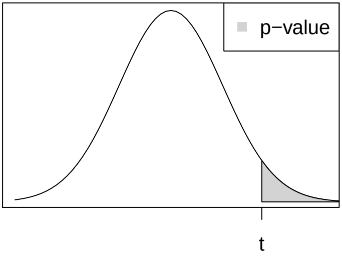
One-sided test (upper-tail):
$H_0 : \mu = \mu_0$
$H_A : \mu > \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

p-value $=$ `1 - pt(t, df = n-1)`

Reject H_0 if p-value $< \alpha$



Computing p-values

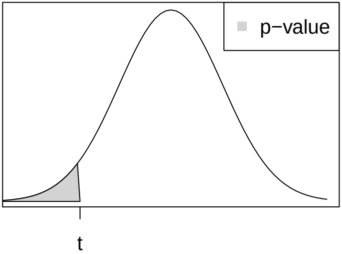
One-sided test (lower-tail):
$H_0 : \mu = \mu_0$
$H_A : \mu < \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

p-value $=$ `pt(t, df = n-1)`

Reject H_0 if p-value $< \alpha$

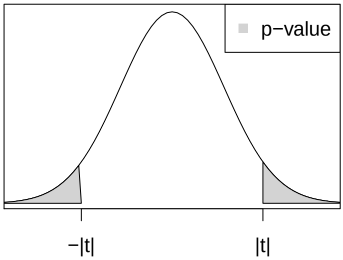


Computing p-values

Two-sided test:
$H_0 : \mu = \mu_0$
$H_A : \mu \neq \mu_0$

Test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$



p-value $=$ `2*pt(-abs(t), df = n-1)`

Reject H_0 if p-value $< \alpha$

Conditions

A hypothesis test for a mean is valid if the following conditions are satisfied:

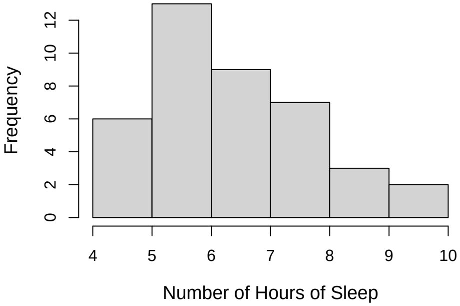
- The data come from a random sample. (This is called the **independence condition** in the textbook.)
- The sample size n is large ($n \geq 30$). Otherwise, if the sample size is small ($n < 30$), the data should have an approximate normal distribution. (This is called the **normality condition** in the textbook.)

Note that these are the same conditions we check for a confidence interval. For small sample sizes ($n < 30$), look at a histogram of the data to check normality.

Example

As part of a survey conducted by the US National Center for Health Statistics, a random sample of $n = 40$ Americans were asked how many hours of sleep they get on a typical weekday. Some summary statistics and a histogram are shown below. Do these data provide evidence that the average amount of sleep Americans get on a typical weekday is significantly less than 8 hours?

n	$\bar{x}$	s	min	max
40	6.8	1.4	4	10

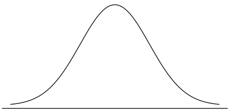


(a) Write the null and alternative hypothesis for a one-sided test.

(b) Check the conditions for the hypothesis test.

(c) Calculate the test statistic.

(d) Calculate the *p*-value and make a decision using $\alpha = 0.05$ significance level.



(e) What is the conclusion of the test in the context of the data?