

Lecture 11:
Inference for Two Means
STAT 310, Spring 2023

Difference Between Two Means


- ▶ In this lecture we discuss how to construct confidence intervals and perform hypothesis tests for the difference between two populations means $\mu_1 - \mu_2$, where the data come from two independent samples.
- ▶ Just as with a single sample, we need to check whether certain conditions are satisfied for the confidence interval or hypothesis test to be valid.
- ▶ An important question we address is whether the difference between the two population means is significantly different than 0.

Confidence Interval

Confidence interval for the difference between two population means $\mu_1 - \mu_2$:

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- ▶ The degrees of freedom for the critical value t^* can be calculated with the formula $df = \min(n_1 - 1, n_2 - 1)$
- ▶ The formula for the degrees of freedom computed using software (`t.test()` function in R) is more complex.¹

¹https://en.wikipedia.org/wiki/Welch%27s_t-test 

Hypothesis Test

Hypothesis test for the difference between two population means:

$$H_0 : \mu_1 = \mu_2$$

$$H_A : \mu_1 \neq \mu_2$$

Test Statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- ▶ The degrees of freedom are the same as the confidence interval.
- ▶ Can also do a one-sided test (e.g., $H_A : \mu_1 > \mu_2$), but we will focus on two-sided tests when comparing two means.

Conditions

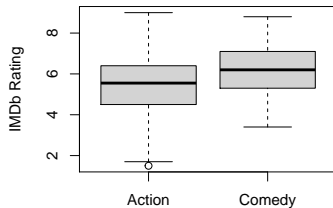
Conditions for a confidence interval or hypothesis test for the difference between two population means:

- ▶ The data in each group comes from a random sample, or randomized experiment. Additionally, the two groups are independent of each other (the cases in the first group are not related to the cases in the second group).
- ▶ The sample sizes are large ($n_1 \geq 30$ and $n_2 \geq 30$). Otherwise, if the samples sizes are small, the data in each group should be approximately normal.

Example

Are action or comedy movies rated higher on IMDb? Below are some summary statistics for a random sample of 50 action movies and 50 comedy movies rated on IMDb. Use a hypothesis test to determine whether there is a statistically significant difference between the two means.

IMDb Rating		
	Action	Comedy
Mean	5.46	6.18
SD	1.55	1.24
n	50	50

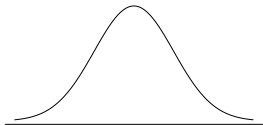


(a) Write the null and alternative hypotheses.

(b) Check the conditions for the test.

(c) Calculate the test statistic.

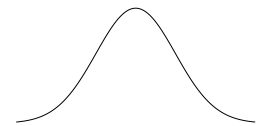
- (d) Calculate the p -value, and make a decision using $\alpha = 0.05$ significance level.



- (e) What is the conclusion of the test in the context of the data?

Example

Calculate and interpret a 95% confidence interval for the difference between the mean ratings of action and comedy movies on IMDb.



Example

Interpretation:

- ▶ We are 95% confident that $\mu_a - \mu_c$ is between -1.28 and -0.16.
- ▶ In context, we are 95% confident that the population mean rating of action movies on IMDb is between 0.16 and 1.28 less than the population mean rating of comedy movies.