Lecture 10: Power and Sample Size Calculations
STAT 630, Fall 2021

Decision \ Truth	H_0 true	H_A true
Reject H_0	Type I error (α)	Correct decision ($1-\beta$)
Do not reject H_0	Correct decision	Type II error (β)

- Type I error: $\alpha = P(\text{Reject } H_0 | H_0 \text{ true})$
  - the probability of falsely rejecting H_0
- Type II error: $\beta = P(\text{Do not reject } H_0 | H_A \text{ true})$
- Power: $1-\beta = P(\text{Reject } H_0 | H_A \text{ true})$
  - the probability of correctly rejecting H_0

$P(A|B)$ probability of A given B

Remarks:

$\uparrow \alpha \quad \downarrow \beta$

- If we increase α, then β decreases (type 1 and 2 errors are inversely related).
- If we increase the sample size n, then the power of the test increases (which implies the probability of a type 2 error β decreases).

$\uparrow n \quad \uparrow$ power $(1-\beta)$

Example: Blood pressure oscillates with the beating of the heart, and the systolic blood pressure is defined as the peak pressure when a person is at rest. The average systolic blood pressure for people in the U.S. is about 130 mmHg with a standard deviation of about 25 mmHg. We are interested in finding out if the average blood pressure of employees at a certain company is greater than the national average, so we randomly sample 100 employees and measure their systolic blood pressure.

(a) What are the null and alternative hypotheses?

(b) Find the values of the sample mean $\bar{x}$ for which the null hypothesis would be rejected. That is, find c such that we reject H_0 if $\bar{x} > c$. Use $\alpha = 0.05$ significance level.

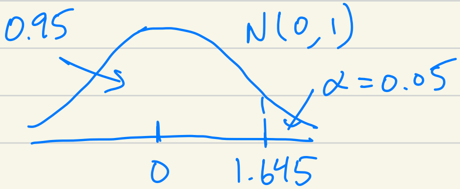
(c) Calculate the power of the test $(1-\beta)$ if the true average blood pressure for employees at this company is 136 mmHg.

(d) How large of a sample is needed to detect a 4 mmHg increase in average blood pressure with 0.9 power ($\beta = 0.1$) and $\alpha = 0.05$?

a) $H_0: \mu = 130$

$H_A: \mu > 130$

b) Find values of $\bar{x}$ where H_0 is rejected:



At $\alpha = 0.05$ significance level we reject H_0 if test statistic $z >$ qnorm(0.95) = 1.645

$\mu_0 = 130$

$\sigma = 25$

$n = 100$

Reject $H_0 \Rightarrow z > 1.645$

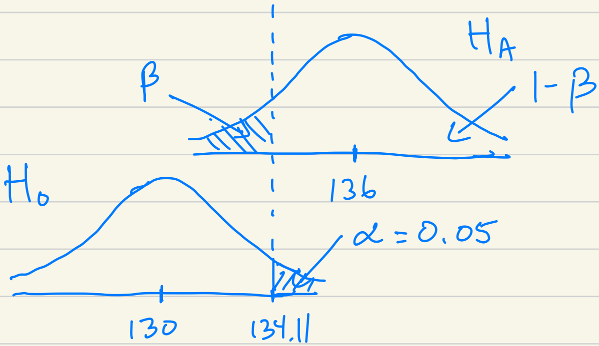
$$\Rightarrow \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} > 1.645$$

$$\Rightarrow \bar{x} > \mu_0 + 1.645 \cdot \sigma/\sqrt{n}$$

$$\Rightarrow \bar{x} > 130 + 1.645 \cdot 25/\sqrt{100}$$

$$\Rightarrow \boxed{\bar{x} > 134.11}$$

c) Calculate power $(1-\beta)$ if true average blood pressure for employees is 136 mmHg



$\mu_0 = 130$
$\mu_A = 136$
$\sigma = 25$, $n = 100$

$$\text{Power} = 1-\beta = P(\text{Reject } H_0 \mid H_A \text{ true})$$

$$= P(\bar{x} > 134.11 \mid \mu = 136)$$

$$= P\left(z > \frac{134.11 - 136}{25/\sqrt{100}}\right)$$

$$= P(z > -0.756)$$

$$= 1 - \text{pnorm}(-0.756) = \boxed{0.775}$$

$\beta = 1 - 0.775 = 0.225$ is probability of type 2 error

Here are the solutions using R:

```r
# part c
# delta = 136-130 = 6
power.t.test(n=100, delta=6, sd=25, sig.level=0.05,
  type="one.sample", alternative="one.sided")

##
##      One-sample t test power calculation
##
##               n = 100
##           delta = 6
##              sd = 25
##       sig.level = 0.05
##           power = 0.7699533
##     alternative = one.sided

# part d
power.t.test(power=0.9, delta=4, sd=25, sig.level=0.05,
  type="one.sample", alternative = "one.sided")

##
##      One-sample t test power calculation
##
##               n = 335.8827
##           delta = 4
##              sd = 25
##       sig.level = 0.05
##           power = 0.9
##     alternative = one.sided
```