Lecture 2:
Sampling Methods
STAT 630, Fall 2020

Topics:

- Sampling design terminology
- Sampling methods:
  - Simple random sampling
  - Stratified sampling
  - Cluster sampling
  - Systematic sampling
- Problems with survey sampling

Sampling Design Terminology

- **Target population:** The complete collection of observations we want to study.
- **Sample:** A subset of the target population.
- **Sampling unit:** The units we actually sample (e.g., people, households).
- **Sampling frame:** The list of sampling units from which the sample was taken (e.g., list of street addresses or telephone numbers).
- A sample is **representative** if it reproduces characteristics of the target population.

Sampling Design Terminology

Example: Public opinion polls (such as Gallop or the Washington Post) are used to predict which candidate will win the next election.

- Target population: all registered voters
- Sampling frame: list of telephone numbers for voters that can be interviewed (may be different than target population)
- Sampling unit: individual voter
- Sample: subset of voters interviewed by telephone

→ Some voters may not have a phone, or be listed in a directory

Sampling Design Terminology

Example: The Environmental Protection Agency (EPA) samples lakes across the conterminous U.S. and assesses their condition (good, fair, or poor according to an aquatic health index).

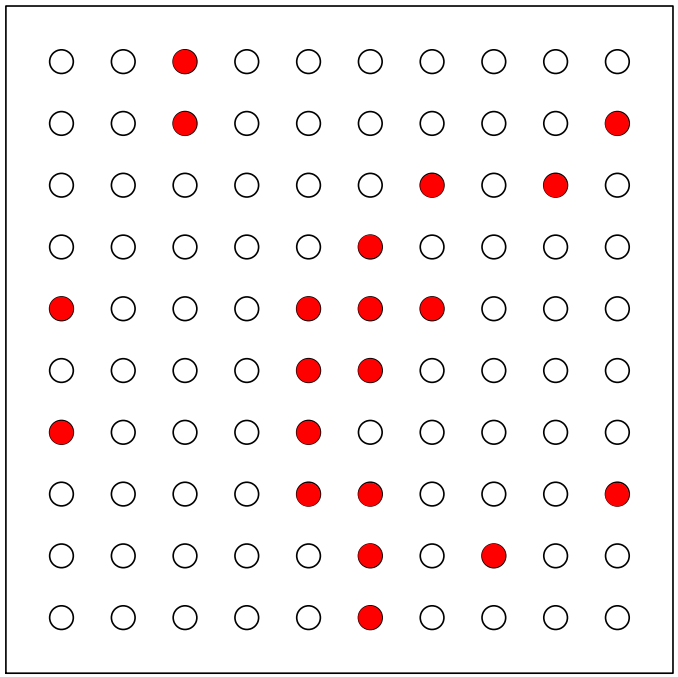
- Target population: all lakes in the conterminous U.S.
- Sampling frame: list of lakes and their locations from a Geographic Information System (GIS) database
- Sampling unit: individual lake site
- Sample: subset of lakes selected from the database

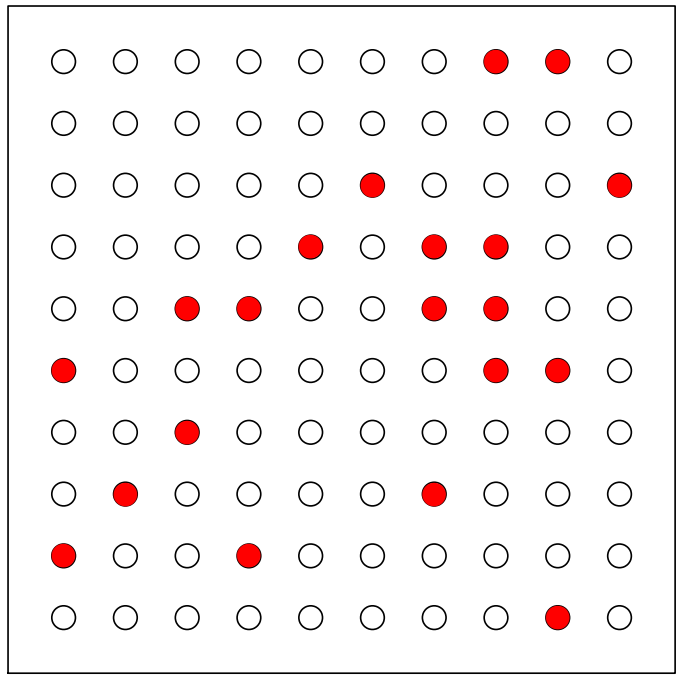
Simple Random Sampling

- A **simple random sample** (SRS) of size n is taken when every possible subset of n distinct units from the population has the same probability of being selected.
- One way to select a SRS of size 10 from a population of size 100: write the numbers $1, \cdots, 100$ on pieces of paper and place in a hat and stir; then select 10 pieces from the hat without replacing any.

Simple Random Sampling

Two simple random samples of size $n = 20$ from a population with $N = 100$ units.





Simple Random Sampling

Using R to take a SRS of size $n = 10$ from a population with $N = 100$ units:

```
> sample(1:100, 10)
 [1] 48 42 49 77 45 96 33 64 98 65

> sample(1:100, 10)
 [1] 78 62 58 33 36 15  6 64 41  2

> sample(1:100, 10)
 [1] 98 40 53 27  8 29  7 84 59 11
```

takes samples without replacement by default:

```
> sample(1:100, 10, replace = FALSE)
```

default argument

Simple Random Sampling

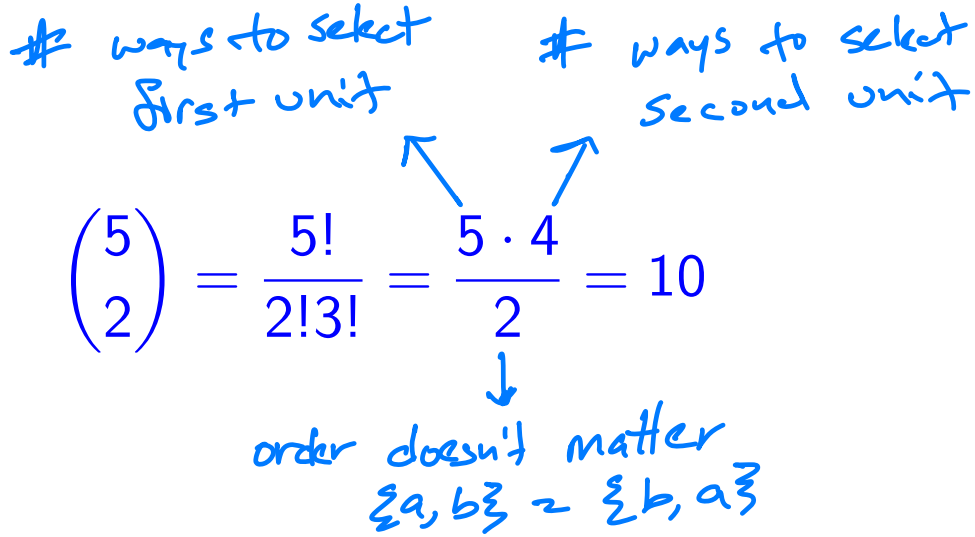
Example: Let {a,b,c,d,e} be a population of size $N = 5$. List all possible samples of size $n = 2$ from this population. For SRS what is the probability of selecting each sample of size $n = 2$?

Solution:
There are 10 possible samples of size $n = 2$:
$\{a, b\}$; $\{a, c\}$; $\{a, d\}$; $\{a, e\}$; $\{b, c\}$; $\{b, d\}$; $\{b, e\}$;
$\{c, d\}$; $\{c, e\}$; $\{d, e\}$

For SRS, 1/10 is the probability of selecting each sample of size $n = 2$.

Simple Random Sampling

Can use combinations to count # of samples:



In general, there are $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ samples of size n from a population with N units.

Simple Random Sampling

Example: Suppose a population consists on $N = 20$ individuals. How many possible samples of size $n = 4$ can we select from this population? Assume sampling is done **without replacement**.

Solution:

$$\binom{20}{4} = \frac{20!}{4!16!} = \frac{20 \cdot 19 \cdot 18 \cdot 17}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{4845}$$

In R:

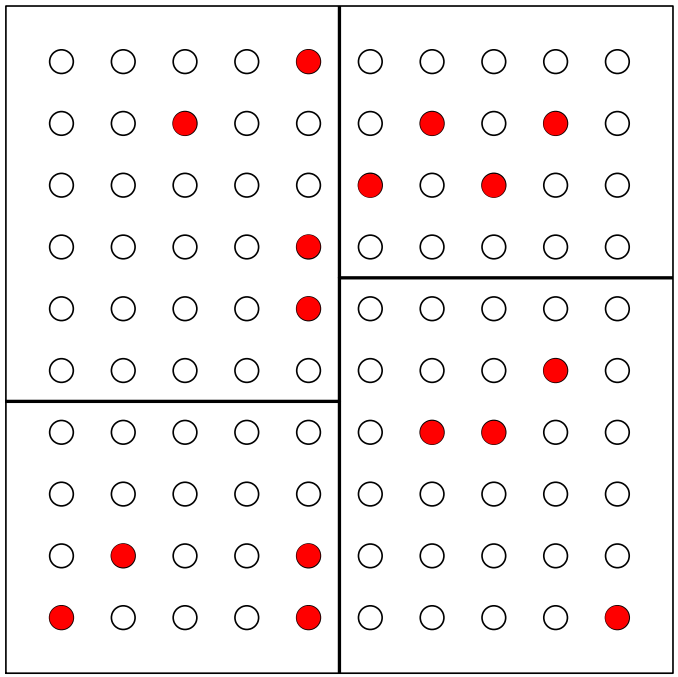
```
> choose(20, 4)
```

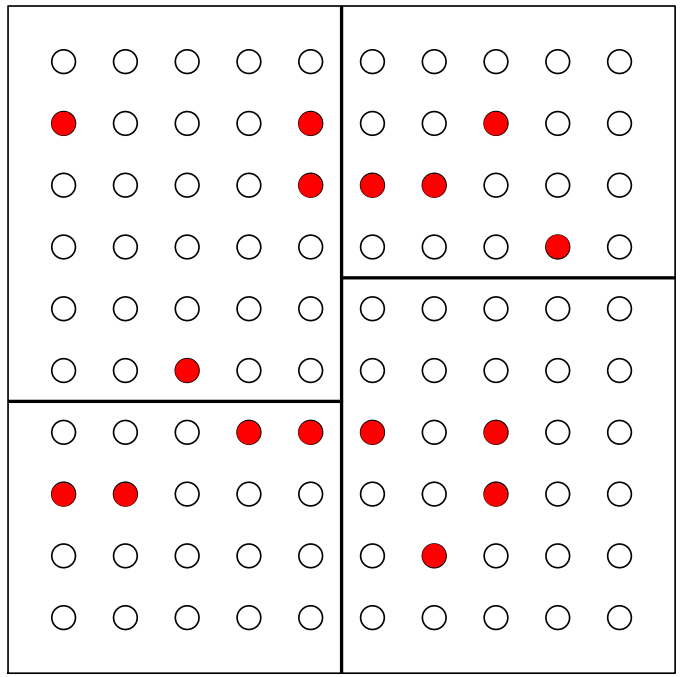
Stratified Sampling

- For **stratified sampling** the population is divided into distinct groups called **strata**. Then a SRS is selected from from each strata.
- The strata are selected so that units within each strata are similar in some way. For example, the strata might be different ethnic or age groups when surveying people.
- Commonly used in geographic sampling where the strata can be states, counties, or ecoregions.

Stratified Sampling

Two stratified random samples. Units are grouped into 4 strata, and a SRS of size 4 is selected within each strata.





Cluster Sampling

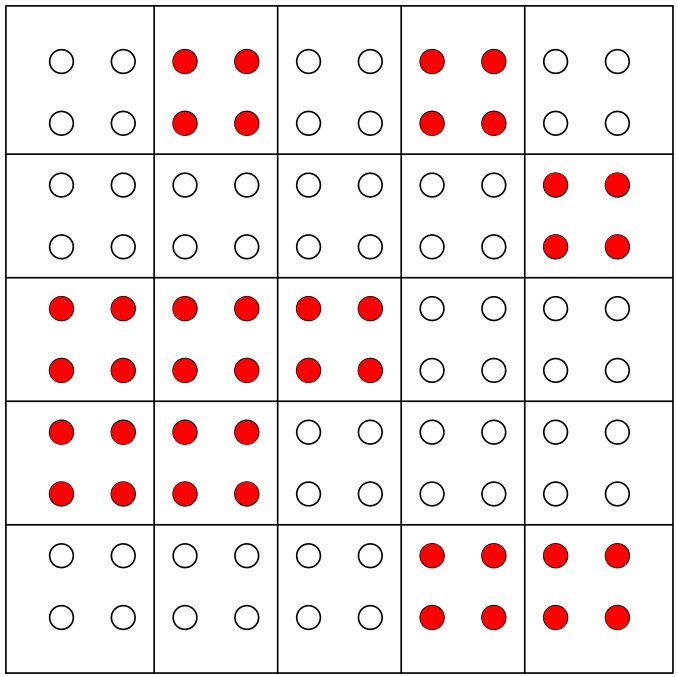
- For **cluster sampling** the population is divided up into groups called clusters. Then a fixed number of clusters are randomly sampled (using SRS) and all units within each of the selected clusters are included in the sample.

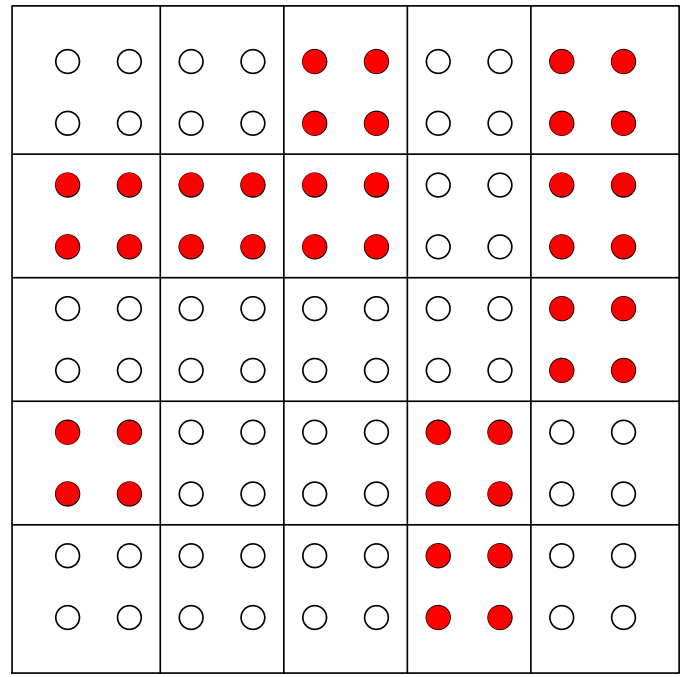
- For example, suppose we want to survey church members. Instead of taking a SRS of individual church members, we take a random sample of churches (the clusters) and sample all individuals in the selected churches.

- Unlike stratified sampling, cluster sampling works best when there is a lot variability within a cluster, and the units within each cluster are representative of the population.

Cluster Sampling

Two cluster samples. There are 25 clusters and 10 clusters are randomly selected. All units within each of the selected clusters are included in the sample.



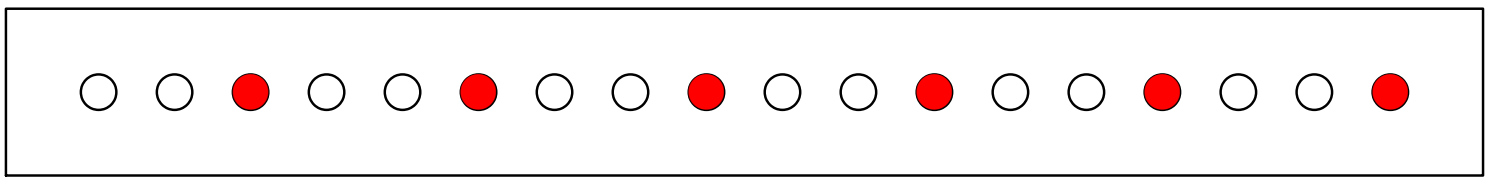


Systematic Sampling

- A **systematic sample** is drawn by selecting units systematically from a sample frame.
- For example, suppose we have a list of names of all students attending CSUEB. We then select a student at the beginning of the list and proceed to select every 10th name thereafter. If the names are alphabetized, it is generally reasonable to assume that the person's name has nothing to do with the characteristic of interest (e.g., height, political opinions).

Systematic Sampling

A systematic sample. Every third unit is included in the sample.



Census

- A **census** is taken if every observation in the population is included in the sample. That is, the sample and the population are the same.
- Taking a census is more costly and time consuming than random sampling.
- For large populations, data collection and processing for a census is complex and may be prone to errors.

Example

Identify the type of sampling design:

- The selection of 200 people to serve as potential jurors in a trail is conducted by assigning a number to each of 140,000 registered voters in the county. The R command `sample(1:140000, 200)` is used to take a sample of 200 numbers between 1 and 140,000. People having these 200 numbers are sent postcards notifying them of jury duty.
  Ans: Simple random sampling (SRS)

- Suppose you are selecting microchips from a production line for inspection. As the chips process past the inspection point, every 100th chip is selected for inspection.
  Ans: Systematic sampling

Example

Identify the type of sampling design:

- In a survey on household income, 1000 households are randomly selected in each of the 50 states in the U.S.
  Ans: Stratified sampling

- A survey is conducted to find the average weight of cows in a region. A list of all farms is available for the region, and 50 farms are selected at random. Then the weight of each cow at the 50 selected farms is recorded.
  Ans: Cluster sampling

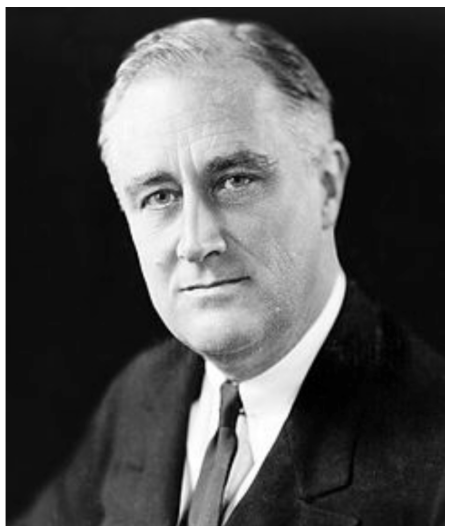
Problems with Survey Sampling

A sample is **biased** if it is not representative of the target population. Statistics from biased samples tend to overestimate or underestimate the population parameter. Some sources of bias for survey sampling include:

- **Nonresponse**: failing to obtain responses from some individuals selected for the sample. There may be differences between those that respond and do not respond to a survey.
- Taking a **sample of convenience** by only including individuals that are easily accessible in the sample.
- Allowing the sample to consist entirely of volunteers.
- Wording a survey question in such a way that it influences the response.
- **Undercoverage**: Using a sample frame that does not include a portion of the target population.

Historical Example: Landon vs. FDR, 1936

- Literary Digest polled 10 million Americans, and 2.4 million responded
- Prediction: 43% for FDR
- Result: 62% for FDR



- The magazine was so discredited by the poll that is was discontinued.

Historical Example: Landon vs. FDR, 1936

What went wrong?

- The magazine had surveyed
  - its own readers,
  - registered automobile owners, and
  - registered telephone users.
- The sample frame consisted of individuals that were wealthier than the majority of voters, and therefore more likely to support the Republicans (example of undercoverage).
- Nonresponse: 10 million sampled, but 2.4 million responded. Persons supporting Landon were more likely to have responded to the survey.