

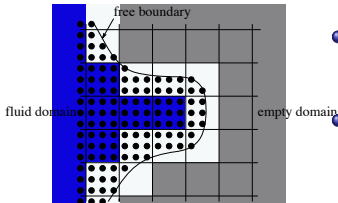
# Free Surface Flows

M. Farahani, E. Wolter, A. Hahn

03.02.2014

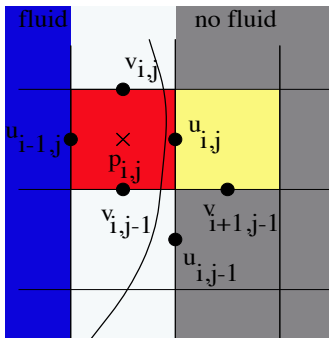


# Theory



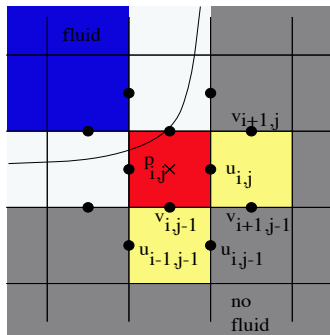
- the stress tensor:  $\sigma = (-P + \lambda \text{div} \vec{u}) I + 2\mu \delta$
- $P + \frac{2}{Re} \left( n_x n_x \frac{\partial u}{\partial x} + n_x n_y \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + n_y n_y \frac{\partial v}{\partial y} \right) = K \kappa$
- $2n_x m_x \frac{\partial u}{\partial x} + (n_x m_y + n_y m_x) \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + 2n_y m_y \frac{\partial v}{\partial y} = 0$

# One empty neighbor



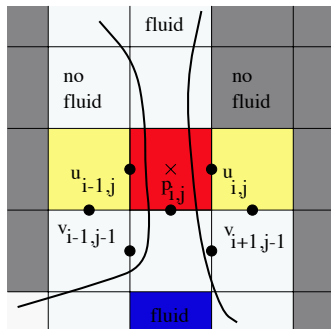
- free boundary lie almost parallel to the grid lines
- $n_y \& m_x = 0 \quad || \quad n_x \& m_y = 0$
- $P = \frac{2}{Re} \frac{\partial u}{\partial x}$
- $\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$
- using continuity equation

# Two empty neighbors - common corner



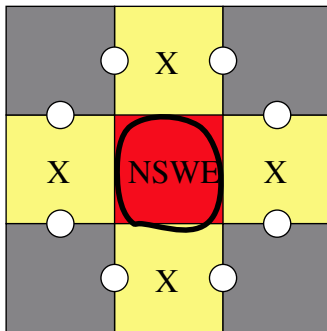
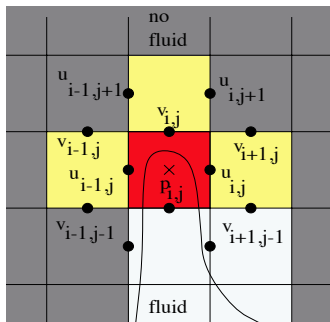
- $n_y = m_x = n_x = m_y$
- $P = \pm \frac{1}{Re} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \right)$
- $\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0$

# Two empty neighbors - opposite side



- $u_{i,j}^{new} = u_{i,j}^{old} + \delta t g_x$
- $u_{i-1,j}^{new} = u_{i-1,j}^{old} + \delta t g_x$
- $v_{i,j}^{new} = v_{i,j}^{old} + \delta t g_y$
- $v_{i,j-1}^{new} = v_{i,j-1}^{old} + \delta t g_y$

# Three and four empty neighbors



# Particle and ParticleTracer

- **Particle(real x, real y, int type)**

Has some functions which can detect its position on the grid

- **ParticleTracer(StaggeredGrid \*grid)**

Has a vector of particles

- **void markCells()**
- **void fillCell(int i, int j, int numParticles, int type)**
- **void addRectangle(real x1, real y1, real x2, real y2, int type)**
- **void addCircle(real x, real y, real r, int type)**
- **void advanceParticles(real const dt)**



# Types and StaggeredGrid

- **Types.hh:**
  - **flag EMPTY**
- **StaggeredGrid.cc:**
  - **int ppc\_**
  - **bool isEmpty(const int x, const int y)**
  - **void setCellToEmpty(int x, int y)**
  - **void refreshEmpty()**

# FluidSimulator

- **FluidSimulator.cc:**
  - `real rectX1_particle_, rectX2_particle_ , ...`
  - `real circR_particle_, circX_particle_ , ...`
  - `void set_UVP_surface(int i, int j , const real &dt, bool compP)`
  - `void one_empty_neighbour(int i , int j , const real &dt, bool compP)`
  - ...
  - `four_empty_neighbour(int i , int j , const real &dt, bool compP)`
  - `void refreshEmpty()`

# Main while-loop

```
while (n <= nrOfTimeSteps)
{
... determineNextDT(safetyfac_);
   particle_tracer_.markCells();
   set_UVP_surface(dt_, true);
   computeFG();
   composeRHS();
   solv().solve(grid_);
   updateVelocities();
   refreshBoundaries();
   set_UVP_surface(dt_, false);
   particle_tracer_.advanceParticles(dt_); ...
}
```

# Examples

- The Breaking Dam - Outflow
- The Breaking Dam - Freeslip
- The Splash of a Liquid Drop

# The Breaking Dam

```
imax = 50,      jmax = 20,  
xlength = 10.0, ylength = 4.0,  
tau = 0.5,      delt = 0.04,      t_end = 5.0,  
eps = 0.001,    omg = 1.7,  
gamma = 0.5,    itermax = 500,  
GX = 0.0,       GY = -1.0,        Re = 10.0,  
UI = 0.0,       VI = 0.0,         PI = 0.0,  
ppc = 16,
```

```
wW = free ,      wE = out ,  
wS = free ,      wN = out
```

# The Splash of a Liquid Drop

```
imax = 40,      jmax = 30,  
xlength = 8.0,  ylength = 6.0,  
tau = 0.2,      delt = 0.01,    t_end = 10.0,  
eps = 0.001,    omg = 1.7,  
gamma = 0.5,    itermax = 500,  
GX = 0.0,       GY = -1.0,      Re = 40.0,  
UI = 0.0,       VI = 0.0,      PI = 0.0,  
ppc = 16,
```

```
wW = free ,      wE = free ,  
wS = free ,      wN = out
```