Free Surface Flows

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Finde eine stückweise zwei mal stetig differenzierbare Bahnkurve der Hinterachse $\Phi \in C^1([0, t^*], \mathbb{R}^2)$, sodass

(I) das Auto zu jedem Zeitpunkt t in einem Gebiet G ist

(II) die Randbedingungen für $\Phi(0), [\Phi(t^*)]_2$ erfüllt sind und

$$\frac{\Phi'(0)}{\|\Phi'(0)\|} = \frac{\Phi'(t^*)}{\|\Phi'(t^*)\|} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

(III) für $-\frac{\Phi'(t)}{\|\Phi'(t)\|_2} = \begin{pmatrix} \cos\beta(t) \\ \sin\beta(t) \end{pmatrix}$

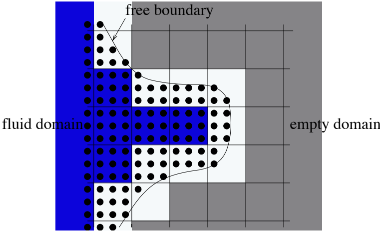
$\beta(t) \in \left[0, \frac{\pi}{2}\right[$ für alle $t \in [0, t^*]$ erfüllt ist

(IV) (Wendekreisbeschränkung erfüllt)

Application

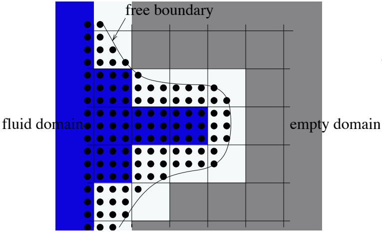



One empty neighbor



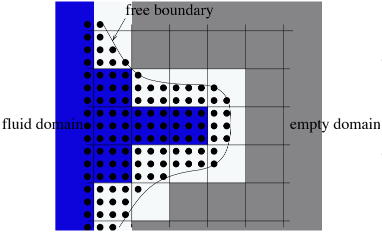
- the stress tensor:
  $\sigma = (-P + \lambda div\vec{u})I + 2\mu\delta$

One empty neighbor



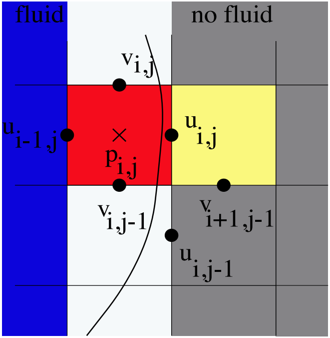
- the stress tensor:
  $\sigma = (-P + \lambda div \vec{u})I + 2\mu\delta$
- $P + \frac{2}{Re}(n_x n_x \frac{\partial u}{\partial x} + n_x n_y (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) + n_y n_y \frac{\partial v}{\partial y}) = K\kappa$

One empty neighbor



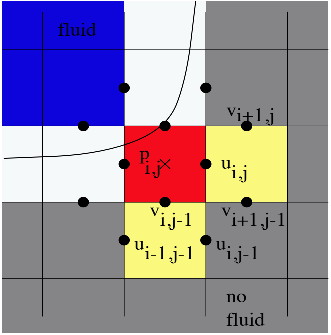
- the stress tensor:
  $\sigma = (-P + \lambda div\vec{u})I + 2\mu\delta$
- $P + \frac{2}{Re}(n_x n_x \frac{\partial u}{\partial x} + n_x n_y (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) + n_y n_y \frac{\partial v}{\partial y}) = K\kappa$
- $2n_x m_x \frac{\partial u}{\partial x} + (n_x m_y + n_y m_x)(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) + 2n_y m_y \frac{\partial v}{\partial y}) = 0$

One empty neighbor



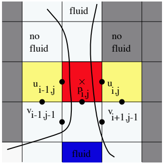
- free boundary lie almost parallel to the grid lines
- $n_y \& m_x = 0 R n_x \& m_y = 0$
- $P = \frac{2}{Re}\frac{\partial u}{\partial x}$
- $\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$
- using continuity equation

Two empty neighbor-common corner



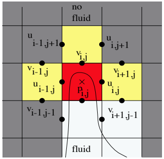
- $n_y = m_x = n_x = m_y$
- $P = \pm \frac{1}{Re}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\right)$
- $\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0$

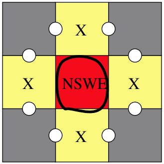
Two empty neighbor-opposite side



- $u_{i,j}^{new} = u_{i,j}^{old} + \delta t g_x$
- $u_{i-1,j}^{new} = u_{i-1,j}^{old} + \delta t g_x$
- $v_{i,j}^{new} = v_{i,j}^{old} + \delta t g_y$
- $v_{i,j-1}^{new} = v_{i,j-1}^{old} + \delta t g_y$

Three empty neighbor





Particle and ParticleTracer

- **Particle(real x, real y, int type)**
  Has some functions which can detect its position on the grid
- **ParticleTracer(StaggeredGrid *grid)**
  Has a vector of particles
  - **void markCells()**
  - **void fillCell(int i, int j, int numParticles, int type)**
  - **void addRectangle(real x1, real y1, real x2, real y2, int type)**
  - **void addCircle(real x, real y, real r, int type)**
  - **void advanceParticles(real const dt)**

Types and StaggeredGrid

- **Types.hh**:
  - **flag EMPTY**
- **StaggeredGrid.cc**:
  - **int ppc_**
  - **bool isEmpty(const int x, const int y)**
  - **void setCellToEmpty(int x, int y)**
  - **void refreshEmpty()**

FluidSimulator

- **FluidSimulator.cc**:
  - **real rectX1_particle_, rectX2_particle_ , ...**
  - **real circR_particle_, circX_particle_, ...**
  - **void set_UVP_surface(int i, int j , const real &dt, bool compP)**
  - **void one_empty_neighbour(int i , int j , const real &dt, bool compP)**
  - ...
  - **four_empty_neighbour(int i , int j , const real &dt, bool compP)**
  - **void refreshEmpty()**

Main while-loop

```
while (n <= nrOfTimeSteps)
{
    ...
    determineNextDT(safetyfac_);
    particle_tracer_.markCells();
    set_UVP_surface(dt_, true);
    computeFG();
    composeRHS();
    solv().solve(grid_);
    updateVelocities();
    refreshBoundaries();
    set_UVP_surface(dt_, false);
    particle_tracer_.advanceParticles(dt_);
```

Breaking dam with outflow at the east wall

$$\Phi(t) = \begin{cases} \Phi(0) - \begin{pmatrix} 1 \\ 0 \end{pmatrix} t & \text{für } t \in [0, t_0] \\ M^1 + r_1 \begin{pmatrix} -\sin\left(\frac{t-t_0}{r_1}\right) \\ \cos\left(\frac{t-t_0}{r_1}\right) \end{pmatrix} & \text{für } t \in [t_0, t_1] \\ \Phi(t_1) + \begin{pmatrix} -\cos\left(\frac{t_1-t_0}{r_1}\right) \\ -\sin\left(\frac{t_1-t_0}{r_1}\right) \end{pmatrix} (t - t_1) & \text{für } t \in [t_1, t_2] \\ M^2 + r_2 \begin{pmatrix} \sin\left(\frac{t^*-t}{r_2}\right) \\ -\cos\left(\frac{t^*-t}{r_2}\right) \end{pmatrix} & \text{für } t \in [t_2, t^*] \end{cases}$$

Breaking dam with free-slip at the east wall

Falling drop

- Umschreibung der Bedingungen in die Variablen t_0, t_1, r_1 und t_2.
- Lösung des entstehenden Optimierungsproblems