Combining Rigid body simulations with oriented particles

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Hiermit versichere ich, die vorliegende Arbeit unter der Betreuung von Prof. Dr. X. Xxxxxx und Dxxx.-Ing. X. Xxxxx nur mit den angegebenen Hilfsmitteln selbst"andig angefertigt zu haben.

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Abstract

The major objective of this study is to explore the possibility to combine a traditional rigid body simulation with a deformable body simulation. The deformable bodies are implemented according to the relatively new approach of using oriented particles. The combined bodies consist of a rigid inner structure and a soft outer structure. The goal is to improve upon traditional system simulating skeletons. The bones can now be modeled with a surrounding cushion of tissue using the presented method. This extends and improves simpler models using only rigid bodies especially in areas where number of contact points is important to achieve a stable simulation. Using the soft deformable tissue enables bodies to surround other bodies more realistically while increasing the number of contact points significantly. This combined simulation model thus can provide a more stable simulation.

The main contribution of this thesis is the modeling and description of the interface between the deformable oriented particles and rigid bodies. The first interface is inside the body itself between the bones and the tissue, which requires an efficient way to closely couple both sub-simulations. The second interface is between these combined bodies and rigid bodies in the simulated world. Here the impulses generated by the tissue onto the rigid bodies has to be defined and calculated.

The results show that the simulation provides realistic results. For the simple scenario of picking up a cylinder only two rigid bodies, modeling the fingers, are needed. A rigid body only simulation would either completely fail at this task or would required multiple bodies to simulate the whole finger. This result promises both more efficient and faster simulations of complete skeleton models.

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1 Introduction

Simulating the physically properties of objects with the help of a computer is a very important topic across a wide range of fields. As computing power is both expensive and limit the approach to simulations is distributed across a spectrum of accuracy at the one end and performance on the other. Some fields require the best and most accurate simulations currently available. These are especially predominant in applications where predictions for the real world behavior of the system are required and later applied to constructing and optimizing real world objects. Example fields for this are automotive or aerospace modeling and simulations. Here extremely detailed models are used to predict the properties and behavior of all kinds of different materials, shapes and bodies. The results of these simulations directly influence the construction and design decisions for their real world counter parts. Simulating the objects first inside the computer enables a faster development cycle and thus provides an efficient and cost effective way of constructing new parts and pieces.

Other fields are not reliant on the direct applicability of their results to the real world. The main focus of these simulations is to provide physically plausible and visually believably results in a timely fashion. Probably the two most popular and widely known application for these types of simulations are Games and Animations. Here the goal is to introduce the viewer into a completely virtual universe where everything is controlled by the media creators. In order to achieve the desired immersion of the viewers realistic behavior of the actors and objects in this universe is really important. Obvious incorrect physical behavior is most of the time immediately recognizable by the consumers and can easily break their immersion in the content.

Today games and animations use a huge number of interacting bodies. These bodies are simulated using a variety of different simulation methods and techniques. For animating characters and objects in movies the main focus lies in reducing the workload of animators, who traditionally had to animate and model everything by hand, a long and tedious process. Physical simulations can aid the animators in automatically generating physically plausible animations to start with. These animations can then be later adjusted and modified as necessary but a huge amount of work can be done by the simulation. These techniques and methods are only secondarily focused on the simulation time, while this is still a consideration to enable faster turn around times for the animator playing with different ideas the primary focus of these systems is the controllability of the output. These system should enable the animators to define their desired behavior and letting the system automatically figure out how to simulate everything in between, always trying to have an at least physical plausible believable result.

Physical simulations in games on the other hand have to primarily focus on the real time aspect of their respective techniques and methods. Today games simulate a huge a amount of objects in an ever increasing world. All these simulations have to calculated in an extremely short period of time. The simulated physics have to share the resources with everything else required to simulate the game world including game logic, artificial intelligence and computer graphics. In order to achieve smooth 60 fps all these different subsystems have to be extremely efficient and fast. If the simulations slow down too much the immersion of the player is immediately broken when things start to move unrealistically.

Another interesting subarea of the games are the so called serious games. Here the focus of the simulated world is not to provide fun and entertainment to the player. Instead the goal is the model the real world as realistically as possible to enable training and orientation for people working in the simulated field of expertise. Prominent examples are military and emergency personnel. Simulations can be used to give these persons a chance to experience real world scenarios in a controlled environment so they can train and prepare themselves for their duties. These types of games are also used to help train medial personnel to perform surgeries on completely simulated patients first before applying the knowledge to real world cases. Providing realistic real time feedback to the users is the main focus of these types of simulations.

For all close to real time and controllable simulations rigid body simulations have been a very popular choice. Rigid body simulations enable fast and efficient simulations of a huge number of interacting bodies. This can be achieved with the assumption that all bodies in the simulation a completely rigid and can never deform. This assumption is plausible for a wide variety of applications ranging from simple car models to more complex ragdoll models. Unfortunately not all possible scenarios can be modeled with simplified rigid bodies. Objects in the real world a very often deformable and bend and stretch under the influence of external forces. Incorporating deformable simulations models into games and animations enable a whole range of new interesting scenarios. However the simulation of deformable bodies requires a lot more computing resources to achieve realistic results. Novel ways to reduce the amount of work associated with deformable simulations of the be found.

One such approach is to combine the two different simulation approaches. Here the goal is combine the very fast rigid body simulations with the slower deformable simulations in order to reap the benefits of both simultaneously. Some approaches try to simulate rigid bodies and deformable bodies side by side for each object, while other approaches try to model a single body using both rigid as well as deformable parts. A real world example of the second time is the human body itself. Here basically rigid bodies, the bones, form an underlying rigid skeleton. However in contrast to simple ragdolls the reality this rigid skeleton is surrounded by a soft layer of tissue. This soft tissue can deform when forces are applied. Modeling this seemingly obvious scenario efficiently is non trivial and a perfect match for a combined simulation model.

The focus of this thesis is to explore this type of simulation, where a rigid inner core is simulated with a soft outer layer of deformable material. The goal is have an efficient simulation with is both fast and robust in scenarios where only using a simplified rigid body simulation would give unsatisfactory results. The thesis furthermore focusses on a specific technique to implement the deformable material. This technique uses oriented particles to simulate the deformations of a body. Oriented particle can be simulated efficiently and in realtime while still giving physically plausible results.

The thesis first gives a short background on how rigid body simulations are implemented and how the simulation is used in the proposed method. After that a more in-depth description of the way oriented particles are implemented is presented, beginning with position based dynamics and shape matching and explaining how these approaches enable oriented particles. The final part describes the method and ideas for combining these two different approaches into one system.

2 Problem Analysis

This thesis focusses on a concrete example scenario. This helps to better understand the implications and advantages of the proposed method. Also the obtained results are more easily compared against the traditional approach.

A widely applicable scenario for modeling bones and soft tissue are fingers. The scenario being used in this thesis involves two fingers grabbing onto and lifting block of the ground. The fingers are positioned to the either side of the block in the model. While applying a continuous pressure onto the block the fingers are lifting it up from the floor and into the air. This is directly applicable to a number of different scenarios involving a robotic arm with attached fingers, which is supposed to pick up blocks are other objects of the ground.

image

illustration of scenario

image

All this has to be able to be simulated in realtime in order to be used games and animations. The simulations also has to be robust against numerically instabilities. These are either caused by simple numerical or rounding errors or by the fingers being slightly off the target when grabbing onto the block. Forces between the inner bone and outer tissue as well as forces between the tissue and other objects have to be modeled so a realistic result can be achieved. This is especially true for the friction caused by the tissue on the block, as otherwise the block would not be lift up at all.

Using a simple rigid body simulation for this is suboptimal. Both fingers and the block would be modeled as rigid bodies. Resolving all the resulting forces acting on the block can quickly get numerically unstable. The blocks can start to move and wiggle incorrectly until the results are no longer visually pleasing and physically plausible.

The problem is further complicated when instead of a simple block any kind of round object like a cylinder would have to be picked up. This would require the fingers to be touch the cylinder perfectly perpendicular and on the exact opposite points. Otherwise the pressure and the resulting forces would simply let the cylinder slip out either in front or the back. This is basically impossible to achieve numerically.

The way to traditionally solve this problem has been to use multiple rigid bodies to simulate the each finger to get a better grip on the object that is lifted up. This way complicates the simulations quite a bit as joints and and many more blocks have to be simulated. However the problem of slowing increasing numerically instabilities would still occur and might still be very hard to handle efficiently.

Solving the problem with only deformable bodies is theoretically possible, however this would not really capture the real world idea of a rigid bone structure with a surrounding soft layer of tissue. Simulating everything as a deformable body would not be very efficient. This would require a lot more computational resources, especially when considering to apply this to a complete skeleton. Great care has also be taken to ensure that some deformable parts are mostly behaving as rigid structures while still giving a physically plausible result.

Therefore the main requirements for the proposed technique of combining an inner core of rigid bodies with a soft layer of surrounding deformable material can be summarized as:

1. realtime simulation
2. low number of bodies
3. only approximate orientation of the fingers relative to the block
4. collision handling/force propagation inside the body between "bone" and "tissue"
5. modeling friction between fingers and object

3 Related Approaches

- Some(**concrete reference missing**) focus on simulations which only aim to simulated both rigid bodies and deformable bodies in the same world. But any body is either rigid or deformable. So the inner forces and collisions between bones and tissue are not really modeled and explored.
- Other(**concrete reference missing**) use non realtime methods to simulated deformable bodies and are thus not applicable to the realtime simulation and problems involved when using oriented particles.
- The state of the art thus does not provide realtime simulations of soft tissue directly attached to rigid bones.

4 Fundamentals

This section describes the fundamental buildings blocks of the algorithm described in this thesis. It provides a quick overview and explanations of how the two different simulation techniques work. It does not describe in any way how the two different simulations might interact but instead describes them in total isolation.

4.1 Rigid Body Simulations

Rigid bodies simulations is very similar to simple particle simulations. Simple particles are modeled as infinitesimal point masses, rigid bodies additional are defined by their dimension. This extension in space remains constant through out the whole simulation. A rigid body can therefore be thought of like a set of many particles. If a force is acting on any one of these point masses all the other connected masses in the rigid body are affected as well. Each point mass has a fixed mass m_i and is located at fixed position r_i inside the system. The total mass of the rigid body is then simply the sum of all the point masses combined

$$M = \sum_{i=1}^{N} m_i$$

The position of the particles is defined relative to a local body coordinate system. The origin of coordinate system is the center of mass of the rigid body. The center of mass is calculated as

$$c = \frac{\sum m_i r_i}{M}$$

In order to describe a rigid body simulation two variables are of central importance. First is the position $x(t)$ of the rigid body in space. The second is the orientation $q(t)$in space. Both these variables are defined in relation to the world coordinate system. There are two different approaches to model the orientation of the rigid body, either a rotation matrix $R(t)$ or a unit quaternion $q(t)$. In practice in almost all cases unit quaternions are used for a number of reasons. One important aspect is the fact that quaternions don't suffer from gimbal lock like simple Euler angles do. Another aspect is that a quaternion consists of only 4 numbers instead of the 9 numbers required for a full rotation matrix. This second point leads to both easier and more efficient calculations, especially for interpolation and normalization.

By definition only the affine transformations of rotation and translation can apply to any rigid body. The main task of the rigid body simulation is therefore to calculate these to variables over time t.

The physics behind the rigid body simulation is based on Newton's laws of motion. Most importantly the first law stating that if no force is acting on an object, the velocity of the object will remain constant. Thus follows that the simulation has to model all forces acting on the body. These forces are the only causes for a change in the bodies state and thus its position and

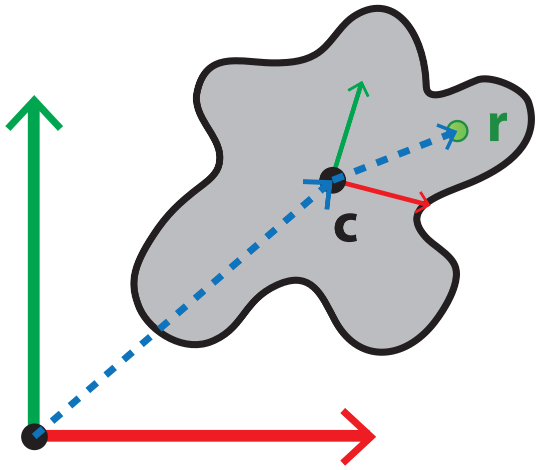


Figure 4.1: Rigid body model illustration

orientation in space. The forces either cause a change in the linear velocity of the body, or an angular velocity results in a rotation of the body.

The position of the rigid body is defined by the position $x(t)$ of its center of mass in world coordinates over time. The change of this position over time defines the linear velocity $v(t)$ for the body. So the linear velocity is simply the following.

$$v(t) = \dot{x}(t)$$

The angular velocity is a little harder to define. First the current rotation of the body has to be stated. The orientation of the rigid body was defined as unit quaternion $q(t)$. The change in orientation can than be calculated with the following .

$$\dot{q}(t) = \omega(t) * q(t)$$

Here $\omega(t)$ denotes the angular velocity of the body at any point in time. Both the linear velocity and the angular velocity are integrated in every time step to simulate the bodies behavior.

So in its most basic form the state of a rigid body body is defined by just the following five variables

$$\begin{aligned}
&m, \text{the mass}\\
&x, \text{the position of the center of mass}\\
&q, \text{the orientation}\\
&v, \text{the linear velocity}\\
&\omega, \text{the angular velocity}
\end{aligned}$$

Until now only the state of the rigid body has been defined. In the real world force are acting upon the body which cause a change in its state according to Newton's laws of motion. The most prominent force usually modeled in a rigid body simulations are for example gravity, collision and friction. In the real world force always act over a defined amount of time. Unfortunately for the simulation an instantaneous change in velocity is more desirable, so that for example in the case of collision the relative velocity of the involved bodies comes to a hold immediately. In order to achieve this goal the new quantity J is introduced. J is called an impulse and represents a force integrated over time.

$$F\Delta t = J$$

Letting F go to infinity and Δt to zero describes how the force would change the velocity if it would cause an immediate change. J thus has the units of momentum.

Again the effects of any impulse has to be evaluated for both the linear velocity as well as the angular velocity. For the linear velocity the change is rather defined as

$$v(t) = \frac{P(t)}{M}$$

In this case $P(t)$ denotes the linear momentum aggregated across all force acting upon the body and M is simply the mass of the rigid body.

The formula for the angular velocity again is a little more involved

$$\omega(t) = L(t) * inv(I(t))$$

Here $L(t)$ denotes the angular momentum. $I(t)$ is called the inertia tensor and is a simple scaling factor between the angular momentum $L(t)$ and $\omega(t)$ just like $inv(M)$ is in the linear case. The inertia tensor describes how the mass of the body is distributed relative to the body's center of mass. It depends on the orientation of the body but not on the translation.

Combining all the state variables with impulses yields the basic foundation for a rigid body simulation. A simplistic version of the simulation loop can for example look like this:

Algorithmus 4.1 Rigid Body Simulation Loop

```
for all bodies b do
    apply gravity
    integrate velocities
    predict state
end for
for all bodies b1 do
    for all bodies b2 do
        perform collision detection between body b1 and body b2
    end for
end for
for all collisions c do
    resolve collision c
end for
for all bodies b do
    integrate position and orientation
end for
```

4.2 Shape Matching

Shape Matching is an geometrically motivated approach to simulating deformable objects. It describes objects as a collection of points and does not need connectivity informations. By trading in physical correctness the technique is able to provide an unconditionally stable simulation.

The main concept of this deformable model is to replace the system of forces and energies between the different particles by simple geometric constraints and distances. All points are moved to goal positions which are obtained by shape matching the original rest state of the object to the currently deformed state of the object. These well-defined goal positions are computationally easy to obtain and provide an unconditionally stable simulation.

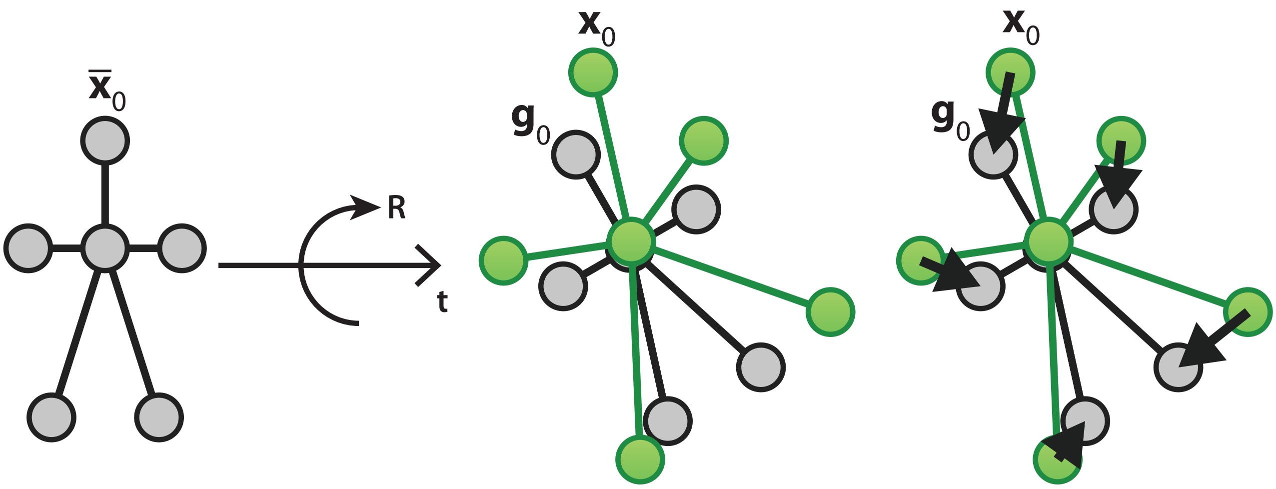


Figure 4.2: Shape matching illustration

The algorithm only takes a set of particles and there initial configuration as inputs. Each particle has four variables

$$\begin{aligned} &m_i, \text{the mass} \\ &\bar{x}_i, \text{the rest position} \\ &x_i, \text{the current position} \\ &v_i, \text{the velocity} \end{aligned}$$

The integration scheme uses the goal positions g_i to move the particles to the desired goal positions. The amount of movement can be directly modeled as a simple stiffness parameter α. A stiffness parameter if 1 thus basically doesn't allow for any deformation as the particles are always moved to their ideal transformed state. By using all these information and the external forces (i.e. gravity) the integration step becomes:

$$v_i(t+h) = v_i(t) + \alpha * \frac{g_i(t) - x_i(t)}{h} + h * f_{ext}(t)/m_i \tag{4.1}$$

$$x_i(t=h) = x_i(t) + h * v_i(t+h) \tag{4.2}$$

The goal position g_i is obtained by shape matching the rest shape to the current deformed shape. The problem shape matching solves is thus defined by finding the rotation matrix R and the translation vector t between the two sets of points $\bar{x}_i$ and x_i. The desired matrix R and vector t minimize the following term:

$$\sum_i m_i(R(\bar{x}_i - \bar{t}) + t - x_i)^2 \tag{4.3}$$

The optimal translation vectors are simply the respective center of masses of the rest shape and the deformed shape.

$$\bar{t} = \bar{c} = \frac{\sum_i m_i \bar{x}_i}{\sum_i m_i}, t = c = \frac{\sum_i m_i x_i}{\sum_i m_i} \tag{4.4}$$

Unfortunately the optimal rotation matrix is not as simple to compute. The equation **??** has to be simplified. The first step is to define relative positions for all points with respect to their center of mass.

$$q_i = \bar{x}_i - \bar{c}, p_i = x_i - c \tag{4.5}$$
$$\sum_i m_i(Rq_i - p_i)^2 \tag{4.6}$$

The next insight is to actually find the optimal linear transformation A instead of the optimal rotation matrix R. Calculating the derivative with respect to all coefficients of A and setting it to zero gives the optimal linear transformation.

$$A = (\sum_i m_i p_i q_i^\top)(\sum_i m_i q_i q_i^\top)^{-1} = A_{pq} A_{qq} \tag{4.7}$$

A_{qq} is a symmetric matrix can therefore contains no rotational part. All the rotation is encapsulated in A_{pp}. The optimal rotation matrix can now be found by calculating the polar decomposition $A = RS$. This allows to finally write down the goal position for each particle as

$$g_i = R(\bar{x}_i - \bar{c}) + c \tag{4.8}$$

All particles are than moved to their respective goal positions by a variable fraction. This fraction effectively models the stiffness of the deformable body. If the stiffness fraction is close to 1 the particle practically snap to their ideal goal positions which by definition are not deformed as they only include translation and rotations of initial rest state. If the stiffness fraction on the other hand is very low the particles stay in their predicted position and bodies is completely deformed.

4.3 Position Based Dynamics

Position based dynamics is a method which omits the traditional force based approach simulate dynamics systems but instead directly works on the positions to simulate deformable objects. This means it also is not physically correct but instead provides an efficient unconditionally stable simulation. Like shape matching it also tries to satisfy constraints in order to arrive at stable end positions. But instead of one global constraint, the shape, position based dynamics handles many different constraints. These constraints either model the physical properties of the object and are thus fixed throughout the simulation, or are generated on demand in the case of collision constraints used to resolve collisions. Also as the name implies position based dynamics only works with the position of the particles updating them directly. The velocity of the particle is solely determined by the difference in positions overtime. This is best illustrated by the the pseudocode of the simulation loop.

Algorithmus 4.2 Position based dynamics simuation

1: **for all** vertices i **do**
2:     initalize $x_i = \bar{x}_i, v_i = \bar{v}_i, w_i = 1/m_i$
3: **end for**
4: **loop**
5:     **for all** vertices i **do**
6:         $v_i \leftarrow v_i + \Delta t w_i f_{ext}(x_i)$
7:     **end for**
8:     dampVelocities($v_1, ..., v_N$)
9:     **for all** vertices i **do**
10:         $p_i \leftarrow x_i + \Delta t v_i$
11:     **end for**
12:     **for all** vertices i **do**
13:         generateCollisionConstraints($x_i \rightarrow p_i$)
14:     **end for**
15:     **while** $i \leq solverIterations$ **do**
16:         projectConstraints($C_1, ..., C_{M+M_{coll}}, p_1, ..., p_N$)
17:     **end while**
18:     **for all** vertices i **do**
19:         $v_i \leftarrow (p_i - x_i)/\Delta t$
20:         $x_i \leftarrow p_i$
21:     **end for**
22:     velocityUpdate($v_1, ..., v_N$)
23: **end loop**

The first key insight into the algorithm occurs in lines 9-11. Here positions for all vertices are estimated using the current velocity and the time step. This step is completely unrestricted and just predicts and estimates a position which is then later redefined. Line 15-17 illustrate this refinement. A Gauss-Seidel type iteration is used to satisfy all constraints defined for the system. These constraints mostly model the inherent structure of object currently simulated but another important aspect handled here are collisions with are just another constraint for the system and generated in each time step after estimating the position. The idea is to iterate over all constraints multiple times so that the particles are projected to valid locations with respect to the given constraint. The final vital piece of the algorithm can be seen in lines 18-21. After all constraints have been satisfied as best as possible the estimated positions alone are used to update the state of all vertices. The velocity of the particle is solely calculated by the difference of the new estimated position and the former position. This integration scheme is unconditionally stable as it doesn't just extrapolate into the future like traditional explicit schemes do. Instead it uses the physically valid positions computed with the constraint solver.

4.4 Oriented Particles

Oriented particles combines the geometrical based approach of shape matching with the idea of updating the velocities based on the positions alone as described in position based dynamics. The specifically improves upon shape matching in sparse regions increasing the overall stability of the shape matching. Modeling the particles as oriented ellipsoids can approximate the surface more accurately than simple sphere which is especially useful in calculating and resolving collisions.

4.4.1 Generalized Shape Matching

The main idea is to use oriented particles that in addition to the position and velocity also store rotation and spin. The particles now all have orientation information associated with them, thus the name of the method. The variables for each particle thus are now extended to encompass the following seven variables.

$$
\begin{aligned}
m_i&, \text{the mass}\\
\bar{x}_i&, \text{the rest position}\\
\bar{q}_i&, \text{the rest orientation}\\
x_i&, \text{the current position}\\
q_i&, \text{the current orientation}\\
v_i&, \text{the linear velocity}\\
\omega_i&, \text{the angular velocity}
\end{aligned}
$$

The advantages to include orientations for each particle become particular evident in sparse regions of the model like for example chains of particles or even single particles. The traditional approach of using spherical particles to model twisting chains would involve adding many particles to capture the twist. In order to model the same scenario with oriented particles only a single particle is required as the twisting is completely captured in the rotation and spin of the particle.

The basic approach of shape matching is directly applied to the generalized shape matching used for the oriented particle but differs in the way the moment matrix is calculated. The original minimization function 4.3 is slightly modified so it becomes the following moment matrix A, but is essentially still the same as in the original shape matching approach.

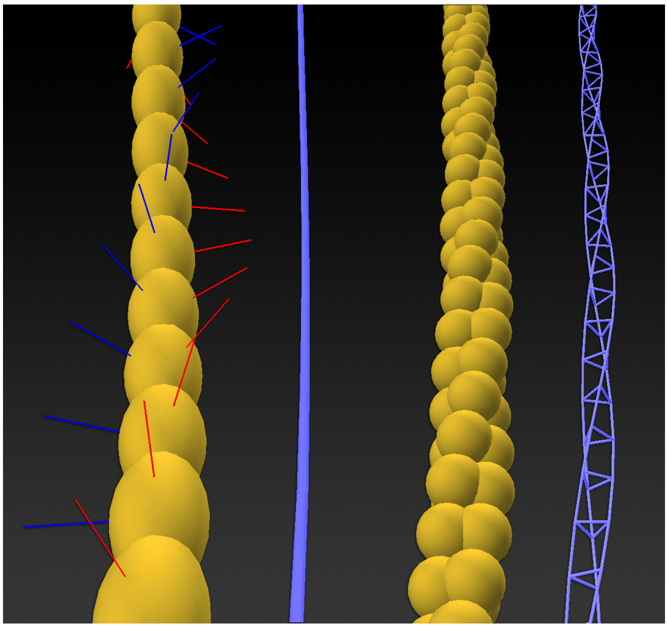


Figure 4.3: Oriented particle twisting chain

$$A = \sum_{i}^{n} m_i (x_i - c)(\bar{x}_i - \bar{c})^\top \in \mathbb{R}^{3x3} \tag{4.9}$$

This moment matrix A in terms of n particles each with a corresponding mass m_i and current position x_i. The center of mass for the current configuration is defined as c. Analog to that the rest position is defined as $\bar{x}_i$ and the rest center of mass as $\bar{c}$. Both center of mass are calculated using the traditional approach as seen in equation 4.6.

As in the original shape matching the goal is to find the optimal translation vector t and rotation matrix R, which map the original shape to the current configuration in a least squares manner. The optimal translation vector can simply be calculated from the differences of the two centers of mass.

$$t = c - \bar{c}$$

The rotation matrix R can be obtained by calculating the polar decomposition of A

$$A = R * S$$

Together with translation vector t and the extracted rotation matrix R the goal positions for each particle can be calculated.

$$g_i = R(\bar{x}_i - \bar{c}) + c$$

Thus far this is still exactly the same as in the original shape matching approach. Unfortunately the moment matrix A can become ill-conditioned are even singular. This happens if the particle are close to co-planar or as in the example of the twisting chain co-linear. If A is ill-conditioned the polar decomposition is not well defined and yields only unusable and unstable results. Thus no reliable R can be obtained. Adding the orientation information for each particle into the calculation can fix this problem.

The solution involves defining a well-conditioned moment matrix for even single particle and consequently adding each moment matrix together in order to arrive at a total moment matrix for the whole particle group. In order to arrive at this result lets begin with two groups of particles, each with their own unknown moment matrix A_1 and A_2 respectively. Both moment matrices are defined with respect to their own center of mass and thus can not simply be added to arrive at the combined moment matrix. As described in Rivers and James equation 4.10 can be reformulated to the following formula which shifts the center of mass to the global one.

$$A = \sum_{i}^{n} m_i x_i \bar{x}_i^\top - M c \bar{c}^\top \tag{4.10}$$

Here M is simply defined as the sum of all particle masses. Now the next step is to apply the same logic to each individual particle forming the group instead of two separate groups. This yields the following equation

$$A = \sum_{i}^{n} (A_i + m_i x_i \bar{x}_i^\top - m_i c \bar{c}^\top) \tag{4.11}$$

A_i denotes the moment matrix for a single particle in the group while the other variables retain their original meaning. This equation can be generalized and improved further.

$$A = \sum_{i}^{n} (A_i + m_i x_i \bar{x}_i^\top) - M c \bar{c}^\top \tag{4.12}$$

Integrating the equation 4.10 over the particle's volume yields the moment matrix for a single particle. Each particle is modeled as an ellipsoid so their moment matrix is defined in terms of their radii and rotation.

$$A_{\text{ellipsoid}} = \frac{1}{5} m \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} R \tag{4.13}$$

The radii of a single particle are simply defined by the length of the semi-axes $r = [a, b, c]^\top$ in all orthonormal directions. Including the orientation R for each single particle causes the total moment matrix A to always be full ranked and thus the polar decomposition to return more reliable results.

Having defined a stable moment matrix for each group the optimal translation vector and rotation matrix can be obtained and thus the goal positions can be calculated. Again analog to the original shape matching the particles are pulled towards their goal positions by a variable stiffness fraction. However the oriented particles way of integrating the position and velocities of the particles is completely different from the way the traditional shape matching method works. Instead the oriented particles are update and integrated by a system closely modeled after the position based dynamics approach but extended by integrating the orientations of the particles.

4.4.2 Generalized Position Based Dynamics

As in PBD the first step is the prediction step. Here a new predicted position x_p is calculated for each particle using a simple integration scheme. However for oriented particles the associated orientation q_p of each particle has also to be predicted over time. This prediction step using an explicit euler integration looks like the following.

$$x_p \leftarrow x + v\Delta t \tag{4.14}$$

$$q_p \leftarrow [\frac{\omega}{|\omega|} sin(\frac{|\omega|\Delta t}{2}), cos(\frac{|\omega|\Delta t}{2})]q \tag{4.15}$$

Now the solver has calculated a predicated state (x_p, q_p) for each single particle. This information is then based onto the second step. The second step is simply the adaptation and modification of the predicted state according to the rules laid out by the generalized shape matching approach outlined above. The last and final step integrating the state for each particle. In accordance with PBD but in difference to shape matching the integration step only takes in the particle's state in the beginning of the iteration and the adjusted predicted state. This way, like in PBD, of calulating the velocities solely based on the positions and orientations provides an unconditionally stable scheme as the adjusted predictions never overshoot. The integration step thus becomes the following equations.

$$v \leftarrow (x_p - x)/\Delta t \tag{4.16}$$

$$x \leftarrow x_p \tag{4.17}$$

$$\omega \leftarrow axis(q_p q^{-1}) \cdot angle(q_p q^- 1)/\Delta t \tag{4.18}$$

$$q \leftarrow q_p \tag{4.19}$$

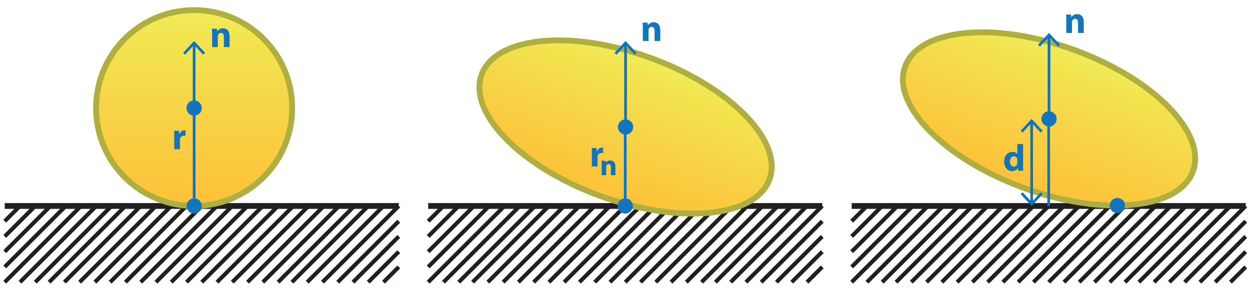
The function $axis(q)$ simply returns the normalized direction of the quaternion q and $angle(q)$ consequently the angle. One caveat to take note of is the observation that there are always two possible rotations $r = q_p q^{-1}$ to transform q into q_p. It is important to choose the smaller rotation of the two to update the angular velocity ω.

The standard simulation model for oriented particles uses an implicit shape matching. For implicit shape matching the particle belonging to an implicit shape matching group are defined by their edges. A group exists for every particle at the center and additional all the particles connected to this center particle by an edge. This results of this usually look like a tetrahedral mesh in more connected regions and thin particle chains in very sparse regions. However the topology of the particles can be completely arbitrary.

In practice a Gauss-Seidel type iteration scheme is used to satisfy all shape matching constraints defined by the implicit groups. This reduces the effect of the inherent bias towards the first groups that are evaluated during the execution. After updating the predicted state multiple times taking into account all groups the predicted position of each particle is moved towards the calculated goal position by a stiffness fraction. The orientation on the other hand is only updated for the respective center particle.

4.4.3 Collision Handling

Another improvement oriented particles provide is a more accurate collision handling. Each oriented particle is no longer a simple sphere but instead is modeled as an ellipsoid.



(a) Sphere collision (b) Incorrect ellipsoid collision (c) Correct ellipsoid collision

Ellipsoids can fit better to the surface area of a model than simplistic spheres. Using sphere might cause unnatural frictions or other visual artifacts. The question now remains how to calculate the correct spot on the surface of the ellipsoid that collided with a plane as can be seen in figure 4.4(c). The simplistic approach used in the spherical case can not be directly transferred as illustrated in figure 4.4(a) and 4.4(b). This would only work if the contact normal is aligned to the principal axis of the ellipsoid. However, in almost all cases this is not the case and the calculation becomes a little bit more involved.

Each potential particle is defined by its principal radii a, b, c and its rotation matrix R. The potential collision plane is simply defined in hessian form as $p = n^\top x = d$. The first step is to compute the contact point on the surface of the ellipsoid with a normal that is parallel to the normal of the plane n. In general the ellipsoid can be defined in terms of the following zero iso-surface.

$$c(x) = x^\top Ax - 1 \tag{4.20}$$

The potential contact point has to meet the following two constraints

$$\bigtriangledown c(x) = \lambda n \tag{4.21}$$
$$c(x) = 0 \tag{4.22}$$

In addition to these two constraints the following two formulas yield enough information to solve for λ and x, though only x is of real interest.

$$\bigtriangledown c(x) = 2Ax \tag{4.23}$$
$$A = R \begin{bmatrix} \frac{1}{a^2} & 0 & 0 \\ 0 & \frac{1}{b^2} & 0 \\ 0 & 0 & \frac{1}{c^2} \end{bmatrix} R^\top \Rightarrow A^{-1} = R \begin{bmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{bmatrix} R^\top \tag{4.24}$$

Thus λ and x can be calculated in the following manner.

$$\lambda = \pm \frac{2}{\sqrt{n^\top A^{-1} n}} \tag{4.25}$$

$$x = \pm \frac{A^{-1} n}{\sqrt{n^\top A^{-1} n}} \tag{4.26}$$

These calculations always lead to two possible positions x on the surface of ellipsoid with have a normal to the plane normal. The correct point to choose is always the point which is closer the plane, this can be easily calculated given the hessian form of the plane. The final step to determine if a collision between the current particle and the plane has occurred is to check if the contact point is below the plane and thus satisfies the condition $n^\top x < d$. Solving this condition also automatically returns the penetration depth of the collision. The depth can be directly utilized to modify the predicted position of particle. The collision detection and resolution step is done for all particles after the positions have been predicted but before the shape matching. Although the collision step should ideally be calculated for every iteration, using Gauss-Seidel, of the shape matching, this is way to expensive as the number of particles and planes can get quite large very quickly. However, just resolving the collisions once before shape matching seems to give satisfactory results.

The oriented particles paper describes a couple of additional features for the simulation with oriented particles. These include explicit shape matching groups defined manually, plastic deformation which cause the rest state to change, friction, torsion resistance, stretching and bending, collision handling between two ellipsoids and using the orientation information for more efficient skinning. However, these features are not needed to achieve the goals and requirements of this thesis and are thus not described here. More details can be found in the original paper.

5 Conceptual Approach

The simulation tries to combine a traditional rigid body simulation with the simulation of deformable bodies using oriented particles. The idea is to have an object which consists of both a rigid core and a surrounding soft layer. Both these layers will be simulated using their respective dynamics system. Both the interaction of such a combined body with the world as well as the interaction between the two layers is explained using the proposed technique.

5.1 Simulation Model

Every object in the simulation is either a combined body or a simple rigid body. The simulation model describes how these different bodies are modeled and constructed in order to enable the simulation and interaction between them. This requires both a model for rigid bodies as their used both alone as well as at the core of the combined body. Oriented particles are used for the deformable parts. Combining these two models into a unified combined body is the main contribution of this thesis.

5.1.1 Rigid Body

Simple rigid bodies are used to simulate objects which do not have or need a soft surrounding layer. Most prominently the object being picked up is for simplicity sake a simple rigid body. Also the ground doesn't need to be deformable and is thus simulated as a rigid body. These usually static rigid bodies are simulating using the traditional rigid body dynamics. They have all the usual properties of rigid bodies. The position, orientation, linear velocity and angular velocities are simulated over time. External forces, like gravity, are acting on the bodies and cause a change in velocity. The velocities are then updated in the collision resolution step using impulses. Finally the position and orientation are integrated and updated.

5.1.2 Oriented Particles

The oriented particles form the deformable tissue of every combined body. They are simulated exactly like the proposed by Mueller et al.. All particles have position and velocity. After updating the velocities shape matching is used to arrive at an optimal configuration of the deformed particles. The position of the particles are than moved towards this configuration. In the final step new velocities are calculated for each particle based solely on the new position of the particle.

5.1.3 Combined Body

Combined bodies are more complex. These bodies are a combination of a rigid inner core with a soft outer shell. The rigid inner body behaves exactly like a traditional rigid body or other non-combined bodies in the simulation. For each combined body a number of variables have to be maintained. First for the inner core the same variables as for any other rigid body are needed. These are the position of the center of mass, the orientation, the linear velocity and the angular velocity. The particles building up the soft tissue have the same variables as original oriented particles, meaning rest position, current position, rest orientation, orientation, linear velocity and angular velocity.

There are two general types of forces acting on the combined body. The first are external force, most prominently gravity. The second are internal forces occurring at the interface between the inner core and the outer tissue. External force are only applied to the inner rigid body, while the resulting changes in the tissue are simply the result of the inner force propagation. How exactly force propagation is implemented will be discussed in section 5.1.5. Another source of changes of the state of the body are collisions which will be described in detail in section 5.1.4.

The particles of the deformable tissue are divided into two types. The first type is called attached particle. Attached particles are evenly distributed across the surface of the inner core. They provide the interface between the outer tissue and the inner bone. These particles are glued to the rigid body and provide the means to have force propagation from the particles to the core if the tissue or inner core are moved relatively to each other.

The second type of particles are simply called outer particles. The outer particles are used to model the shape and structure of the surrounding tissue. They can be positioned and arranged in any desirable way. The simulation places to limitations on the shape of the outer tissue. The only obvious requirement is that the outer particles are well connected to the layer of the attached particles on the surface of the rigid core, as otherwise the tissue could simply separated from the bone.

All particles, regardless if attached or outer particle, are simulated according to the approach described in the oriented particles paper, no modifications are needed. This also holds true for the shape matching constraints, which are defined implicitly. However the simulation again doesn't place any limitations on the way connections are modeled.

5.1.4 Collision Handling

There are three different types of collisions handled by the current simulation model.

1. Rigid Body ⟷ Rigid Body
2. Combined Body ⟷ Rigid Body
3. Outer Tissue ⟷ Inner Bone

The first type of collision between two rigid bodies is handled in the traditional way. For each pair of rigid bodies a set of contact points is calculated if they are colliding. Taking into account the velocities of both bodies at each contact point and the resulting relative velocity the collision is resolved using impulses. The impulses cause a immediate change in velocity for each body. Both the resulting normal force separating the two bodies and the friction force are taken into account.

The second type of collision occurs between a combined body and a rigid body. Actually this type of collision is divided into many subcollision checks comparing the particles with the planes of the rigid body. The subcollisions are only evaluated with regard to the outer particles. This is motivated by the assumption that the tissue can be compressed to such a degree that the attached particle could ever come into contact with the outer world. For each outer particle the contact points are calculated, taking advantage of the ellipsoid shape of the particles. This happens in exactly the same way as described in the original oriented particles paper, with the addition to have to compare every outer particle with every face of the rigid body.

However, the collision resolution can not directly be applied to the rigid body. The simulation model proposes a novel hybrid approach, combining the position based approach for particles with an impulse based approach for the rigid body. The collision response for the particle is exactly the same as for the general case. The penetration depth of the particle is calculated and the predicted position of the particle is simply moved out of the rigid body along the contact normal.

The effect on the rigid body on the other hand is calculated using the traditional rigid body simulation. The exact same algorithm used for rigid and rigid collisions is employed. The colliding particle is simply interpreted as a rigid body and fed to the exact same algorithm used for rigid-to-rigid body collisions. The resulting force is therefore both calculated for normal direction as well as for friction. However, these forces are only applied to the rigid body and not to the particle. This approach of course is not physically correct, but yields satisfactory and plausible results.

The third type of collisions between the outer soft tissue and the inner rigid bone of each combined body is modeled in a similar way to the approach taking in Position Based Dynamics. This approach and the application to the combined body is discussed in section 5.1.5. Another theoretical possible collision type not mentioned thus far is between two combined bodies which comes down to collisions between particles. This type of collision is not modeled in the current simulation model, as it lies outside the requirements.

5.1.5 Force Propagation

Force propagation defines and models the forces occurring between the outer soft tissue and inner rigid bone. During the simulation the surrounding tissue and the inner core will move relative to each other, which will have to be corrected as otherwise the two parts would simply separate. The approach is modeled in a similar manner to the way Position Based Dynamics models the collision of particles with rigid bodies. The impulse used in PBD is calculated using the following formula.

$$J = m_i \Delta p_i / \Delta t \tag{5.1}$$

This impulse is applied to the rigid body at the contact point in case there was a collision. However, for the inner force of combined body the problem is not that simple. In case of the combined body only the attached particles have to be evaluated, again under the assumption that the tissue can not be deformed enough to cause the outer particles to collide with the inner core. After all shape matching iterations have finished the attached particles have a final predicted position. However, this position is most likely not on the surface of inner rigid body. They either end up inside the core or further away. The case of being inside is exactly the same as the collision case described in PBD which results in a push force on the rigid body. The case of settling away from the surface, however should also result in a force, more specifically the core should be pulled towards the attached particle. In order to calculate the correct impulse for both cases the attached particles are realigned to their position on the surface which causes a response in the rigid body. Calculating the correct impulse first requires Δp_i, which can be obtained by comparing the current predicted position x_p with the rest position x_r on the surface of core relative to the core's current transformation in world space T.

$$x_r = T\bar{x}_r \tag{5.2}$$

$$\Delta p_i = x_p - x_r \tag{5.3}$$

Here $\bar{x}_r$ denotes the relative position of the particle relative to the resting transform of the cube. The resulting impulse is than directly applied to the x_r on the body. In the final step the predicted position of all attached particles is forcefully set to x_r, so that they appear glued to the surface of the inner rigid core.

5.2 Requirements

In chapter 2 a number of main requirements were defined. All of these requirements can now be compared and validated against the proposed simulation model, as described in chapter 5.

1. Realtime Simulation

As arguably the most important requirement, the real time aspect of the overall simulation is directly dependent on the performance of the individual subsimulations. The system includes a complete, although simplistic, rigid body simulation. Rigid body simulations are relatively straightforward and have been around for a long time. A huge number of bodies can be simulated reliable and in real time. So this subcomponent meets the requirement implicitly. The other subcomponent is a direct application of the concepts introduced by the oriented particles paper. The goal of this paper was also the real time applicability of the deformable simulation. This important characteristic was proven by the authors quite successfully. So another component passes the requirement implicitly.

The two major additions are the collision resolution between outer particles and rigid bodies and the force propagation inside the combined body. The force propagation is of linear complexity and has only to be calculated for a rather limited number of attached particles. Thus it doesn't have any significant impact on the real time requirement.

The collision resolution between concerning the outer particles is the most computationally intensive component. Each particle has to be compared against every face of the rigid body. Fortunately this can basically be reduced to a collision detection between to rigid bodies with all possible optimizations. The calculations are in fact even easier than that because the particles are perfect ellipsoids with reduces the required comparisons significantly.

All components contributing to the overall simulation are capable of real time. Thus the overall simulation will run in realtime and the requirement is fulfilled.

2. Low Number of Bodies

The first step in reducing the number of bodies in the simulation was already taken by the oriented particles approach, which allows to simulate fewer particles overall and especially in sparse regions. By using the rigid core for the combined body the number of particles can be further reduced as the internal structure no longer has to be modeled using particles. In the practice these inner particles might not even be deformed at all as the external force are not large enough and are thus needlessly consume resources. Using the soft surrounding layer enables the tissue to wrap around the other object. This increases the contact area between the two bodies and reduces the need to model the same scenario with multiple rigid bodies connected by joints.

3. Only Approximate Orientation of Finger Relative to Object

By enabling the soft tissue to wrap and deform around other objects the contact area between the bodies is increased which allows for more stable simulations. Using rigid bodies for the same tasks can result in numerical instabilities because of the way collisions are detected and resolved. This is especially for completely non trivial shapes where because of the rasterization the perfect solutions can not always be found. The deformable tissue solves this as it can smooth out these instabilities over time.

4. Force Propagation Inside Body Between Tissue and Bone

The force propagation inside the body is explicitly modeled and described in section 5.1.5. Permanently attached particles on the surface of the bone propagate the forces resulting from changes of the surrounding tissue. Thus the tissue is always firmly glued to the bone and nothing can drift apart.

5. Friction Between Finger and Object

The ways force are handled between oriented particles and rigid bodies is explicitly modeled and describe in section 5.1.4. Friction is only acting on the rigid body but not on the particles. Although this is not at all physically it behaves plausible and believable.

6 Solution Details

- Implementation combines rigid body simulation and oriented particles
- Using bullet library for the managing the rigid body state
- Includes forces, integrating velocities, and predicting unconstrained motion
- Bullet is also used for applying impulses generated by the tissue and finally updating the rigid bodies state
- Oriented particles implementation is completely custom

6.1 Setup

- Settings up the environment
- In total four bodies are simulated
- ground, 2 fingers, cylinder
- ground is a simple static rigid body and does not move are react to force during the simulation
- fingers and cylinder are all combined bodies
- Setting up a combined body
- Each inner rigid body has a arbitrarily chosen mass
- The underlying geometric structure is used to evenly distribute attached particles across its surface
- The density of the attached particles can be controlled by a configurable $density$parameter
- For each attached particle on the surface a corresponding outer particle is generated
- The outer particle is positioned along the same vector as its corresponding attached particle relative to the center of mass of the rigid inner core
- The distance between attached and outer particle can be controlled by a configurable $extrude$ parameter
- After all the particles have been added to the surrounding tissue the implicit shape matching groups are generated
- The size of each group is a configurable $groupSize$ parameter

- For each particle the $n = groupSize - 1$ closest particles using simple euclidean distance are added to an implicit group with this particle at the center
- In addition to the closest particles the corresponding attached or outer particle is also added to the group
- Thus the group size for each particle becomes exactly $groupSize$

6.2 Loop

- For all active bodies gravity is applied and the velocities updated
- After that an unconstrained motion is predicted
- Both the rigid body is temporarily moved to a new position as well as all the particles belonging to the body
- This predicts both the positions and orientations
- Predicted position of particles are adjusted in case of potential collisions
- Gauss-Seidel type iterations are performed a configurable number of times in order to resolve the shape matching constraints
- After arriving at the final shape matched configuration of predicted position and orientation for the particles, the error between the fixed position of attached particle and their predicted position is evaluated
- For each error an impulse is applied to the predicted transform of the rigid body
- After applying the impulses the predicted position and orientation of the attached particles is forcefully set to their fixed position and orientation relative to position of the rigid body
- In the final step the the position/orientation of the rigid body and all particles is updated to their new values

7 Evaluation

- Describing scenario
- Two fingers try to pick up a cylinder of the floor
- Pressure onto the cylinder results in friction and the ability to pick up the cylinder
- Comparing traditional simple rigid body simulation with new combined body approach
- Simulation using three simple blocks of rigid bodies
- Simulation fails because the cylinder is pushed out of the two fingers
- Simulation using the proposed combined bodies
- Tissue wraps around are cylinder providing a larger contact area
- The friction at contact area enables fingers to pick up cylinder

8 Conclusions

- friction on oriented particles
- collision between oriented particles
- extending structure of the tissue