PHYS14X Notes

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Contents

1 Course Structure

Before each lecture:
• Do the assigned reading
• Complete the lecture reading discussion
• Make notes about questions you have

During Lecture:
• Professor will assume I have done reading and covered selected points only
• There will be frequent "Clicker questions" via LearningCatalytics
• Ask questions in the Chat when I am uncertain/confused/curious

After Lecture:
• Notes lecture recordings will be posted
• Don't rely on these as a substitute for attending class

Course Overview:
• Lectures, labs, and tutorials all require *interaction*
• All components involve constructing and applying *models*
• All components are mutually reinforcing, but:
- Note every topic is covered in every component
- Labs generally emphasize observations, tutorials emphasize concepts, homework emphasizes applying models
• Homework is not just for practice
• Usually a topic is introduced in lecture first, but there are exceptions

Difference between honors and normal physics:
• Honors has live lectures, normal is prerecorded
• The topics:
- Cover some material more quickly (e.g., motion in 1D)
- Cover some topics in much more depth (e.g., special relativity)
- Deviate more from the textbook
• The grade distribution
• The students in honors physics have a stronger physics background and are more likely to consider a physics major/minor
• Students in honors are more inquisitive, like to talk about important ideas, enjoy friendly competition, are great at collaborating

Major Differences with physics courses you may have taken before:
• Everything is in 1D first (e.g. no projectile motion until Chapter 10)
• Conservation laws are emphasized (Momentum and energy are covered before forces and Newton's laws)
• Inductive reasoning is emphasized (Figuring out how to explain observations, not just figuring out how to apply known laws)
• Every chapter covers concepts first, then quantitative tools

In-class quizzes aka "clicker" questions:
• Questions asked a few times each class
• Change your answer as often as you want
• Discuss in the zoom chat if you want
• 1 point for any answer (correct or not)
• If you get 80% of the possible points, you get a perfect class participation score

2 Foundations

2.1 Representations

Visual representations are a vital part of understanding a problem and developing a model. Reprsentations may include pictures, sketches, diagrams, graphs, and a multitude of context-specific procedures. To avoid cluttering a reprsentation, it is best to oversimplify a real-life situation and gradually construct less idealized models. It our oversimplified model reproduces the main features of its real-world counterpart, then we know we have chosen adequate essential attributes.

In this respect, mathematical symbols also act as representations of more complex phrases. Take, for example, the statement below.

> "The magnitude of the acceleration of an object is directly proportional to the magnitude of the vector sum of the forces exerted on the object and inversely proportional to the object's inertia. The direction of the acceleration is the same as the direction of the vector sum of the forces."

This can be expressed concisely and more clearly with the mathematic equation,

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

2.2 Physical quantities and units

Physical quantities and their symbols:

Physical Quantity	Symbol
length	l
time	t
mass	m
speed	v
volume	V
energy	E
temperature	T

Physical quantities are expressed as the product of a number and a unit of measurement. For example, the length l of an object that is $1.2m$ long can be expressed as $l = 1.2m$. The global unit system used in science and engineering is the **Systéme International (SI)**. There are seven base units in the SI system from which all other units can be derived.

The Seven SI Base Units:

Name of Unit	Abbreviation	Physical Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	thermodynamic temperature
mole	mol	amount of substance
candela	cd	luminous intensity

To work conveniently with very large or very small numbers, we modify the unit name with prefixes representing integer powers of ten, conventionally powers of ten that are multiples of 3. For example, a billionth of a second is denoted by 1 ns and pronounced "one nanosecond", where 1 ns = 10^{-9}s.

SI Prefixes:

10^n	Prefix	Abbreviation
10^{24}	yotta-	Y
10^{21}	zetta-	Z
10^{18}	exa-	E
10^{15}	peta-	P
10^{12}	tera-	T
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	k
10^{0}	–	–
10^{-3}	milli-	m
10^{-6}	micro-	μ
10^{-9}	nano-	n
10^{-12}	pico-	p
10^{-15}	femto-	f
10^{-18}	atto-	a
10^{-21}	zepto-	z
10^{-24}	yocto-	y

A **mole** is currently defined as the number of atoms in 12×10^{-3} kg of carbon-12. This number is referred to as **Avogrado's number (N_A)**, where the currently accepted measurement of Avogradro's number is

$$N_A = 6.0221413 \times 10^{23}$$

Density is the physical quantity measuring how much of some substance exists in a given volume. **Number density** is the number of objects per unit volume. If there are N objects in a volume V, then the number density n of these objects is

Number Density

$n = \frac{N}{V}$

Mass density (ρ) is the amount of mass m per unit volume:

Mass Density

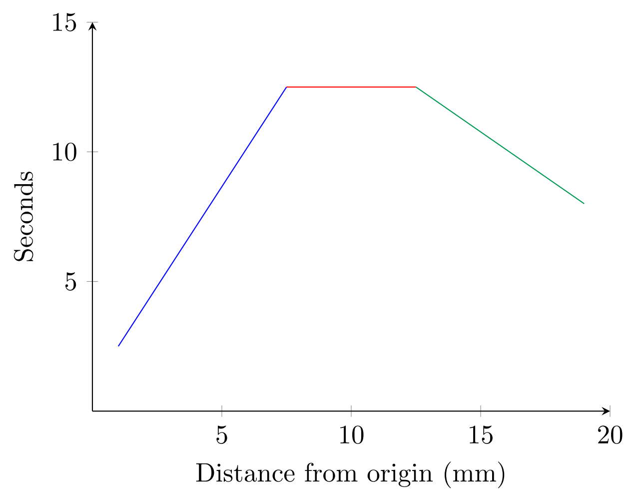
$\rho = \frac{m}{V}$

The easiest way to convert measurements to other units is to write the **conversion factor** as a fraction. Then multiply what you're to express by the conversion ratio, cancelling out the units. Below is an example of converting 4.5 inches to millimeters:

$$4.5 \text{ in.} \;= (4.5 \cancel{\text{ in.}}) \left(\frac{25.4 \text{ mm}}{1 \cancel{\text{ in.}}}\right) = 4.5 \times 25.4 \text{ mm} \;= 1.1 \times 10^2 \text{ mm}$$

2.3 From Reality to Model

If the position of an object is not changing, it is said to be **at rest**. A **Position-Time Graph** describes the position of an object as a function of a unit of time. Below is a position-time graph for walking.

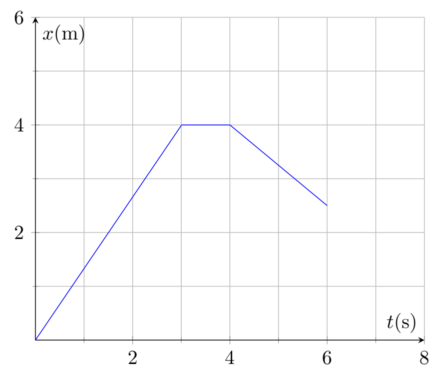


Here, the blue section of the graph describes when the person is walking forwards, the red section is when the person is pausing, and the green section is when the person is walking backwards.

3 Motion in One Dimension and Changes in Velocity

3.1 Position and Displacement

Position-versus-time graph: a graph that represents position (x) as a function of time (t).



Here, the object moves forward at a constant velocity between $t = 0$ and $t = 3$ as demonstrated by the even slope of the curve when $0 \leq t \leq 3$. Then the object is at rest between $t = 3$ and $t = 4$, hence the curve is a flat line in that interval. Lastly, the object turns around and moves backward towards the origin, moving at a constant velocity again demonstrated by the even slope of the curve when $4 \leq t \leq 6$.

The **displacement** of an object is the vector change in position. The x **component of displacement** is given by

$$\Delta x = x_2 - x_1$$

The particular wording of "x component" reminds us that the x component of displacement is measured along some specific x-axis. The x component of displacement can be *either* positive or negative. It is positive for displacements in the direction of increasing x and negative for displacements in the direction of decreasing x. Whereas *distance traveled* refers to the distance covered by a moving object along the path of its motion, *displacement* describes a vector quantity with no reference position or axis. Only when motion is always in the same direction is the distance traveled equal to the distance between the initial and final positions.

3.2 Representing Motion

A curve in a position-versus-time graph is called an $\boldsymbol{x(t)}$ **curve** because it can be represented by a mathematical function $x(t)$.

3.3 Average Speed and Average Velocity

The slope of an $x(t)$ curve is the speed of the object at that time interval. Hence, in a position-versus-time graph, *the steeper the slope, the higher the speed.*

$$s_{avg} = \frac{\Delta x}{\Delta t}$$

Velocity is a vector quantity expressing both speed and the direction of travel, where

$$x \text{ component of object's } v_{avg} = \frac{x \text{ component of its displacement}}{\text{time interval for that displacement}}$$

$$\Delta x \text{ of object's } v_{avg} = \frac{\Delta x \text{ of displacement}}{\Delta t \text{ for displacement}}$$

A **velocity-versus-time graph** represents velocity (v) as a function of time (t). A curve plotted in such a graph is called a $\boldsymbol{v(t)}$ curve. We denote the x component of an object's average velocity by v_x.

3.4 Scalars and Vectors

Scalars are physical quantities specified by a positive or negative magnitude and a unit of measurement.
• e.g. temperature, volume, density, speed, energy, mass, time
• written as a lowercase letter

Vectors are physical quantities specified by a positive or negative magnitude, a unit of measurement, and a direction in space.
• e.g. force, velocity, acceleration, displacement, momentum

• written as a lowercase letter with a right arrow over it, e.g. $\overrightarrow{v}$
The **magnitude of a vector** is denoted by

$$|\overrightarrow{b}| \text{ or } b$$

To describe vectors mathematically, we use a **unit vector**, a vector whose sole purpose is to define a direction in space. Unit vectors do not have units and they have a magnitude of 1. Hence,

$$|\hat{\imath}| = 1$$

Unit vectors are denoted by a caret/hat ($\hat{}$) to distinguish them from normal vectors. Unit vectors pointing in the direction of increasing x along the x-axis are denoted by $\hat{\imath}$ (pronounced *i-hat*). **Unit vector notation** expresses a vector along the x-axis as the product of a scalar and the unit vector:

$$\overrightarrow{b} = b_x \hat{\imath} \text{ (one dimension)}$$

where b_x is a scalar known as the x component of $\overrightarrow{b}$. If we take the absolute value of both sides of the above equation, we find that, for vectors in one dimension, the magnitude of the vector $\overrightarrow{b}$ equals the magitude of b_x:

$$\begin{aligned} \overrightarrow{b} &= b_x \hat{\imath} \\ |\overrightarrow{b}| &= |b_x \hat{\imath}| \\ &= |b_x||\hat{\imath}| \\ &= |b_x| \end{aligned}$$

If $b_x > 0$ then $\overrightarrow{b}$ is in the same direction as $\hat{\imath}$. If $b_x < 0$ then $\overrightarrow{b}$ is in the opposite direction to $\hat{\imath}$.

3.5 Position and Displacement Vectors

The most basic vector is displacement ($\Delta\overrightarrow{r}$). Recall that the x component of displacement is equal to the change in the x coordinate such that

$$\Delta r_x = \Delta x = x_f - x_i$$

The distance d between two points is given by

$$d = |x_1 - x_2| \text{ (one dimension)}$$

A vector that has zero magnitude is equal to the **zero vector**, denoted by a zero with an arrow ($\overrightarrow{0}$). In diagrams, the zero vector is represented by a dot. Rearranging the definition for Δx, we get

$$x_f = x_i + \Delta x$$

An object's position can also be represented by a vector:

$$\Delta x \hat{\imath} = (x_f - x_i)\hat{\imath} = x_f \hat{\imath} - x_i \hat{\imath}$$

where the left side of the equation represents the displacement

$$\Delta x \hat{\imath} = \Delta r_x \hat{\imath} = \Delta \overrightarrow{r}$$

The vector $x_i \hat{\imath}$ represents the **position vector**, a vector that lets us determine the position of a point in space relative to some chosen origin. Position is also represented by $\overrightarrow{r}$:

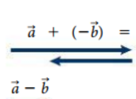
$$\overrightarrow{r} = x\hat{\imath} \text{ one dimension}$$

Hence, we can see that the x component of position is equal to the coordinate such that

$$r_x = x$$

To subtract a vector from another vector, we reverse the direction of the vector being subtracted and add the reversed vector to the other vector:

The vector $\overrightarrow{a} - \overrightarrow{b}$ can be graphically represented like so:



Scalar multiplication of a vector:

$$c\overrightarrow{a} = c\left(a_x \hat{\imath}\right) = (ca_x)\,\hat{\imath}$$

3.6 Velocity as a Vector

The x component of the average velocity is given by

$$v_{x,av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

We can multiply the first and middle terms of the above equation by the unit vector i and substitute $\Delta \overrightarrow{r} = \Delta x \hat{i}$ to get

$$\overrightarrow{v_{av}} = v_{x,av}\hat{i} = \frac{\Delta \overrightarrow{r}}{\Delta t}$$

Because Δt is always positive, we see that for one dimensional motion, $\overrightarrow{v_{av}}$ always points in the same direction as displacement.

3.7 Motion at Constant Velocity

By convention, the phrase "average" is dropped from references to constant velocity. Thus, the x component of the average velocity is simply v_x and we see that the slope of the $x(t)$ curve is numerically equal to the x component of the velocity:

$$\frac{\Delta x}{\Delta t} = v_x \text{ constant velocity}$$

The x component of the displacement during any time interval is given by the area under the $v_x(t)$ curve between the beginning and the end of the time interval. We can use the fact that $\Delta x = x_f - x_i$ to rewrite the above equation in terms of x_f such that

$$x_f = x_i + v_x \Delta t \text{ constant velocity}$$

3.8 Instantaneous Velocity

Instantaneous Velocity: the velocity of an object at any given instant. The x component of the velocity at instant t is given by

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The velocity is then

$$\overrightarrow{v} = v_x \hat{i} = \left(\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right) \hat{i}$$

Because the unit vector is constant, we can rewrite the velocity as

$$\overrightarrow{v} = \lim_{\Delta t \to 0} \frac{\Delta x \hat{i}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \overrightarrow{r}}{\Delta t} = \frac{d\overrightarrow{r}}{dt}$$

Hence, speed is given by the magnitude of this vector:

$$v = |\overrightarrow{v}|$$

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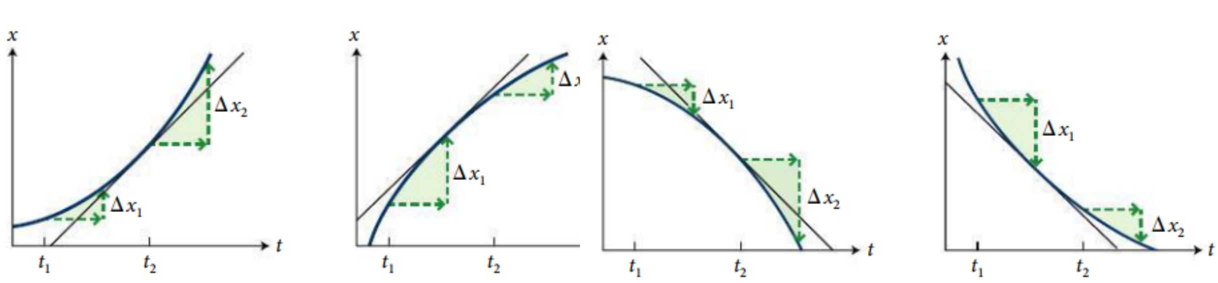
3.9 Changes in Velocity

Acceleration: the rate of change of velocity with respect to time. The **x component of average acceleration** is the change in the x component of the velocity divided by the time interval during which this change took place. In physics, the word "*deceleration*" has no meaning is better avoided, as *acceleration* refers to *any* change in velocity, regardless of increasing or decreasing speed.

When an object's velocity and acceleration is in the same direction, the object speeds up. When they are in opposite directions, the object slows down. It is important to note confuse the positive x component of acceleration with speeding up when working with objects traveling in the negative x direction. An object speeds up when the magnitude of its velocity (aka speed) increases, regardless of the direction of motion. Whether or not the x component of an object's acceleration is positive, however, dependson hte direction in which the object is moving.

Position-versus-time graphs for objects accelerating along the positive x axis:



The curvature of an $x(t)$ curve is a measure of the x component of acceleration. An upward curvature corresponds to a positive x component of acceleration, whereas a downward curvature corresponds to a negative x component of acceleration.

3.10 Acceleration Due to Gravity

In a vacuum, a feather and a stone drop at the same rate. Yet in air, the stone drops faster. This result has been attributed to air resistance pushing harder on the feather than the stone. **Free fall** is the motion of objects moving solely under the influence of gravity. A falling object inside a vacuum is in free fall because it is influenced only by gravity, but an object falling in the air is not in free fall because its motion is also affected by air resistance. However, the effect of air resistance on a stone outside of a vacuum is negligible. This property is true for many objects that are not too light and not dropped from too great of a height. Hence, if an object is not extremely light or dropped from a substantive height, we can ignore air resistance and consider all dropped objects to be in free fall.

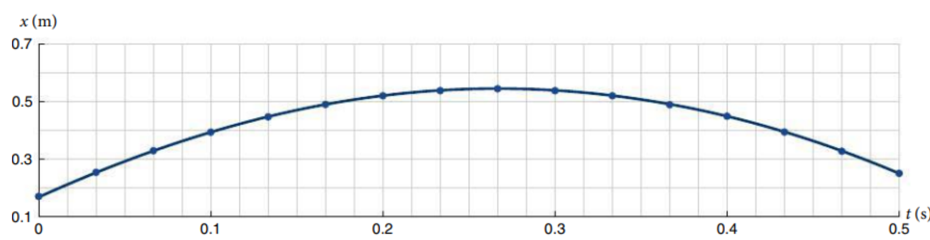
In the absence of air resistance, the **magnitude of the acceleration of all objects due to gravity** is **9.8** m/s^2 and consequently the amount of *time it takes to fall from a certain height is the same for all falling objects.*

3.11 Projectile Motion

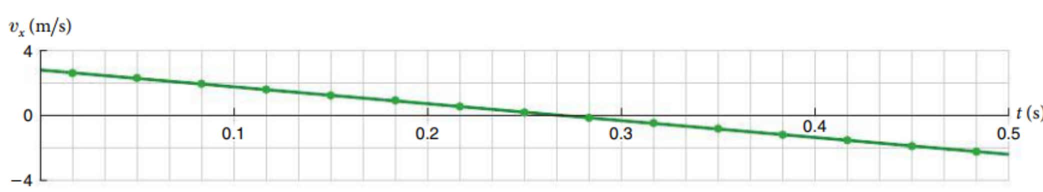
An object that is launched but not self-propelled is a **projectile**, and its motion is called **projectile motion**. The path a projectile follows is called its **trajectory**. The motion of an object in free fall is a special case of projectile motion.

Consider a ball launched straight up. The ball slows down as it moves upward and hence its acceleration and velocity point in opposite directions. The ball's velocity changes **linearly**, changing by equal amounts in equal time intervals, thus its acceleration is constant. In the position-versus-time and velocity-versus-time graphs below, it can be seen that the slope of the $v_x(t)$ curve is about -10 m/s^2, which is the acceleration due to gravity. Thus, both on the way up and way down, the acceleration of a projectile is equal *both in magnitude and in direction* to that of a freely falling object.

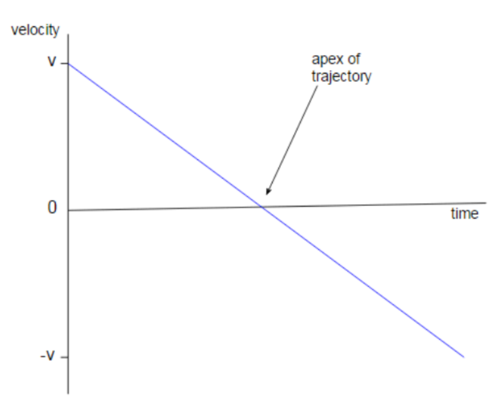
$x(t)$ curve for a ball launched straight up:



$v_x(t)$ curve for a ball launched straight up:



For an object launched upwards, the launch affects only the initial velocity. Once the object is released, the rest of its motion is determined by gravity alone (free fall). At the top of a trajectory of a ball launched upwards, there is instant at which the x component of the velocity is zero. *The x component of the acceleration is not zero at the top.* To conceptualize this, consider the graph below.



Though velocity is zero at the apex of the trajectory, acceleration is defined as

$$a = \frac{dv}{dt}$$

The gradient of the line in the graph is nonzero and happens to be -9.81 m/s^2. Hence, the magntiude of the acceleration does not depend on v and is equal to 9.81 m/s^2 for all values of v.

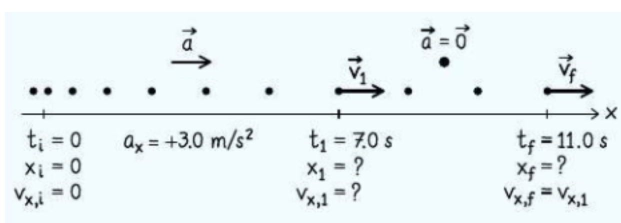
3.12 Motion Diagrams

Motion diagrams represent the positions of a moving object at equally spaced time intervals. We can make a motion diagrams by the following steps:

1. Use dots to represent the moving object at even time intervals. If the object moves at constant speed then the dots are evenly spaced. Conversely, if the object speeds up, the spacing between the dots increases. If the object slows down, the spacing decreases.
2. Choose an x (position) axis convenient for the problem. This is usually an axis that has its origin at the initial or final position of the object and is oriented in the direction of motion or acceleration.
3. Specify the position and velocity at all relevant interests, particularly, the **initial conditions**— position and velocity at the beginning of the time interval of interest, and the **final positions**— position and velocity at the end of that time interval. Additionally, specify all positions where the velocity reverses direction or the acceleration changes. Label any unknown parameters with a question mark.
4. Indicate the acceleration of the object between all instants specified in step 3.
5. To consider the motion of more than one object, draw separate diagrams side by side, one for each object, using one common x-axis.
6. If the object reverses direction, separate the motion diagram into two parts, one for each direction of travel.

An acronym for a useful checklist of a motion diagram is "**VENUS**":
- **V**ectors and scalars used appropriately
- **E**verything answered
- **N**o unknowns left
- **U**nits correct
- **S**ignificant digits justified

A Motion Diagram:



3.13 Summary of Symbols and Meanings in This Topic

Symbol	Meaning
t	clock reading = instant in time
$\Delta t = t_f - t_i$	change in clock reading = interval of time
x	x-coordinate
$d = \lvert x_1 - x_2 \rvert$	straight-line distance between two points (never has sign)
$\hat{i}$	unit vector defining direction of x-axis (vector magnitude 1)
$\vec{b} = v_x\hat{i}$	vector and unit vector notation
b_x	x component of vector $\vec{b}$ (can be positive or negative)
$b = \lvert \vec{b} \rvert = \lvert b_x \rvert$	magnitude of vector $\vec{b}$ (always positive)
$g = \lvert \vec{a}_{\text{free fall}} \rvert = 9.8$ m/s^2	acceleration due to gravity near Earth's surface

Vector	Meaning	Component	Meaning
$\Delta\vec{r} = \Delta r_x\hat{i}$	displacement	$\Delta r_x = \Delta x$	x component of displacement = change in x-coordinate
$\vec{r} = r_x\hat{i}$	position	$r_x = x$	x component of position = x-coordinate
$\vec{v_{av}} = v_{x,av}\hat{i}$	average velocity	$v_{x,av} = \frac{\Delta x}{\Delta t}$	x component of average velocity
$\vec{v} = v_x\hat{i}$	(instantaneous) velocity	$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	x component of (instantaneous) velocity
$\vec{a} = a_x\hat{i}$	acceleration	$a_x = \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	x component of acceleration

Average Velocity: The constant velocity required to achieve the same displacement in the same time interval as the actual motion.

$$v_{avg} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)dt$$

Note that this is just the average value of a function integral.

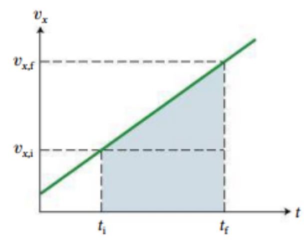
4 Constant Acceleration

4.1 Motion with Constant Acceleration

For an object moving with constant acceleration, the $v_x(t)$ curve is a straight line. We know Δv_x and $v_{x,f}$ to be

$$\Delta v_x = v_{x,f} - v_{x,i} = a_x \Delta t \qquad \text{(constant acceleration)}$$
$$v_{x,f} = v_{x,i} + a_x \Delta t \qquad \text{(constant acceleration)}$$

We also know displacement to be the area under a velocity curve:



Hence, the displacement is given by the area of the triangle and the area of the rectangle. The area of the triangle is

$$\frac{1}{2}(v_{x,f} - v_{x,i})(t_f - t_i) = \frac{1}{2}\Delta v_x \Delta t = \frac{1}{2}a_x(\Delta t)^2$$

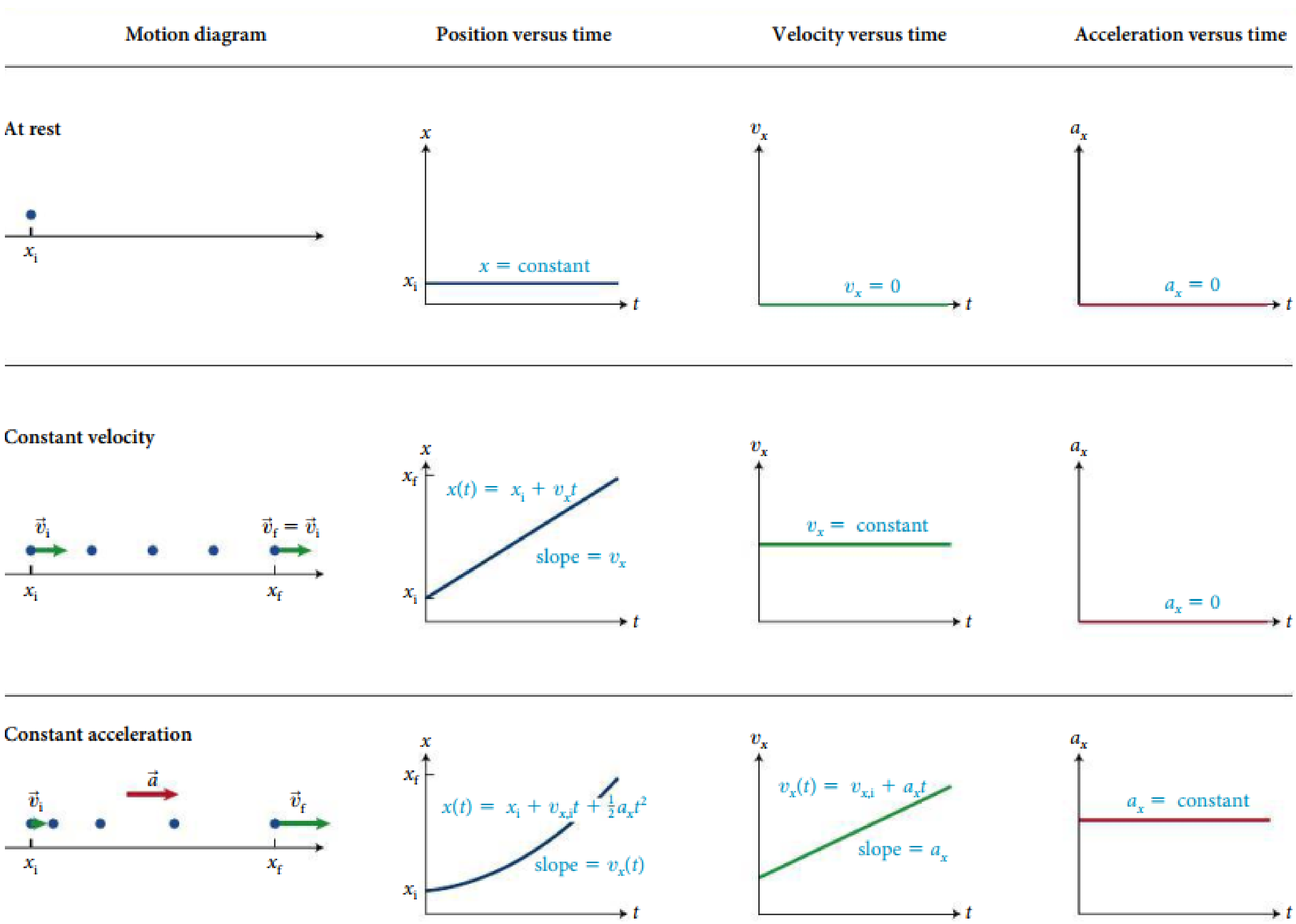
and the area of the rectangle is

$$(v_{x,i} - 0(t_f - t_i) = v_{x,i}\Delta t$$

Thus, the combined displacement is

$$x_f - x_i = v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \text{ (constant acceleration)}$$

Below are **kinematics graphs for the three basic types of motion:**



4.2 Free-fall Equations

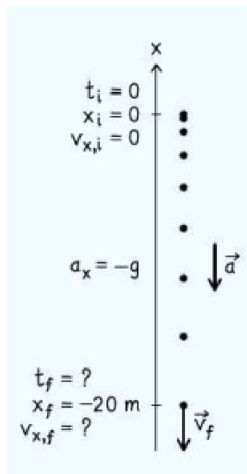
Magnitude of **acceleration due to gravity (g)** is given by

$$g = |\overrightarrow{a}_{\text{free fall}}|$$

If an object is dropped from a certain height with zero initial velocity along an upward-pointing x axis, then,

$$x_f = x_i - \frac{1}{2}gt_f^2$$
$$v_{x,f} = -gt_f$$

Example of a Free-fall Diagram:



4.3 Inclined Planes

When a ball rolls down an incline starting from rest, the ratio of the distance traveled to the square of the amount of time needed to travel that distance is constant. For example, if a ball released from position $x_i = 0$ at $t_1 = 0$ reaches position x_1 at instant t_1, x_2 at t_2, and x_3 at t_3, then

$$\frac{x_1}{t_1^2} = \frac{x_2}{t_2^2} = \frac{x_3}{t_3^2}$$

Recall that for an object moving with constant acceleration and starting from rest, we have

$$x_f = \frac{1}{2}a_x t_f^2$$

Hence,

$$\frac{x_f}{t_f^2} = \frac{1}{2}a_x$$

As incline increases, so does the ratio $\frac{x_0}{t_0^2}$. Through experimentation, we can determine the component of acceleration along the incline to be related to g by the equation

$$a_x = g\sin\theta$$

4.4 Instantaneous Acceleration

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

Hence,

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

We can then express changes in velocity and position as:

$$\Delta v_x = \int_{t_1}^{t_f} a_x(t)dt$$
$$\Delta x = \int^{t_f} t_i v_x(t)dt$$

4.5 Summary of Topic

Free fall: the motion of an object subject to only the influence of gravity
Motion diagram: a diagram that shows the position of a moving object at equally spaced time intervals and summarizes what is known about the object's initial and final conditions (its position, velocity, and acceleration)
Projectile motion: the motion of an object that is luanched but not self-propelled (*projectile*)
• The launch only affects the object's initial velocity
• Once the object is launched, its motion is determined by gravity only, hence *the object is in free fall on the way up as well as on the way down*
Trajectory: the path taken by a projectile

For motion at constant acceleration, the x-coordinate and the x-component of the velocity of an object are given by

$$x_f = x_i + v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$
$$v_{x,f} = v_{x,i} + a_x\Delta t$$

5 Momentum

5.1 Friction

Friction: the resistance to motion that one surface or object encounters when moving over another.
In the absence of friction, objects moving along a horizontal track keep moving without slowing down.

5.2 Inertia

Inertia: a measure of an object's tendency to resist any change in its velocity.
• The motion of larger objects made of the same material is harder to change than the motion of smaller objects
• The ratio of the inertias of the two carts is equal to the inverse of the ratio of their velocity changes

5.3 What Determines Inertia?

The inertia of an object is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object.

5.4 Systems

System: any object or group of objects that we can separate, in our minds, from the surrounding environment
Extensive quantities: quantities whose value is proportional to the size or "extent" of the system
• Only four values can change the value of an extensive quantity: input, output, creation, and destruction
• If we can divide the system into a number of pieces then the sum of an extensive quantity for all the separate pieces is equal to the value of that quantity for the entire system
• The number of trees in a park is extensive because if we divide the park into two parts and add the number of trees in each part then we obtain the number of trees in the park
• The price per gallon of gasoline is not an extensive quantity because we can divide a tankful of gas into two parts and add the price per gallon for the two parts, thus obtaining twice the price per gallon for the entire tank

Intensive Quantities: quantities that do not depend on the extent of the system
System Diagrams: diagrams that show a system's intitial and final conditions

The change in the number of trees over a certain time interval is not

$$\text{change} \;=\; \text{final tree count} \;-\; \text{initial tree count}$$

but rather

$$\text{change} \;=\; \text{input - output + creation - destruction}$$

Conserved: any extensive quantity that cannot be created nor destroyed
Hence, the value of a conserved quantity can only change by

$$\text{change} \;=\; \text{input - output}$$

5.5 Inertial Standard

Inertia:
• A scalar quantity
• Denoted by m
• Represented in kilograms

Recall that the ratio of the inertias of two colliding objects is the inverse of the ratio of the magnitude of their velocity changes. Hence,

$$\frac{m_u}{m_s} = -\frac{\Delta v_{s,x}}{v_{u,x}}$$

therefore

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}} m_s.$$

Because in the case of colliding objects, the velocities are in opposite directions, the minus sign cancels out and *inertia is then always a positive quantity.* We can substitute the standard quantity $m_s = 1$ kg into the equation to get

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}} \text{ kg}$$

5.6 Momentum

Momentum: the product of the inertia and the velocity of an object, represented by $\vec{p}$ and the SI units "kg · m/s":

$$\vec{p} = m\vec{v}$$

The x component of the momentum is the product of the inertia and the x component of the velocity:

$$p_x = mv_x$$

Inertia is an intrinsic property of an object (inertia cannot be changed without changing the object) but the value of the momentum of an object can change. With the definition of momentum, we have also the equation below, which describes how the change in momentum for an object is always the negative of the change in momentum for another object in a collision.

$$\Delta p_{u,x} + \Delta p_{s,x} = 0$$

This can also be written in vector form:

$$\Delta\vec{p}_u + \Delta\vec{p}_s = \vec{0}$$

5.7 Isolated Systems

The momentum of a system of two moving carts is the sum of the momenta of the two individual carts:

$$\vec{p} = \vec{p}_1 + \vec{p}_2.$$

Interaction: two objects acting on each other in such a way that one or both are accelerated
External Interactions: interactions across the boundary of a system
Internal Interactions: interactions between two objects inside the system
Isolated System: a system for which there are no external interactions

5.8 Conservation of Momentum

In an isolated system, the momentum of the system is not changing:

$$\Delta\vec{p} = \vec{0} \text{ (isolated system)}$$

Thus, for any object that collides with the inertial standard on a level, low-friction track, regardless of initial velocities and values of inertias m_1 and m_2, we have

$$\Delta\vec{p}_1 + \Delta\vec{p}_s = \vec{0} \text{ (by definition)}$$
$$\Delta\vec{p}_2 + \Delta\vec{p}_2 = 0 \text{ (by definition)}$$

In a system that is not isolated, we have

$$\Delta\vec{p} = \vec{J},$$

where $\vec{J}$ represents the transfer of momentum from the environment to the system, also known as the **impulse** delivered to the system. Like momentum, impulse is a vector and has units of kg · m/s.

5.9 Lecture 6 Notes

Inertia is actually the same thing as gravitational mass.

For collisions, we have

$$\frac{m_1}{m_2} = -\frac{\Delta\vec{v}_2}{\Delta\vec{v}_1}$$

When you choose a system, be consistent and keep note of:
• Interactions: pushing/pulling that can change motion of object, if the interactions cross the system boundary then they are *external*
• Isolation: no external interactions or external interactions are mutually cancelled or external interactions are negligible (at least temporarily)

Whereas systems in biology have to be interacting, systems in physics can be literally anything. Systems in physics don't need to be interacting or even have the same properties.

$$\Delta\text{Quantity } = J + A,$$

where J is transfer (I-O) and A is creation - destruction. If the system is isolated, then $J = 0$ (nothing crosses system boundary). If a quantity cannot be created nor destroyed, it is said to be conserved.

We have

$$\Delta \overrightarrow{p}_{sys} = \overrightarrow{J},$$

where $\overrightarrow{J}$ is impulse. Because impulse is simply a change in momentum, the units for impulse are the same as momentum.

In general,

$$\overrightarrow{p}_{sys} = \sum_{i=1}^{n} \overrightarrow{p}_i$$

5.10 Topic Summary and Equations

For collisions between identical objects on level surfaces,

$$\Delta \overrightarrow{v}_1 = -\Delta \overrightarrow{v}_2$$

6 Kinetic and Internal Energy

6.1 Classification of Collisions

Relative velocity:

$$\vec{v}_{12} = \vec{v}_2 - \vec{v}_1 \text{ is velocity of cart 2 relative to cart 1}$$
$$\vec{v}_{21} = \vec{v}_1 - \vec{v}_2 \text{ is velocity of cart 1 relative to cart 2}$$

v_{12} is velocity of cart 2 and v_{21} is velocity of cart 2. **Relative speed** is the magnitude of the relative velocity.
Elastic Collision: a collision in which the relative speed before the collision is the same as the relative speed after the collision
• collisions between hard objects are usually elastic

Inelastic Collision: a collision for which the relative speed after the collision is lower than that before the collision
Totally Inelastic Collision: a special case of an inelastic collision in which the two objects move together after the collision so that their relative speed is reduced to zero

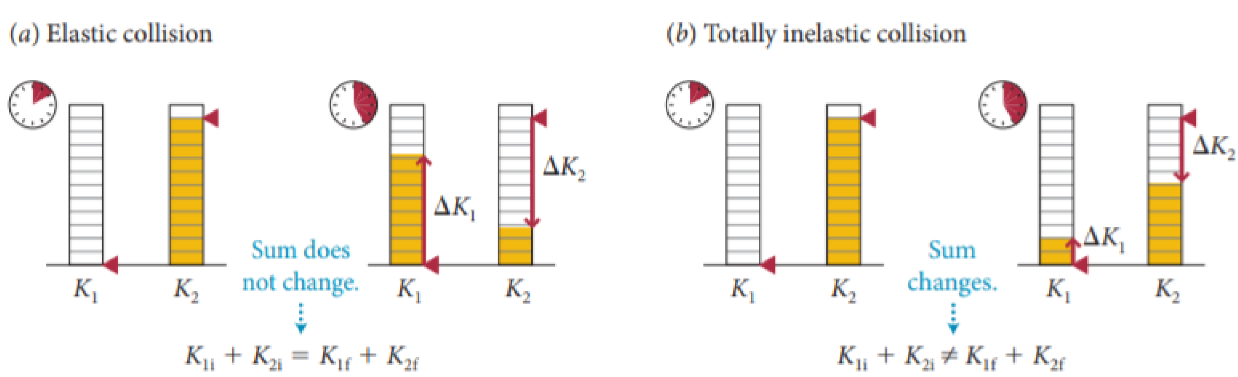
6.2 Kinetic Energy

Kinetic energy: energy associated with a single object's motion, extensive property (depend on amount of matter being measured), given by

$$K = \frac{1}{2}mv^2$$

In a typical elastic collision, the sum of the kinetic energies of the objects before the collision is the same as the sum of the kinetic energies after the collision.

Because kinetic energy is a scalar extensive quantity, bar diagrams are a good way to visually represent changes in this quantity:



6.3 Internal Energy

State: the condition of an object as specified by some complete set of physical parameters: shape temperature, whatever - *every possible physical variable that defines the object.*
In inelastic collisions, objects deform and heat up. A **process** is a transformation of a system from an initial state to a final state. Inelastic collisions are **irreversible processes**. Elastic collisions are **reversible processes**.

The energy of a system is given by the sum of the *kinetic* and *internal* energies:

$$E = E_K + E_{int}$$

Table 5.2 Elastic and inelastic collisions

Collision type	Relative speed	State
elastic	unchanged	unchanged
inelastic	changed	changed
totally inelastic	changed (becomes zero)	changed

In any inelastic collision, the states of the colliding objects change and the sum of their internal energies increases by an amount equal to the decrease in the sum of their kinetic energies. The energy of a system of two colliding objects does not change during the collision.

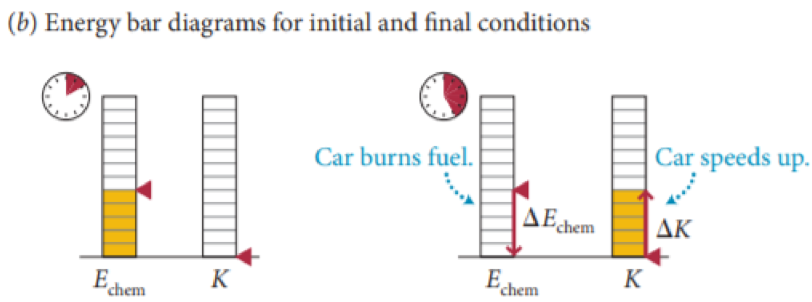
Law of Conservation of Energy: Energy can be transferred from one object to another or converted from one form to another, but it cannot be destroyed or created.

7 Conservation of Energy

7.1 Closed Systems

Closed system: any system to or from which no energy is transferred
• a closed system need not be isolated

Below are some energy bar diagrams for initial and final conditions:



7.2 Elastic Collisions

For elastic collisions, the objects *relative speed*,

$$v_{12} = |\vec{v}_2 - \vec{v}_1|$$

is the same before and after the collision. For two objects moving along the x axis, we can write this as

$$v_{2x,i} = v_{1x,i} = -(v_{2x,f} - v1x, f)$$

Recall that kinetic energy is given by

$$K = \frac{1}{2}mv^2$$

Thus, for elastic collisions,

$$K_i = K_F \equiv \Delta K = 0$$

Kinetic energy is represented in **joules**, where

$$1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1 \text{ J}$$

7.3 Inelastic Collisions

The majority of collisions are between the two extremes of elastic and totally inelastic. For these cases, we can define the ratio of relative speeds as the **coefficient of restitution** (e):

$$e = \frac{v_{12f}}{v_{12i}}$$

Because e is a ratio of speeds which are always positive, e is also always positive. We generally write e with a negative value because the relative velocity changes sign after the collision:

$$e = -\frac{v_{2x,f} - v_{1x,f}}{v_{2x,i} - v_{1x,i}} = -\frac{v_{12,f}}{v_{12x,i}}$$

Process	Relative speed	Coefficient of restitution
totally inelastic collision	$v_{12f} = 0$	$e = 0$
inelastic collision	$0 < v_{12f} < v_{12i}$	$0 < e < 1$
elastic collision	$v_{12f} = v_{12i}$	$e = 1$
explosive separation*	$v_{12f} > v_{12i}$	$e > 1$

7.4 Conservation of Energy

The combined kinetic and internal energies of a system is given by

$$E = K + E_{int}$$

For a closed system, we have

$$\Delta E_{int} = -\Delta K$$

7.5 Explosive Separations

Explosive separation: where objects separate or break apart from each other
• kinetic energy increases and internal energy decreases in these cases

7.6 Topic Summary

Closed system: a system to or from which no energy is transferred
Coefficient of restitution (e) (unitless): A scalar equal to the ratio of relative speeds after and before a collision of two objects:

$$e = \frac{v_{12f}}{v_{12i}}$$

Conservation of energy: energy can be transferred from one object to another or converted from one form to another, but it cannot be created or destroyed. The energy of a closed system cannot change:

$$\Delta E = 0 \text{ (closed system)}$$

Elastic, inelastic, totally inelasic collisions: collisions between two objects are classified according to what happens to the relative speed

$$v_{12} = |\vec{v}_2 - \vec{v}_1|$$

of the two objects.

Energy, E(J): A scalar that provides a quantitative measure of the state or motion of an object or system. Energy appears in many different forms. The energy of an object or system always refers to the sum of all forms of energy in that object or system.
Explosive separation: a process in which objects break apart from one another and the relative speed of the objects increases.
Internal Energy, E_{int} (J): any energy not associated with the motion of an object or system. Internal energy is a quantitative measure of the state of the object or system.
Irreversible process: a process involving changes that cannot undo themselves spontaneously.
Joule (J): The derived SI unit of energy, defined as

$$1J = 1 \text{ kg } \cdot \text{ m}^2/\text{s}^2$$

Process: the transformation of a system from an intial state to a final state
Relative Velocity, $\vec{v}_{12}$ (m/s): The velocity of one object relative to another:

$$\vec{v}_{12} = \vec{v}_2 - \vec{v}_1$$

The magnitude of this velocity is called the **relative speed**:

$$v_{12} = |\vec{v}_2 - \vec{v}_1$$

Reversible process: a process that can run backward so that the initial state is restored
State: The condition of an object (or a system) as specified by a complete set of variables

8 Relativity

8.1 Relativity of Motion

Observer: the person doing the measuring in a discussion of motion:
• The velocity measured for an object depends on the motion of the observer
Reference Frame: the collective term for the axis and origin
Earth Reference Frame: a reference frame at rest relative to the surface of Earth

If an object moves at constant velocity in the Earth reference frame, its motion observed from *any reference frame moving at constant velocity relative to the Earth* is also at constant velocity.

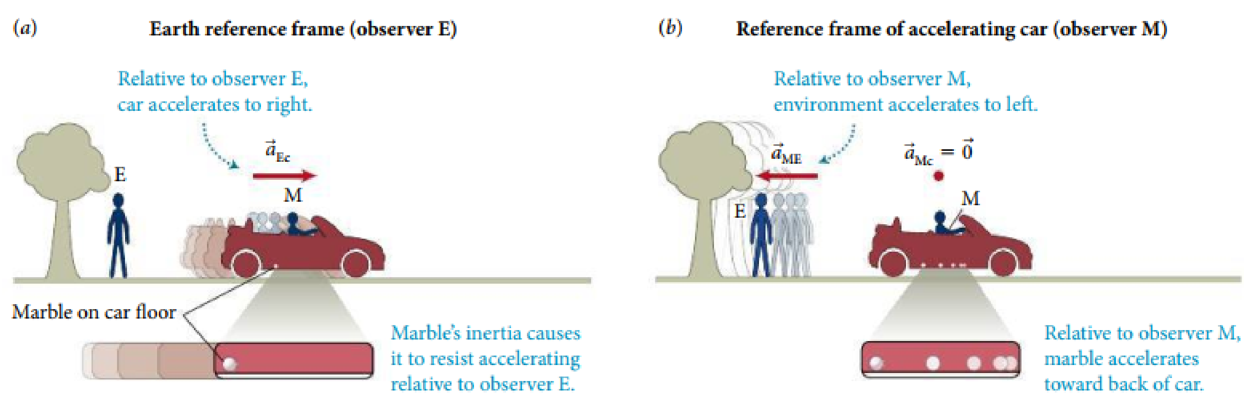
8.2 Inertial Reference Frames

Inertial Reference Frame: any reference frame moving at constant velocity relative to Earth
• every inertial reference frame satisfies the **Law of Inertia**:

Law of Inertia

"In an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object in motion keeps moving at a constant velocity.

Example of Inertial Reference Frames:



Non-inertial Reference Frame: any reference frame that is not inertial

8.3 Principle of Relativity

The kinetic energy of a system of two elastically colliding objects does not change in any inertial reference frame.

Principle of Relativity

The laws of the universe are the same in all inertial reference frames moving at constant velocity relative to each other

To determine your velocity relative to Earth's surface, you need to take measurements in two reference frames. *A consequence of the principle of relativity is that it is not possible to deduce from measurements taken entirely in one reference frame the motion of that reference frame relative to other reference frames.*

9 Centre of Mass

9.1 Centre of Mass

Because phenomena happening in related reference frames are equal such that

$$m_{A0} = m_{B0} = m_0,$$

the **momenta of an object measured in two reference frames** A and B are related by:

$$\begin{aligned}\vec{p}_{A0} &= m_0\vec{v}_{A0}\\ &= m_0(\vec{v}_{AB} + \vec{v}_{B0})\\ &= m_0\vec{v}_{AB} + \vec{p}_{B0}\end{aligned}$$

Similarly,

$$\vec{p}_{Asys} = m\vec{v}_{AB} + \vec{p}_{Bsys}$$

and

$$\vec{p}_{Bsys} = \vec{p}_{Asys} - m\frac{\vec{p}_{Asys}}{m} = \vec{0}$$

In other words, reference frame B is a zero-momentum reference frame. Together, these equations imply that relative to Earth, the velocity of the zero-momentum reference frame Z for a system of objects is equal to the system's momentum measured in the Earth reference frame divided by the inertia of the system:

$$\vec{v}_{EZ} = \frac{\vec{p}_{Esys}}{m} \text{ (zero-momentum reference frame)}$$

The velocity of the zero-momentum reference frame is related to the position of the **center of mass** of a system. This position is defined as

$$\vec{r}_{\mathrm{cm}} = \frac{m_1\vec{r}_1 + m_2{\rightarrow}r_2 + \dots}{m_1 + m_2 + \dots},$$

where $\vec{r}_1, \vec{r}_2$ represent the positions of the objects of inertia m_1, m_2 in any system. Differentiating both sides of the above equation, we have that:

$$\vec{v}_{\mathrm{cm}} = \frac{d\vec{r}_{\mathrm{cm}}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \dots}{m_1 + m_2 + \dots},$$

where $\vec{v}_{\mathrm{cm}}$ is the system's **centre-of-mass velocity**. Note that this velocity is preceisely the velocity of the zero-momentum reference frame in the equation for $\vec{v}_{EZ}$.

Because real objects are *extended* rather than *pointlike*, the term "position" of a system or of a real object is not a precise statement unless a fixed reference point in the system or object is specified. The centre of mass allows us to specify this fixed position in a system according to the exact prescription given by earlier equation for $\vec{r}_{\mathrm{cm}}$. The **centre of mass** is also an important tool for simplifying situations, even for systems with complex motion.

9.2 Convertible Kinetic Energy

Recall that

$$\vec{v}_{E0} = \vec{v}_{EZ} + \vec{v}_{Z0}$$

and that for the zero-momentum reference frame,

$$\vec{v}_{EZ} = \vec{v}_{\mathrm{m}}.$$

Then, we can derive an expression for a system of objects that gives the kinetic energy for the system measured in the Earth reference frame in terms of the corresponding kinetic energy K_{Zsys} measured in the zero-momentum reference frame:

$$\begin{aligned}K_{Esys} &= \frac{1}{2}m_1v_{E1x}^2 + \frac{1}{2}m_2v_{E2x}^2 + \dots\\ &= \frac{1}{2}m_1(v_{\mathrm{cm}\ x} + v_{Z1x})^2 + \frac{1}{2}m_2(v_{\mathrm{cm}} + v_{z2x})^2 + \dots\end{aligned}$$

This can be simplified to the **Kinetic Energy of a system relative to the Earth reference frame:**

$$K_{Esys} = \frac{1}{2}mv_{\mathrm{cm}}^2 + K_{Zsys}$$

The first term on the right in this equation is called the system's **translational kinetic energy**. It is the kinetic energy associated with the motino of the center of mass of the system:

$$K_{\mathrm{cm}} = \frac{1}{2}mv_{\mathrm{cm}}^2$$

The other term on the right of the equation is the system's **convertible kinetic energy**, the amount that can be converted to internal energy without changing the momentum of the system. It is equal to the system's kinetic energy minus the (nonconvertible) translational kinetic energy:

$$K_{conv} = K_{Zsys} = K_{Esys} - \frac{1}{2}mv_{\text{cm}}^2$$

The kinetic energy of a system can be split into a convertible part and a nonconvertible part. The nonconvertible part is the system's translational kinetic energy $K_{cm} = \frac{1}{2}mv_{\textbf{cm}}^2$. The remainder of the kinetic energy is convertible.

We can use mu to simplify our equation; if μ is given by

$$\mu = \frac{m_1 m_2}{m_1 + m_2},$$

then the convertible kinetic energy for a two object system is:

$$K_{conv} = \frac{1}{2}\mu v_{12}^2 \text{ (two-object system)},$$

where

$$\overrightarrow{v}_{12} = \overrightarrow{v}_2 - \overrightarrow{v}_1 istherelativevelocityofthetwoobjects.$$

The letter mu (μ) is called the **Reduced inertia** or "reduced mass" because it is less than the inertia of either of the two colliding objects.

For an inelastic collision, v_{12} changes and K does too. Since the change of kinetic energy relative to the centre of mass reference frame is 0 because the system is isolated, its translation kientic energy cannot change. We can write this expression in terms of the coefficient of restitution e:

$$\begin{aligned}\Delta K &= \frac{1}{2}\mu v_{12i}^2 \left(\frac{v_{12f}^2}{v_{12i}^2} - 1\right) \\ &= \frac{1}{2}\mu v_{12i}^2 (e^2 - 1)\end{aligned}$$

The maximum change in the system's kinetic energy occurs in a totally inelastic collision ($e = 0$):

$$\Delta K = -\frac{1}{2}\mu v_{12i}^2 \text{ (totally inelastic collision)}$$

9.3 Conservation Laws and Relativity

Changes in the momentum of a system are the same in any two reference frames moving at constant velocity relative to each other:

$$\Delta\overrightarrow{p}_{Asys} = \Delta\overrightarrow{p}_{Bsys}$$

By the principle of relativity, if a system is isolated in reference frame A, then it is also isolated in reference frame B. Because internal energy is a quantitative measure of a change in state and state changes cannot depend on the motion of the observer, we can conclude that any change in internal energy is independent of the reference frame:

$$\Delta E_{Aint} = \Delta E_{Bint}$$

Similarly,

$$\Delta K_B = \Delta K_A$$

and thus,

$$\Delta E_{Asys} = \Delta E_{Bsys}$$

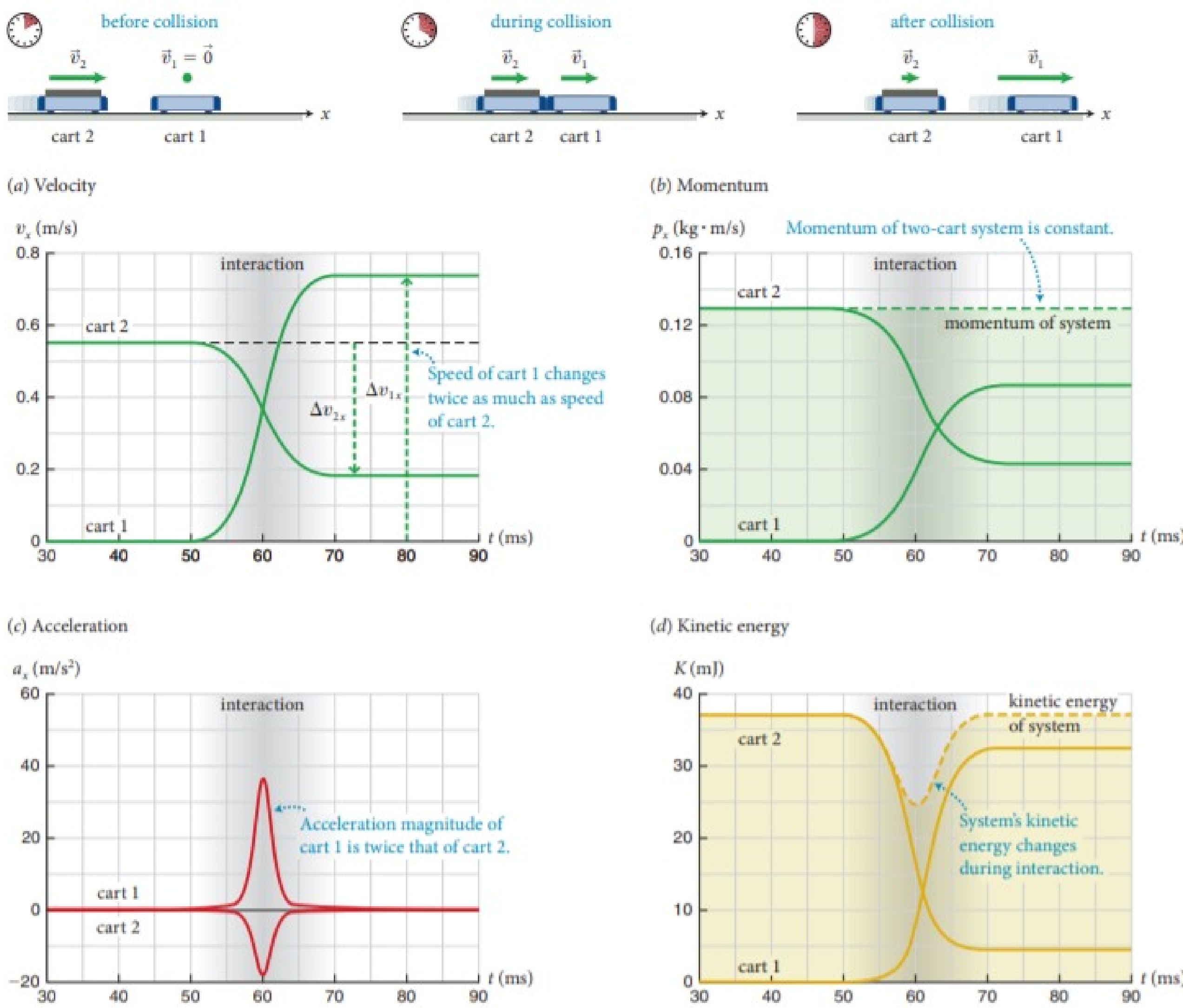
10 Transfer of Energy

10.1 The Effects of Interactions

Interactions: mutual influences between two objects that produce either physical change or a change in motion. An interaction that causes objects to accelerate can be either repulsive or attractive. A **repulsive interaction** is one in which the interacting bodies accelerate away from each other; an **attractive interaction** is one in which the interacting bodies accelerate toward each other.

Whenever two objects interact, the ratio of the x components of their accelerations is equal to the negative inverse of the ratio of their inertias.

Below are some figures describing the conservation of momentum and kinetic energy in an elastic collision between two carts on a low-friction track. The ienrtia of cart 1 is 0.12 kg, and that of cart 2 is 0.24 kg.



Even in an elastic collision between two objects, there is an instant at which the carts have the same velocity, which means that at that instant their relative velocity is zero. For a tennis ball hitting a wall, at the instant the ball has zero velocity, all its kinetic energy has been used to deform the ball (and to a lesser extent, the wall). Thus, this energy is temporarily stored as *elastic potential energy.*

Summary of the Characteristics of an Interaction:
1. *Two objects* are needed
2. The *momentum* of a system of interacting objects is the same before, during, and after the interaction (provided the system is isolated)
For Interactions that affect the motion of objects:

1. The ratio of the x components of the acceleration of the interacting objects is the negative inverse ratio of their inertias. Because the velocities of both objects change in an interaction, the individual momenta and kinetic energies change.
2. The system's *kinetic energy changes during the interactions.* Part of it is converted to (or from) some internal energy. In an elastic collision, all of the converted energy reappears as kinetic energy *after* the collision. In an inelastic collision, some of the converted kinetic energy reappears as kinetic energy; in an totally inelastic collision, none of the converted kinetic energy reappears.

10.2 Potential Energy

Potential Energy: a form of internal energy, represented by U. Potential energy is stored in reversible changes in the **configuration state** of the system in the context of the spatial arrangement of the system's interacting components.
• can be converted entirely to kinetic energy

Gravitational Potential Energy: the potential energy an object has because of its position in a gravitational field, most commonly Earth's gravitational atmosphere.

10.3 Energy Dissipation

The part of the converted kinetic energy that does not reappear after an inelastic collision is said to be **dissipated**, i.e. irreversibly converted. Deformation can take place in a **coherent** manner, meaning that at the atomic level, there is a pattern in the displacements of the atoms: they move orderly in rows, with each successive row experiencing a small displacement in the same direction as adjacent rows. **Incoherent** manner refers to how atoms are displaced in random directions.

Mechanical (coherent) energy: the sum of a system's kinetic energy and potential energy of a system. An important part of a system's incoherent energy is its **thermal energy**. **Internal Energy** is the sum of the system's incoherent energy and its potential energy.

10.4 Source Energy

Source energy: the energy obtained from fossil and mineral fuels, nuclear fuel, biomass fuel, water reservoirs, solar radiation, and wind

The four types of source energy: 1. **chemical energy**: energy associated with the configuration of atoms inside molecules released in such chemical reactions as the burning of oil, coal, gas, and wood and the metabolizing of food
2. **nuclear energy**: (energy associated with the configuration of the nuclei of atoms) released in nuclear reactions
3. **solar energy:** delivered by radiation from the Sun
4. **stored solar energy:** wind and hydroelectric energy
All energy can be divided into four categories: kinetic energy K, potential energy U, source energy E_s, and thermal energy E_{th}.

Nondissipative interactions are reversible, whereas **dissipative** interactions are irreversible.