

PHYS14X Notes

Eric Xia

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# 1 Course Structure

Before each lecture:

- Do the assigned reading
- Complete the lecture reading discussion
- Make notes about questions you have

During Lecture:

- Professor will assume I have done reading and covered selected points only
- There will be frequent "Clicker questions" via LearningCatalytics
- Ask questions in the Chat when I am uncertain/confused/curious

After Lecture:

- Notes lecture recordings will be posted
- Don't rely on these as a substitute for attending class

Course Overview:

- Lectures, labs, and tutorials all require *interaction*
- All components involve constructing and applying *models*
- All components are mutually reinforcing, but:
  - Note every topic is covered in every component
  - Labs generally emphasize observations, tutorials emphasize concepts, homework emphasizes applying models
- Homework is not just for practice
- Usually a topic is introduced in lecture first, but there are exceptions

Difference between honors and normal physics:

- Honors has live lectures, normal is prerecorded
- The topics:
  - Cover some material more quickly (e.g., motion in 1D)
  - Cover some topics in much more depth (e.g., special relativity)
  - Deviate more from the textbook
- The grade distribution
- The students in honors physics have a stronger physics background and are more likely to consider a physics major/minor
- Students in honors are more inquisitive, like to talk about important ideas, enjoy friendly competition, are great at collaborating

Major Differences with physics courses you may have taken before:

- Everything is in 1D first (e.g. no projectile motion until Chapter 10)
- Conservation laws are emphasized (Momentum and energy are covered before forces and Newton's laws)
- Inductive reasoning is emphasized (Figuring out how to explain observations, not just figuring out how to apply known laws)
- Every chapter covers concepts first, then quantitative tools

In-class quizzes aka "clicker" questions:

- Questions asked a few times each class
- Change your answer as often as you want
- Discuss in the zoom chat if you want
- 1 point for any answer (correct or not)
- If you get 80% of the possible points, you get a perfect class participation score

## 2 Foundations

### 2.1 Representations

Visual representations are a vital part of understanding a problem and developing a model. Representations may include pictures, sketches, diagrams, graphs, and a multitude of context-specific procedures. To avoid cluttering a representation, it is best to oversimplify a real-life situation and gradually construct less idealized models. If our oversimplified model reproduces the main features of its real-world counterpart, then we know we have chosen adequate essential attributes.

In this respect, mathematical symbols also act as representations of more complex phrases. Take, for example, the statement below.

”The magnitude of the acceleration of an object is directly proportional to the magnitude of the vector sum of the forces exerted on the object and inversely proportional to the object’s inertia. The direction of the acceleration is the same as the direction of the vector sum of the forces.”

This can be expressed concisely and more clearly with the mathematic equation,

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

### 2.2 Physical quantities and units

Physical quantities and their symbols:

Physical Quantity	Symbol
length	$l$
time	$t$
mass	$m$
speed	$v$
volume	$V$
energy	$E$
temperature	$T$

Physical quantities are expressed as the product of a number and a unit of measurement. For example, the length  $l$  of an object that is  $1.2m$  long can be expressed as  $l = 1.2m$ . The global unit system used in science and engineering is the **Système International (SI)**. There are seven base units in the SI system from which all other units can be derived.

The Seven SI Base Units:

Name of Unit	Abbreviation	Physical Quantity
meter	m	length
kilogram	kg	mass
second	s	time
ampere	A	electric current
kelvin	K	thermodynamic temperature
mole	mol	amount of substance
candela	cd	luminous intensity

To work conveniently with very large or very small numbers, we modify the unit name with prefixes representing integer powers of ten, conventionally powers of ten that are multiples of 3. For example, a billionth of a second is denoted by 1 ns and pronounced ”one nanosecond”, where  $1\text{ ns} = 10^{-9}\text{s}$ .

SI Prefixes:

$10^n$	Prefix	Abbreviation
$10^{24}$	yotta-	Y
$10^{21}$	zetta-	Z
$10^{18}$	exa-	E
$10^{15}$	peta-	P
$10^{12}$	tera-	T
$10^9$	giga-	G
$10^6$	mega-	M
$10^3$	kilo-	k
$10^0$	-	-
$10^{-3}$	milli-	m
$10^{-6}$	micro-	$\mu$
$10^{-9}$	nano-	n
$10^{-12}$	pico-	p
$10^{-15}$	femto-	f
$10^{-18}$	atto-	a
$10^{-21}$	zepto-	z
$10^{-24}$	yocto-	y

A **mole** is currently defined as the number of atoms in  $12 \times 10^{-3}$  kg of carbon-12. This number is referred to as **Avogadro’s number** ( $N_A$ ), where the currently accepted measurement of Avogradro’s number is

$$N_A = 6.0221413 \times 10^{23}$$

**Density** is the physical quantity measuring how much of some substance exists in a given volume. **Number density** is the number of objects per unit volume. If there are  $N$  objects in a volume  $V$ , then the number density  $n$  of these objects is

Number Density

$$n = \frac{N}{V}$$

**Mass density** ( $\rho$ ) is the amount of mass  $m$  per unit volume:

Mass Density

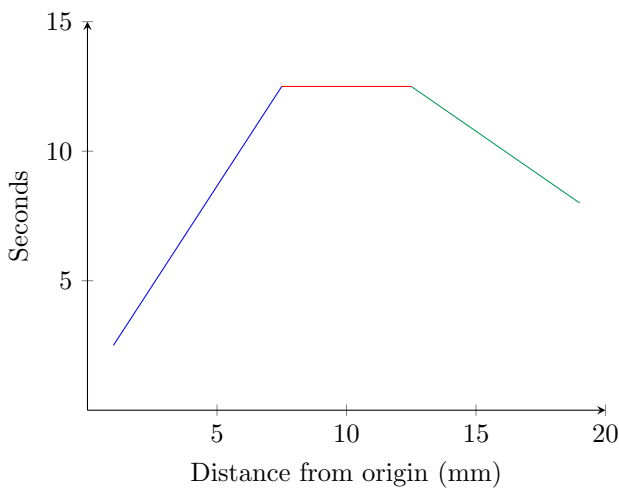
$$\rho = \frac{m}{V}$$

The easiest way to convert measurements to other units is to write the **conversion factor** as a fraction. Then multiply what you’re to express by the conversion ratio, cancelling out the units. Below is an example of converting 4.5 inches to millimeters:

$$4.5 \text{ in.} = (4.5 \cancel{\text{in.}}) \left( \frac{25.4 \text{ mm}}{1 \cancel{\text{in.}}} \right) = 4.5 \times 25.4 \text{ mm} = 1.1 \times 10^2 \text{ mm}$$

### 2.3 From Reality to Model

If the position of an object is not changing, it is said to be **at rest**. A **Position-Time Graph** describes the position of an object as a function of a unit of time. Below is a position-time graph for walking.

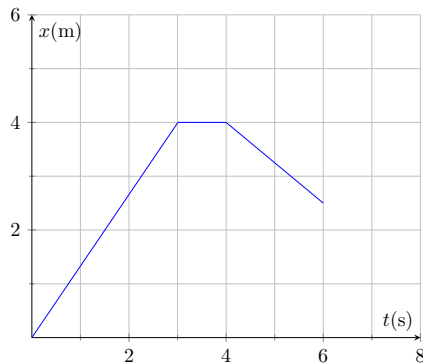


Here, the blue section of the graph describes when the person is walking forwards, the red section is when the person is pausing, and the green section is when the person is walking backwards.

### 3 Motion in One Dimension and Changes in Velocity

#### 3.1 Position and Displacement

**Position-versus-time graph:** a graph that represents position ( $x$ ) as a function of time ( $t$ ).



Here, the object moves forward at a constant velocity between  $t = 0$  and  $t = 3$  as demonstrated by the even slope of the curve when  $0 \leq t \leq 3$ . Then the object is at rest between  $t = 3$  and  $t = 4$ , hence the curve is a flat line in that interval. Lastly, the object turns around and moves backward towards the origin, moving at a constant velocity again demonstrated by the even slope of the curve when  $4 \leq t \leq 6$ .

The **displacement** of an object is the vector change in position. The  $x$  **component of displacement** is given by

$$\Delta x = x_2 - x_1$$

The particular wording of "x component" reminds us that the  $x$  component of displacement is measured along some specific  $x$ -axis. The  $x$  component of displacement can be *either* positive or negative. It is positive for displacements in the direction of increasing  $x$  and negative for displacements in the direction of decreasing  $x$ . Whereas *distance traveled* refers to the distance covered by a moving object along the path of its motion, *displacement* describes a vector quantity with no reference position or axis. Only when motion is always in the same direction is the distance traveled equal to the distance between the initial and final positions.

#### 3.2 Representing Motion

A curve in a position-versus-time graph is called an  **$x(t)$  curve** because it can be represented by a mathematical function  $x(t)$ .

#### 3.3 Average Speed and Average Velocity

The slope of an  $x(t)$  curve is the speed of the object at that time interval. Hence, in a position-versus-time graph, *the steeper the slope, the higher the speed*.

$$s_{avg} = \frac{\Delta x}{\Delta t}$$

**Velocity** is a vector quantity expressing both speed and the direction of travel, where

$$\begin{aligned} x \text{ component of object's } v_{avg} &= \frac{x \text{ component of its displacement}}{\text{time interval for that displacement}} \\ \Delta x \text{ of object's } v_{avg} &= \frac{\Delta x \text{ of displacement}}{\Delta t \text{ for displacement}} \end{aligned}$$

A **velocity-versus-time graph** represents velocity ( $v$ ) as a function of time ( $t$ ). A curve plotted in such a graph is called a  **$v(t)$  curve**. We denote the  $x$  component of an object's average velocity by  $v_x$ .

#### 3.4 Scalars and Vectors

**Scalars** are physical quantities specified by a positive or negative magnitude and a unit of measurement.

- e.g. temperature, volume, density, speed, energy, mass, time
- written as a lowercase letter

**Vectors** are physical quantities specified by a positive or negative magnitude, a unit of measurement, and a direction in space.

- e.g. force, velocity, acceleration, displacement, momentum

- written as a lowercase letter with a right arrow over it, e.g.  $\vec{v}$

The **magnitude of a vector** is denoted by

$$|\vec{b}| \text{ or } b$$

To describe vectors mathematically, we use a **unit vector**, a vector whose sole purpose is to define a direction in space. Unit vectors do not have units and they have a magnitude of 1. Hence,

$$|\hat{i}| = 1$$

Unit vectors are denoted by a caret/hat (^) to distinguish them from normal vectors. Unit vectors pointing in the direction of increasing  $x$  along the  $x$ -axis are denoted by  $\hat{i}$  (pronounced *i-hat*). **Unit vector notation** expresses a vector along the  $x$ -axis as the product of a scalar and the unit vector:

$$\vec{b} = b_x \hat{i} \text{ (one dimension)}$$

where  $b_x$  is a scalar known as the  $x$  component of  $\vec{b}$ . If we take the absolute value of both sides of the above equation, we find that, for vectors in one dimension, the magnitude of the vector  $\vec{b}$  equals the magnitude of  $b_x$ :

$$\begin{aligned} \vec{b} &= b_x \hat{i} \\ |\vec{b}| &= |b_x \hat{i}| \\ &= |b_x| |\hat{i}| \\ &= |b_x| \end{aligned}$$

If  $b_x > 0$  then  $\vec{b}$  is in the same direction as  $\hat{i}$ . If  $b_x < 0$  then  $\vec{b}$  is in the opposite direction to  $\hat{i}$ .

### 3.5 Position and Displacement Vectors

The most basic vector is displacement ( $\Delta \vec{r}$ ). Recall that the  $x$  component of displacement is equal to the change in the  $x$  coordinate such that

$$\Delta r_x = \Delta x = x_f - x_i$$

The distance  $d$  between two points is given by

$$d = |x_1 - x_2| \text{ (one dimension)}$$

A vector that has zero magnitude is equal to the **zero vector**, denoted by a zero with an arrow ( $\vec{0}$ ). In diagrams, the zero vector is represented by a dot. Rearranging the definition for  $\Delta x$ , we get

$$x_f = x_i + \Delta x$$

An object's position can also be represented by a vector:

$$\Delta x \hat{i} = (x_f - x_i) \hat{i} = x_f \hat{i} - x_i \hat{i}$$

where the left side of the equation represents the displacement

$$\Delta x \hat{i} = \Delta r_x \hat{i} = \Delta \vec{r}$$

The vector  $x_i \hat{i}$  represents the **position vector**, a vector that lets us determine the position of a point in space relative to some chosen origin. Position is also represented by  $\vec{r}$ :

$$\vec{r} = x \hat{i} \text{ one dimension}$$

Hence, we can see that the  $x$  component of position is equal to the coordinate such that

$$r_x = x$$

To subtract a vector from another vector, we reverse the direction of the vector being subtracted and add the reversed vector to the other vector:

The vector  $\vec{a} - \vec{b}$  can be graphically represented like so:

$$\begin{array}{c} \vec{a} + (-\vec{b}) = \\ \overrightarrow{\hspace{1.5cm}} \\ \overleftarrow{\hspace{1.5cm}} \\ \vec{a} - \vec{b} \\ \overrightarrow{\hspace{1.5cm}} \end{array}$$

Scalar multiplication of a vector:

$$c \vec{a} = c (a_x \hat{i}) = (ca_x) \hat{i}$$

### 3.6 Velocity as a Vector

The  $x$  component of the average velocity is given by

$$v_{x,av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

We can multiply the first and middle terms of the above equation by the unit vector  $\hat{i}$  and substitute  $\Delta \vec{r} = \Delta x \hat{i}$  to get

$$\vec{v}_{av} = v_{x,av} \hat{i} = \frac{\Delta \vec{r}}{\Delta t}$$

Because  $\Delta t$  is always positive, we see that for one dimensional motion,  $\vec{v}_{av}$  always points in the same direction as displacement.

### 3.7 Motion at Constant Velocity

By convention, the phrase "average" is dropped from references to constant velocity. Thus, the  $x$  component of the average velocity is simply  $v_x$  and we see that the slope of the  $x(t)$  curve is numerically equal to the  $x$  component of the velocity:

$$\frac{\Delta x}{\Delta t} = v_x \text{ constant velocity}$$

The  $x$  component of the displacement during any time interval is given by the area under the  $v_x(t)$  curve between the beginning and the end of the time interval. We can use the fact that  $\Delta x = x_f - x_i$  to rewrite the above equation in terms of  $x_f$  such that

$$x_f = x_i + v_x \Delta t \text{ constant velocity}$$

### 3.8 Instantaneous Velocity

**Instantaneous Velocity:** the velocity of an object at any given instant. The  $x$  component of the velocity at instant  $t$  is given by

$$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The velocity is then

$$\vec{v} = v_x \hat{i} = \left( \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \right) \hat{i}$$

Because the unit vector is constant, we can rewrite the velocity as

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x \hat{i}}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

Hence, speed is given by the magnitude of this vector:

$$v = |\vec{v}|$$

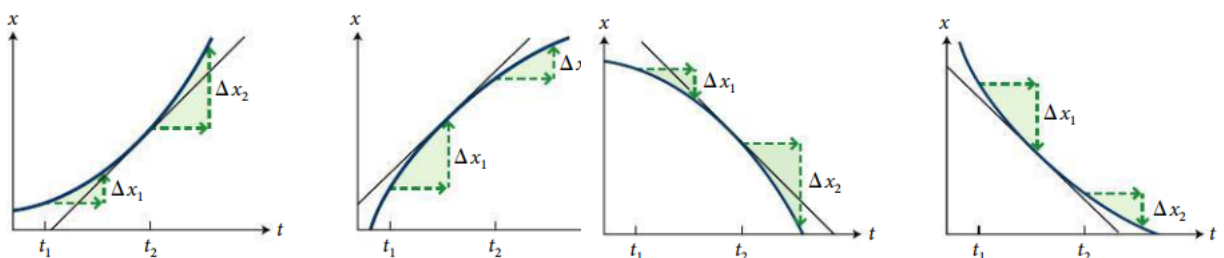
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### 3.9 Changes in Velocity

**Acceleration:** the rate of change of velocity with respect to time. The  **$x$  component of average acceleration** is the change in the  $x$  component of the velocity divided by the time interval during which this change took place. In physics, the word "*deceleration*" has no meaning is better avoided, as *acceleration* refers to *any* change in velocity, regardless of increasing or decreasing speed.

When an object's velocity and acceleration is in the same direction, the object speeds up. When they are in opposite directions, the object slows down. It is important to note confuse the positive  $x$  component of acceleration with speeding up when working with objects traveling in the negative  $x$  direction. An object speeds up when the magnitude of its velocity (aka speed) increases, regardless of the direction of motion. Whether or not the  $x$  component of an object's acceleration is positive, however, dependson hte direction in which the object is moving.

**Position-versus-time graphs for objects accelerating along the positive  $x$  axis:**



The curvature of an  $x(t)$  curve is a measure of the  $x$  component of acceleration. An upward curvature corresponds to a positive  $x$  component of acceleration, whereas a downward curvature corresponds to a negative  $x$  component of acceleration.

### 3.10 Acceleration Due to Gravity

In a vacuum, a feather and a stone drop at the same rate. Yet in air, the stone drops faster. This result has been attributed to air resistance pushing harder on the feather than the stone. **Free fall** is the motion of objects moving solely under the influence of gravity. A falling object inside a vacuum is in free fall because it is influenced only by gravity, but an object falling in the air is not in free fall because its motion is also affected by air resistance. However, the effect of air resistance on a stone outside of a vacuum is negligible. This property is true for many objects that are not too light and not dropped from too great of a height. Hence, if an object is not extremely light or dropped from a substantive height, we can ignore air resistance and consider all dropped objects to be in free fall.

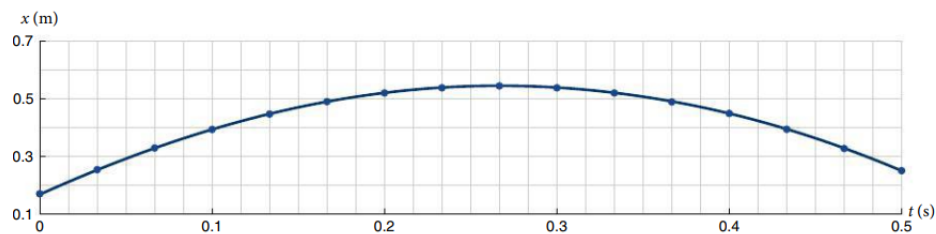
In the absence of air resistance, the **magnitude of the acceleration of all objects due to gravity** is **9.8 m/s<sup>2</sup>** and consequently the amount of *time it takes to fall from a certain height is the same for all falling objects*.

### 3.11 Projectile Motion

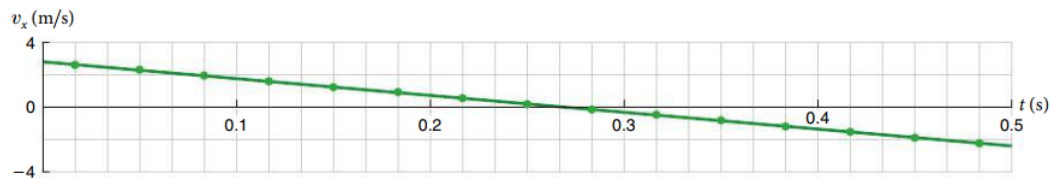
An object that is launched but not self-propelled is a **projectile**, and its motion is called **projectile motion**. The path a projectile follows is called its **trajectory**. The motion of an object in free fall is a special case of projectile motion.

Consider a ball launched straight up. The ball slows down as it moves upward and hence its acceleration and velocity point in opposite directions. The ball’s velocity changes **linearly**, changing by equal amounts in equal time intervals, thus its acceleration is constant. In the position-versus-time and velocity-versus-time graphs below, it can be seen that the slope of the  $v_x(t)$  curve is about -10 m/s<sup>2</sup>, which is the acceleration due to gravity. Thus, both on the way up and way down, the acceleration of a projectile is equal *both in magnitude and in direction* to that of a freely falling object.

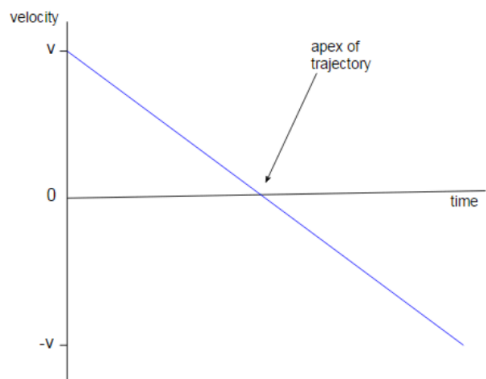
$x(t)$  curve for a ball launched straight up:



$v_x(t)$  curve for a ball launched straight up:



For an object launched upwards, the launch affects only the initial velocity. Once the object is released, the rest of its motion is determined by gravity alone (free fall). At the top of a trajectory of a ball launched upwards, there is instant at which the  $x$  component of the velocity is zero. *The  $x$  component of the acceleration is not zero at the top*. To conceptualize this, consider the graph below.



Though velocity is zero at the apex of the trajectory, acceleration is defined as

$$a = \frac{dv}{dt}$$



The gradient of the line in the graph is nonzero and happens to be  $-9.81\text{ m/s}^2$ . Hence, the magntiude of the acceleration does not depend on  $v$  and is equal to  $9.81\text{ m/s}^2$  for all values of  $v$ .

### 3.12 Motion Diagrams

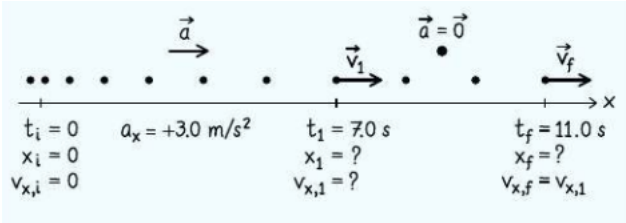
**Motion diagrams** represent the positions of a moving object at equally spaced time intervals. We can make a motion diagrams by the following steps:

1. Use dots to represent the moving object at even time intervals. If the object moves at constant speed then the dots are evenly spaced. Conversely, if the object speeds up, the spacing between the dots increases. If the object slows down, the spacing decreases.
2. Choose an  $x$  (position) axis convenient for the problem. This is usually an axis that has its origin at the initial or final position of the object and is oriented in the direction of motion or acceleration.
3. Specify the position and velocity at all relevant interests, particularly, the **initial conditions**— position and velocity at the beginning of the time interval of interest, and the **final positions**— position and velocity at the end of that time interval. Additionally, specify all positions where the velocity reverses direction or the acceleration changes. Label any unknown parameters with a question mark.
4. Indicate the acceleration of the object between all instants specified in step 3.
5. To consider the motion of more than one object, draw separate diagrams side by side, one for each object, using one common  $x$ -axis.
6. If the object reverses direction, separate the motion diagram into two parts, one for each direction of travel.

An acronym for a useful checklist of a motion diagram is ”**VENUS**”:

- **V**ectors and scalars used appropriately
- **E**verything answered
- **N**o unknowns left
- **U**nits correct
- **S**ignificant digits justified

A Motion Diagram:



### 3.13 Summary of Symbols and Meanings in This Topic

Symbol	Meaning
$t$	clock reading = instant in time
$\Delta t = t_f - t_i$	change in clock reading = interval of time
$x$	$x$ -coordinate
$d =  x_1 - x_2 $	straight-line distance between two points (never has sign)
$\hat{i}$	unit vector defining direction of $x$ -axis (vector magnitude 1)
$\vec{b} = v_x \hat{i}$	vector and unit vector notation
$b_x$	$x$ component of vector $\vec{b}$ (can be positive or negative)
$b =  \vec{b}  =  b_x $	magnitude of vector $\vec{b}$ (always positive)
$g =  \vec{a}_{\text{free fall}}  = 9.8\text{ m/s}^2$	acceleration due to gravity near Earth’s surface

Vector	Meaning	Component	Meaning
$\Delta \vec{r} = \Delta r_x \hat{i}$	displacement	$\Delta r_x = \Delta x$	$x$ component of displacement = change in $x$ -coordinate
$\vec{r} = r_x \hat{i}$	position	$r_x = x$	$x$ component of position = $x$ -coordinate
$\vec{v}_{av} = v_{x,av} \hat{i}$	average velocity	$v_{x,av} = \frac{\Delta x}{\Delta t}$	$x$ component of average velocity
$\vec{v} = v_x \hat{i}$	(instantaneous) velocity	$v_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$x$ component of (instantaneous) velocity
$\vec{a} = a_x \hat{i}$	acceleration	$a_x = \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	$x$ component of acceleration

**Average Velocity:** The constant velocity required to achieve the same displacement in the same time interval as the actual motion.

$$v_{avg} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v(t) dt$$

Note that this is just the average value of a function integral.

# 4 Constant Acceleration

## 4.1 Motion with Constant Acceleration

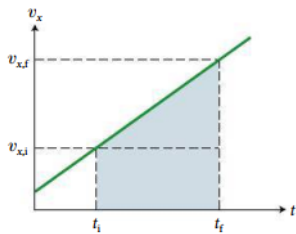
For an object moving with constant acceleration, the  $v_x(t)$  curve is a straight line. We know  $\Delta v_x$  and  $v_{x,f}$  to be

$$\Delta v_x = v_{x,f} - v_{x,i} = a_x \Delta t$$
$$v_{x,f} = v_{x,i} + a_x \Delta t$$

(constant acceleration)

(constant acceleration)

We also know displacement to be the area under a velocity curve:



Hence, the displacement is given by the area of the triangle and the area of the rectangle. The area of the triangle is

$$\frac{1}{2}(v_{x,f} - v_{x,i})(t_f - t_i) = \frac{1}{2}\Delta v_x \Delta t = \frac{1}{2}a_x(\Delta t)^2$$

and the area of the rectangle is

$$(v_{x,i} - 0)(t_f - t_i) = v_{x,i} \Delta t$$

Thus, the combined displacement is

$$x_f - x_i = v_{x,i} \Delta t + \frac{1}{2}a_x(\Delta t)^2 \text{ (constant acceleration)}$$

Below are **kinematics graphs** for the three basic types of motion:

Motion diagram	Position versus time	Velocity versus time	Acceleration versus time
<div>At rest</div>			
<div>Constant velocity</div>			
<div>Constant acceleration</div>			

## 4.2 Free-fall Equations

Magnitude of **acceleration due to gravity (g)** is given by

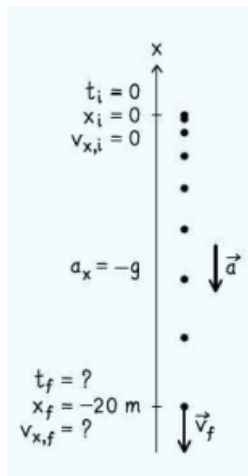
$$g = |\vec{a}_{\text{free fall}}|$$

If an object is dropped from a certain height with zero initial velocity along an upward-pointing  $x$  axis, then,

$$x_f = x_i - \frac{1}{2}gt_f^2$$

$$v_{x,f} = -gt_f$$

Example of a Free-fall Diagram:



## 4.3 Inclined Planes

When a ball rolls down an incline starting from rest, the ratio of the distance traveled to the square of the amount of time needed to travel that distance is constant. For example, if a ball released from position  $x_i = 0$  at  $t_1 = 0$  reaches position  $x_1$  at instant  $t_1$ ,  $x_2$  at  $t_2$ , and  $x_3$  at  $t_3$ , then

$$\frac{x_1}{t_1^2} = \frac{x_2}{t_2^2} = \frac{x_3}{t_3^2}$$

Recall that for an object moving with constant acceleration and starting from rest, we have

$$x_f = \frac{1}{2}a_x t_f^2$$

Hence,

$$\frac{x_f}{t_f^2} = \frac{1}{2}a_x$$

As incline increases, so does the ratio  $\frac{x_0}{t_0^2}$ . Through experimentation, we can determine the component of acceleration along the incline to be related to  $g$  by the equation

$$a_x = g \sin \theta$$

## 4.4 Instantaneous Acceleration

$$a_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t}$$

Hence,

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2 x}{dt^2}$$

We can then express changes in velocity and position as:

$$\Delta v_x = \int_{t_1}^{t_f} a_x(t) dt$$

$$\Delta x = \int_{t_1}^{t_f} v_x(t) dt$$

## 4.5 Summary of Topic

**Free fall:** the motion of an object subject to only the influence of gravity

**Motion diagram:** a diagram that shows the position of a moving object at equally spaced time intervals and summarizes what is known about the object's initial and final conditions (its position, velocity, and acceleration)

**Projectile motion:** the motion of an object that is launched but not self-propelled (*projectile*)

- The launch only affects the object's initial velocity
- Once the object is launched, its motion is determined by gravity only, hence *the object is in free fall on the way up as well as on the way down*

**Trajectory:** the path taken by a projectile

For motion at constant acceleration, the  $x$ -coordinate and the  $x$ -component of the velocity of an object are given by

$$x_f = x_i + v_{x,i}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$
$$v_{x,f} = v_{x,i} + a_x\Delta t$$

## 5 Momentum

### 5.1 Friction

**Friction:** the resistance to motion that one surface or object encounters when moving over another.  
In the absence of friction, objects moving along a horizontal track keep moving without slowing down.

### 5.2 Inertia

**Inertia:** a measure of an object's tendency to resist any change in its velocity.

- The motion of larger objects made of the same material is harder to change than the motion of smaller objects
- The ratio of the inertias of the two carts is equal to the inverse of the ratio of their velocity changes

### 5.3 What Determines Inertia?

*The inertia of an object is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object.*

### 5.4 Systems

**System:** any object or group of objects that we can separate, in our minds, from the surrounding environment

**Extensive quantities:** quantities whose value is proportional to the size or "extent" of the system

- Only four values can change the value of an extensive quantity: input, output, creation, and destruction
- If we can divide the system into a number of pieces then the sum of an extensive quantity for all the separate pieces is equal to the value of that quantity for the entire system
- The number of trees in a park is extensive because if we divide the park into two parts and add the number of trees in each part then we obtain the number of trees in the park
- The price per gallon of gasoline is not an extensive quantity because we can divide a tankful of gas into two parts and add the price per gallon for the two parts, thus obtaining twice the price per gallon for the entire tank

**Intensive Quantities:** quantities that do not depend on the extent of the system

**System Diagrams:** diagrams that show a system's initial and final conditions

The change in the number of trees over a certain time interval is not

$$\text{change} = \text{final tree count} - \text{initial tree count}$$

but rather

$$\text{change} = \text{input} - \text{output} + \text{creation} - \text{destruction}$$

**Conserved:** any extensive quantity that cannot be created nor destroyed

Hence, the value of a conserved quantity can only change by

$$\text{change} = \text{input} - \text{output}$$

### 5.5 Inertial Standard

Inertia:

- A scalar quantity
- Denoted by  $m$
- Represented in kilograms

Recall that the ratio of the inertias of two colliding objects is the inverse of the ratio of the magnitude of their velocity changes. Hence,

$$\frac{m_u}{m_s} = -\frac{\Delta v_{s,x}}{v_{u,x}}$$

therefore

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}} m_s.$$

Because in the case of colliding objects, the velocities are in opposite directions, the minus sign cancels out and *inertia is then always a positive quantity*. We can substitute the standard quantity  $m_s = 1$  kg into the equation to get

$$m_u = -\frac{\Delta v_{s,x}}{\Delta v_{u,x}} \text{ kg}$$

## 5.6 Momentum

**Momentum:** the product of the inertia and the velocity of an object, represented by  $\vec{p}$  and the SI units "kg · m/s":

$$\vec{p} = m \vec{v}$$

The  $x$  component of the momentum is the product of the inertia and the  $x$  component of the velocity:

$$p_x = mv_x$$

Inertia is an intrinsic property of an object (inertia cannot be changed without changing the object) but the value of the momentum of an object can change. With the definition of momentum, we have also the equation below, which describes how the change in momentum for an object is always the negative of the change in momentum for another object in a collision.

$$\Delta p_{u,x} + \Delta p_{s,x} = 0$$

This can also be written in vector form:

$$\Delta \vec{p}_u + \Delta \vec{p}_s = \vec{0}$$

## 5.7 Isolated Systems

The momentum of a system of two moving carts is the sum of the momenta of the two individual carts:

$$\vec{p} = \vec{p}_1 + \vec{p}_2.$$

**Interaction:** two objects acting on each other in such a way that one or both are accelerated

**External Interactions:** interactions across the boundary of a system

**Internal Interactions:** interactions between two objects inside the system

**Isolated System:** a system for which there are no external interactions