

PHYS122 Notes

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1 Electric Interactions

1.1 Static Electricity

Examples of Static Electricity (**Electric Interactions**):

- Plastic wrap is attracted to anything that gets close
- Styrofoam peanuts are attracted to you when you open a box of them
- If you rub a balloon against a wool sweater then the balloon will then be attracted to the wall if held close to the wall

Objects involved in electric interactions exert **electric forces** on each other. Electric forces are *field forces*, hence the magnitude of the electric force depends on distance, decreasing as you increase the separation of the involved objects. This relation can be expressed mathematically by **Coulomb's Law**, given below and covered later in more detail.

$$F^E \propto \frac{1}{r^2}$$

where r is the distance.

Strips of tape just pulled out of a dispenser will repel each other. The repulsive force is great enough to keep the strips apart even when they are weighed down by paper clips:



1.2 Electrical Charge

Electrical charge: the attribute objects have that are responsible for the electric interaction

Charge carrier: any microscopic objects (e.g. electrons and ions) that carry an electrical charge

Negative charge: the state of having a surplus of electrons, hence a lower electric potential

- a plastic comb passed through hair several times will be negatively charged
- Electrically neutral:** objects with equal amounts of positive and negative charge; *discharged* objects fall under this term
- When two neutral objects touch, the result is that both of the objects are no longer neutral. Without further information, we *cannot tell whether positive charge has been transferred between objects, negative charge has been transferred between objects, or a combination of the two.*

Objects that carry like charges repel each other whereas objects that carry opposite charges attract each other.

1.3 Mobility of Charge Carriers

Charge can be transferred from one object to another by bringing the two into contact.

Electrical insulators: materials through which charge carriers cannot flow easily or do not flow at all

- any charge transferred to an insulator remains near the spot at which it was deposited
- glass, rubber, wood, plastic
- air, particularly dry air, is an insulator, though the presence of large amounts of charge can cause charge carriers to "jump" between objects, causing sparks

If a charged rod is brought into contact with an uncharged metal rod, then *all* points on the surface of the metal rod interact electrically with other objects, indicating that the charge has spread out on the object—a phenomenon that can be observed with a **electroscope**.

Electrical conductors: materials through which charge carriers can flow

- the human body due to its water content
- metals are the only solid materials that are conductors at room

temperature

- *not pure water*, but water with minute amounts of impurities
- earth except for the outer layer of soil

Grounding: discharging objects by connecting them to Earth by a wire

Conduction: the flow of charge through conductors

Charge is an inherent property of subatomic particles, thus, all electrical charge comes in whole number multiples of the electrical charge on the electron and there is no such thing as a discharged electron.

Elementary charge: the magnitude of the charge on the electron, designated by e .

- Charge on an electron: $-e$
- Charge on a proton: $+e$

Ions: charged atoms/molecules

- always immobile in solids but can move freely in liquids

Electrons that can move freely inside some object, e.g. metal, are called **free electrons** and are responsible for the easy flow of charge through a metal.

When making charged pieces of tape from neutral pieces of tape, it is important to rub vigorously or separate strips of tape quickly because with friction comes the breaking of chemical bonds, and if the breaking of chemical bonds occurs slowly, then the electrons migrate back and no surplus charge builds up. Another point is that any two dissimilar materials become charged when brought into contact with each other. When they are separated rapidly, small amounts of opposite charge may be left behind on each material.

Principle of Conservation of Charge

Electrical charge can be created or destroyed only in identical positive-negative pairs such that the charge of a closed system always remains constant.

When an electron (charge $-e$ collides with a subatomic particle known as a positron (charge $+e$), both particles are destroyed, leaving only a flash of highly energetic radiation.)

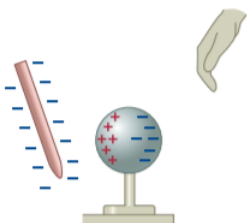
1.4 Charge Polarization

Polarization: any separation of charge carriers in an object

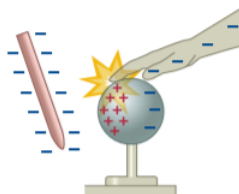
- explains why charged and neutral have some reaction (repel/attract)

The below image shows how polarization can be applied to charge neutral conducting objects in a process called **charging by induction**.

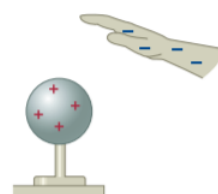
1 Charged rod induces polarization in metal sphere.



2 When you touch sphere, negative charge gets farther from rod by spreading onto you.



3 When you let go, you retain surplus of one type of charge and sphere retains surplus of opposite type of charge.



1.5 Coulomb's Law

Electric force (electrostatic force): the attractive or repulsive interaction between charged bodies

- is sometimes referred to as electrostatic force because the interaction between charged particles becomes more complicated when particles are not at rest
- Coulomb's Law:** gives the magnitude of the electric force exerted by two charged particles separated by a distance r_{12} and carrying charges q_1 and q_2 as:

$$F_{12}^E = k \frac{|q_1||q_2|}{r_{12}^2}$$

Coulomb (C): the derived SI unit for charge, defined as the quantity of electrical charge transported in one second by a current of one ampere.

- $1 \text{ C} = 6.24 \cdot 10^{18}$ electrons.
- $e = \frac{1}{6.24} \cdot 10^{18} \text{ C} = 1.60 \cdot 10^{-19} \text{ C}$.

Experimentally, the value of k can be found to be

$$k = 9.0 \cdot 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

The electric force is *central*, meaning that its line of acceleration is along the line connecting the two interacting charged particles. We can define a unit vector pointing from particle 1 to particle 2 as follows:

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{r_{12}}$$

Charge distribution: the manner in which a collection of charge carriers is spread over a macroscopic object.

- two charged spheres separated by a small distance have uniform charge distributions with the center of each charge distribution coinciding with the center of the sphere and so r_{12} is well-defined
- when the spheres are brought closer together, the like charge carriers repel one another and move to the far side of each sphere

1.6 Force Exerted By Distributions of Charge Carriers

Coulomb's Law only deals with *pairs* of charged objects, and so to calculate the force exerted by an assembly of objects carrying charges q_2, q_3, q_4, \dots on an object that is carrying a charge q_1 , we take a vector sum of all the forces exerted on an object 1 by each of the other charged objects independently:

$$\sum \vec{F}_1^E = k \frac{q_2 q_1}{r_{21}^2} \hat{r}_{21} + k \frac{q_3 q_1}{r_{31}^2} \hat{r}_{31} + k \frac{q_4 q_1}{r_{41}^2} + \dots$$

Note that the above law is valid only for charged *particles*.

1.7 Chapter Glossary

Charge: (electrical) q (C) A scalar that represents the attribute responsible for electromagnetic interactions, including electric interactions. There are two types of charge: *positive* ($q > 0$) and *negative* ($q < 0$). Two objects that carry the same type of charge exert repulsive forces on each other; objects that carry different types of charge exert attractive forces on each other

Charge carrier: Any microscopic object that carries an electrical charge

Charge distribution: The way in which a collection of charge carriers is distributed in space

Charge polarization: A spatial separation of the positive and negative charge carriers in an object. The polarization of neutral objects induced by the presence of external charged objects is responsible for the electric interaction between charged and neutral objects.

Charging by induction: a method of charging a neutral object using a charged object, with no physical contact between them.

Conduction: The flow of charge carriers through a material.

Conservation of charge: The principle that the charge of a closed system cannot change. Thus charge can be transferred from one object to another and can be created or destroyed only in identical positive-negative pairs.

Coulomb (C): The derived SI unit of charge equal to the magnitude of the charge on about $6.24 \cdot 10^{18}$ electrons. (The coulomb is defined as the quantity of electrical charge transported in one second by a current of one ampere, a unit defined later.)

Coulomb's Law: The force law that gives the direction and magnitude of the electric force between two particles at rest carrying charges q_1 and q_2 separated by a distance r_{12} :

$$\vec{F}_{12}^E = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

The constant k has the value $k = 9.0 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$.

Electric force \vec{F}^E (N): The force that charge carriers (and macroscopic objects that carry a surplus electrical charge) exert on each other. The magnitude and direction of this force are given by Coulomb's law.

Electric interaction: a long-ranged interaction between charged particles or objects that carry a surplus electrical charge and that are at rest relative to the observer.

Elementary charge: The smallest observed quantity of charge, corresponding to the magnitude of the charge of the electron: $e = 1.60 \cdot 10^{-19} \text{ C}$.

Grounding: The process of electrically connecting an object to Earth ("ground"). Grounding permits the exchange of charge carriers with Earth, a huge reservoir of charge carriers. A charged, conducting object that is grounded will retain no surplus of either type of charge, assuming no other nearby electrical influences.

Insulator (electrical): Any material or object through which charge cannot flow easily.

Ion: An atom or molecule that contains unequal numbers of electrons and protons and therefore carries a surplus charge.

Negative charge: The type of charge acquired by a plastic comb that has been passed through hair a few times.

Neutral: The electrical state of objects whose charge is zero. Electrically neutral macroscopic objects contain the same number of positively and negatively charged particles (protons and electrons).

Positive charge: The type of charge acquired by hair after a plastic comb has been passed through it a few times.

2 The Electric Field

2.1 The Field Model

In the field model, an interacting object fills the space around itself with a field.

Interaction field: a physical quantity represented by a number or tensor that has a value for each point in space and time.

- the field of an object always exist even when the object is not interacting with anything else

Gravitational field: the space around any object that has mass

- exert forces on objects that have mass
- for any object A located in a gravitational field created by an object S, the magnitude of the field felt by A depends only on the properties of S and on the position of A relative to S; the field magnitude does not depend in any way on the properties of A
- at any given location in the space surrounding a source object S, the magnitude of the gravitational field created by S is the magnitude of the gravitational force exerted on an object B placed at that location divided by the mass of B.
- is a **vector field**, where each position has both magnitude and direction

Electric field: the space around any electrically charged object

- exert forces on objects that either carry a charge or can be polarized

Temperature field: temperature across the surface of a region has a specific value at each location

- is a **scalar field**, where each position has a magnitude but not a direction

Test particle: an idealized particle whose mass is small enough that its presence does not perturb the object whose gravitational field we are measuring

- can be used to determine the magnitude and direction of a gravitational field in the space surrounding Earth
- Earth's gravitational field always points towards the center of Earth and its magnitude decreases with increasing distance away from Earth.

2.2 Electric Field Diagrams

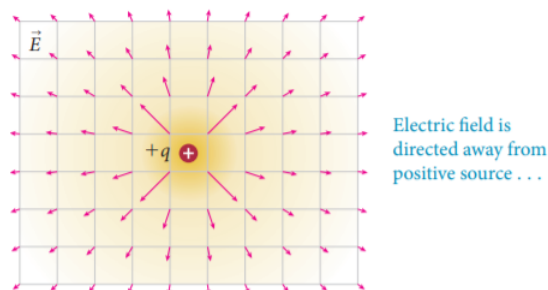
At any given location in the space surrounding a source object S, the electric field created by S is the electric force exerted on a charged test particle placed at that location divided by the charge of the test particle:

$$\vec{E}_S \equiv \frac{\vec{F}_{St}^E}{q_t}$$

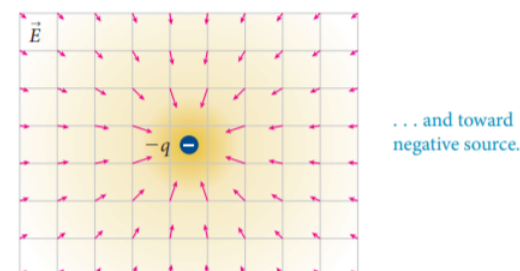
The direction of the electric field at a given location is the same as the direction of the electric force exerted on a positively charged object at that location:

Figure 23.7 Vector field diagrams for positively and negatively charged particles. The lengths of the vectors show that the electric field magnitude decreases with increasing distance from the source.

(a) Electric field of positively charged particle



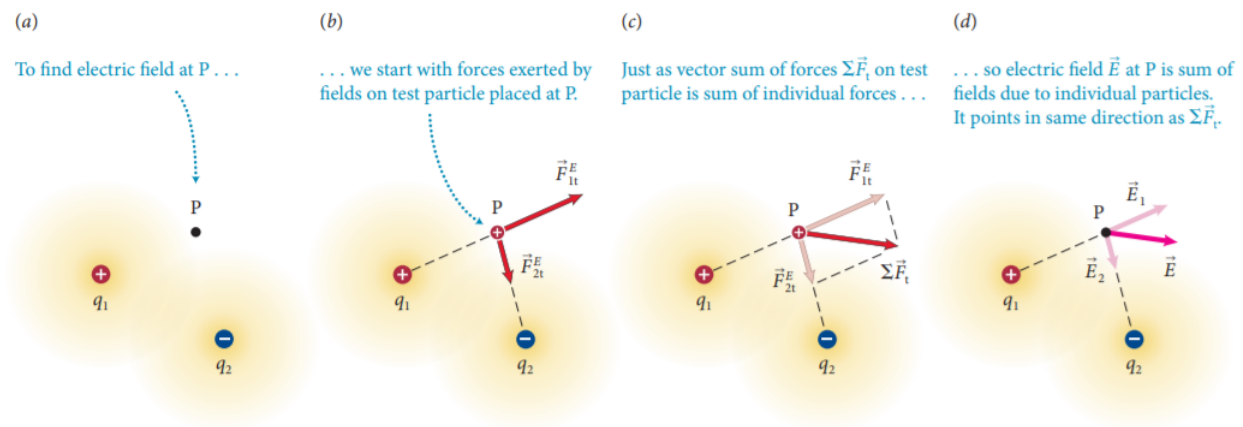
(b) Electric field of negatively charged particle



2.3 Superposition of Electric Fields

Superposition: the combined electric field created by a collection of charged objects is equal to the vector sum of the electric fields created by the individual objects

Figure 23.9 The electric field due to multiple charged objects (here, a pair of charged particles) is the vector sum of the fields created by the individual objects.



2.4 Electric Fields and Forces

Uniform Electric Field: an electric field for which the direction and magnitude are the same everywhere

- no electric field is ever uniform throughout all space but it is possible to create regions of space with uniform electric fields

Nonuniform electric field: an electric field for which the direction and magnitude vary between positions

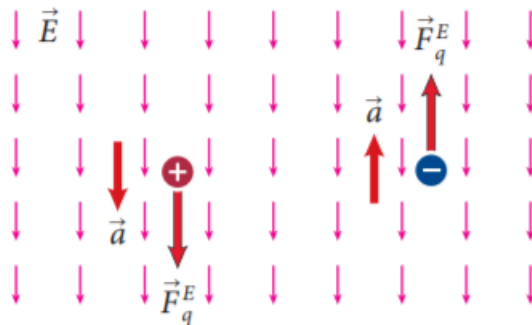
Constant Acceleration in a Uniform Electric Field

In a uniform electric field, the force \vec{F}_p^E exerted by an electric field \vec{E} on a particle carrying a charge q is $\vec{F}_p^E = q\vec{E}$. Because \vec{E} is the same everywhere, the force \vec{F}_p^E exerted on the particle is constant and so it undergoes a constant acceleration

$$\vec{a} = \frac{\vec{F}_p^E}{m} = \frac{q\vec{E}}{m} = \frac{q}{m}\vec{E},$$

where m is the particle's mass. Thus, **a charged particle placed in a uniform electric field undergoes constant acceleration.**

If the particle carries a positive charge, then $q > 0$, \vec{F}_p^E and \vec{a} point in the same direction as \vec{E} . If $q < 0$, \vec{F}_p^E and \vec{a} point in the direction opposite the direction of the electric field:



Note from $\vec{a} = \frac{q}{m}\vec{E}$ that the magnitude of the acceleration depends on the magnitude of the electric field and on the charge-to-mass ratio $\frac{q}{m}$ of the particle. A large charge q causes a greater force to be exerted on the particle and therefore a greater acceleration; a larger mass m means that the particle has greater inertia and therefore the acceleration is smaller.

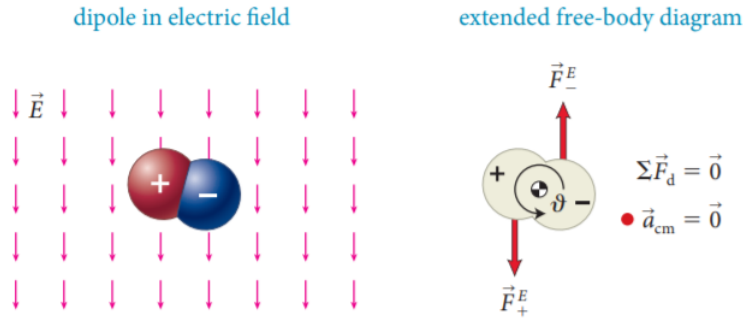
In general, the trajectory of charged particles in uniform electric fields is parabolic. In the special case where the initial velocity of a charged particle is parallel to the direction of the electric field, the trajectory is a straight line.

A positively charged particle placed in a nonuniform electric field has an acceleration in the same direction as the electric field; a negatively charged particle placed in such a field has an acceleration in the opposite direction.

Electric dipole: equal amounts of positive and negative charge separated by a small distance

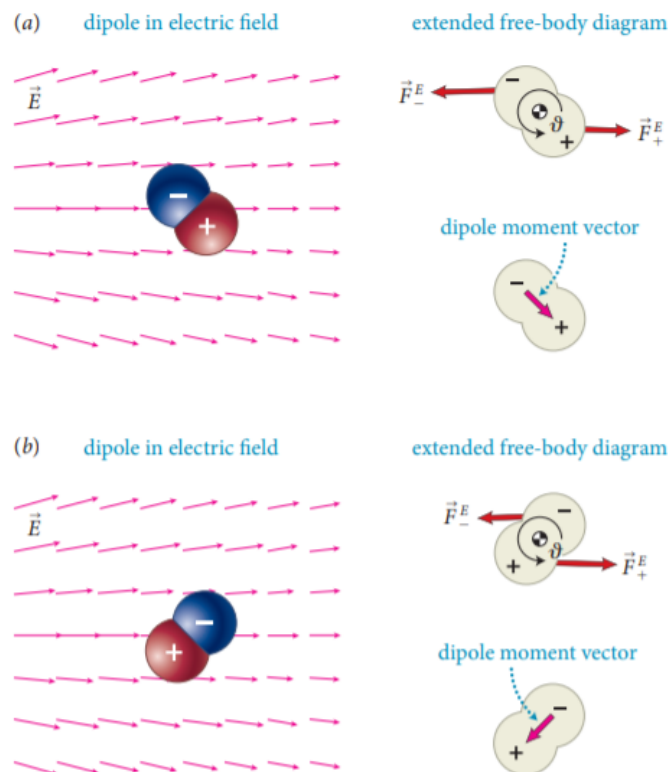
- many molecules such as water are *permanent dipole*, in which the centers of positive and negative charge are kept separated by some internal mechanism
- for a dipole in a uniform field, the magnitude of the charge on the positive end of the dipole is equal to the magnitude of the charge on the negative end and so they cancel each other and their vector sum is zero. However, these forces cause a torque:

Figure 23.17 Extended free-body diagram for a permanent dipole placed in a uniform electric field.



The orientation of a dipole can be characterized by the **dipole moment**, a vector that by definition points from the center of negative charge to the center of positive charge. A permanent electric dipole placed in an electric field is subject to a torque that tends to align the dipole moment with the direction of the electric field. If the field is uniform, the dipole has zero acceleration; if the electric field is nonuniform, the dipole has a nonzero acceleration.

Figure 23.18 Extended free-body diagrams for permanent dipoles in nonuniform electric fields. The electric field shown is due to a positively charged particle to the left of the figure.



2.5 Electric Field of a Charged Particle

Relating Force

The force exerted on any particle carrying charge q placed at a position with a known electric field is given by:

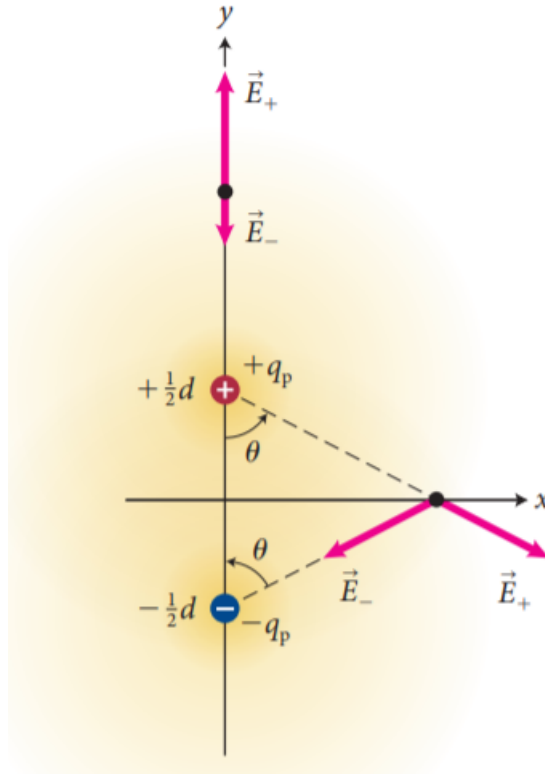
$$\vec{F}_p^E = q\vec{E}$$

2.6 Dipole Field

For a dipole consisting of a particle carrying a charge $+q_p$ at $x = 0, y = +\frac{1}{2}d$, and another particle carrying a charge $-q_p$ at $x = 0, y = -\frac{1}{2}d$, where d is the distance between the two particles. The charge q_p of the positively charged pole is called the **dipole charge**, and the distance d is called the **dipole separation**. Each particle creates an electric field and so the two fields overlap everywhere. The combined electric field at any position can be found by adding the two fields vectorially.

Along the x axis, which bisects, the dipole, the magnitudes of the electric fields due to the two ends of the dipole are equal s.t.

$$E_+ = E_- = k \frac{q_p}{x^2 + \left(\frac{d}{2}\right)^2}$$



The x components of the two electric fields point in opposite directions and so add to zero. The magnitude of the combined electric field is thus equal to the sum of the y components:

$$\begin{aligned}
 E_y &= E_{+y} + E_{-y} \\
 &= -(E_+ + E_-) \cos \theta \\
 &= -\left(2k \frac{q_p}{x^2 + \left(\frac{d}{2}\right)^2}\right) \left(\frac{\frac{d}{2}}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}\right) \\
 &= -k \frac{q_p d}{\left[x^2 + \left(\frac{d}{2}\right)^2\right]^{\frac{3}{2}}}
 \end{aligned}$$

The electric field far from the dipole is given by

- $E_y \approx -k \frac{p}{|x|^3}$ (along the x -axis)
- notice the dipole moment vector is toward the y -direction
- (along the x -axis) means your interest point is along the x -axis
- $E_y \approx 2k \frac{p}{|y|^3}$ (along the y -axis)
- Notice the dipole moment vector is toward the y -direction
- (along the y -axis) means your interest point is along the y -axis

This can be rewritten and simplified to

$$E_y = 2k \frac{p}{y^3}, \text{ where } y > \frac{d}{2}$$

2.7 Electric Fields of Continuous Charge Distributions

For the charged macroscopic object shown below, we can use Coulomb's law to obtain the infinitesimal portion of the electrical field at point P contributed by a segment:

$$d\vec{E}_s(P) = k \frac{dq_s}{r_{sP}^2} \vec{r}_{sP}$$

Using the principle of superposition, we can sum the contributions of all the segments making up the object:

$$\vec{E} = \int d\vec{E}_s = k \int \frac{dq_s}{r_{sP}^2} \vec{r}_{sP}$$

To evaluate this integral, dq_s , $\frac{1}{r_{sP}^2}$, and \vec{r}_{sP} must be expressed in terms of the same coordinate(s). To do, we express the charge on the object in terms of a **charge density**, the amount of charge per unit of length, per unit of surface area, or per unit of volume. For a 1D object, e.g. a thin, charged wire of length l carrying a charge q uniformly distributed, the **linear charge density** is given by

$$\lambda = \frac{q}{l}, \frac{\text{C}}{\text{m}}$$

For uniformly charged 2D objects, the **surface charge density** of an object with area A carrying a uniformly distributed charge q is

$$\sigma = \frac{q}{A}, \frac{\text{C}}{\text{m}^2}$$

For a uniformly charged 3D object, the **volume charge density** is given by

$$\rho = \frac{q}{V}, \frac{\text{C}}{\text{m}^3}$$

2.8 Dipoles in Electric Fields

An electric field will exert forces on the charged ends of the dipole equal in magnitude but opposite in direction, such that the torque is given by the cross product of the dipole moment and the electric field force:

$$\sum \vec{\tau} = \vec{p} \times \vec{E}$$

Along a dipole axis, the magnitude of the electric field created by the dipole is $2k\frac{p}{y^3}$, and so the magnitude of the force exerted by the dipole on the particle is

Magnitude of Force Exerted by Dipole and Particle

Magnitude of force exerted by dipole on particle:

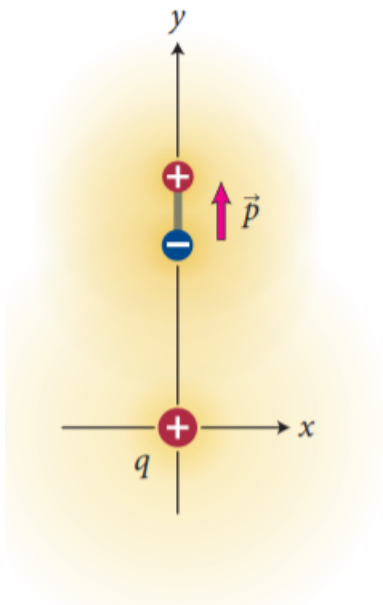
$$F_{dp}^E = qE_d = 2k\frac{pq}{y^3}$$

Magnitude of force exerted by particle on dipole:

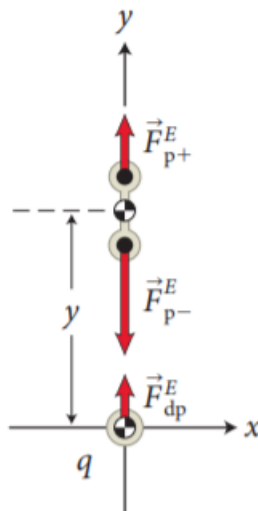
$$F_{pd}^E = F_{p-}^E - F_{p+}^E = 2k\frac{pq}{y^3}$$

The torque on the dipole is maximized when the dipole moment is perpendicular to the electric field and zero when it is parallel or anti-parallel to the electric field

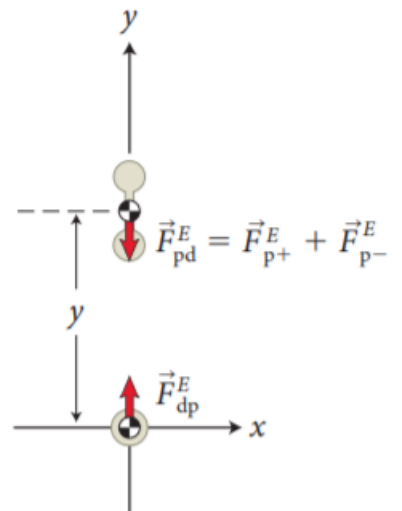
(a) Fields of charged particle and dipole



(b) Extended free-body diagrams



(c) Interaction pair



Induced dipole: a weak attraction that results when a polar molecule induces a dipole in an atom or in a nonpolar molecule by disturbing the arrangement of electrons in the nonpolar species

- when a neutral atom is placed in an electric field \vec{E} , as long as the electric forces exerted by that field on the charged particles in the atom are not too large, the induced dipole separation d_{ind} in the atom obeys Hooke's law.
- this means that the induced dipole separation is proportional to the magnitude of the applied electric force, $F_d^E = cd_{ind}$, where c is the "spring constant" of the atom.

Because d_{int} is proportional to the magnitude E of the electric field at the position of the dipole, the **induced dipole moment** is proportional to the field at the position of the dipole:

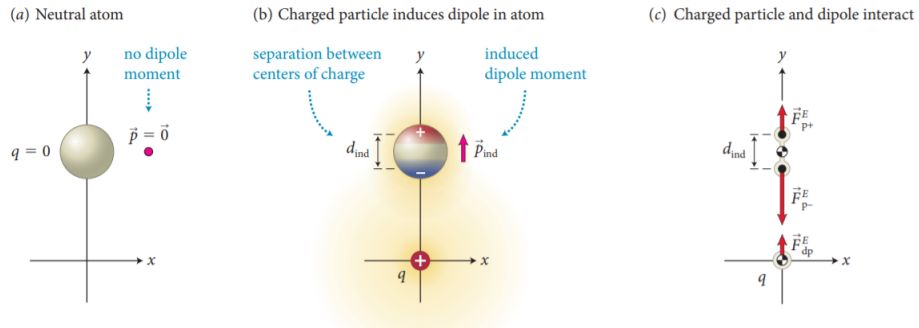
$$\vec{p}_{ind} = \alpha \vec{E}, (\vec{E} \text{ not too large}),$$

where α , the **polarizability** of the atom, is a constant that expresses how easily the charge distribution in the atom are displaced from each other.

- SI unit of polarizability: $\text{C}^2 \cdot \text{m}/\text{N}$.

A charged particle induces a dipole in an electrically neutral atom:

Substituting the induced-dipole result of the equation for p_{ind} into an earlier equation, we find that



Force exerted by charged particle on induced dipole

$$F_{pd}^E = 2k \frac{p_{ind} q}{y^3} = \alpha \frac{2k^2 q^2}{y^5}$$

From the equation above, we find that the interaction between a charged particle and a polarized object depend much more strongly on the distance between them $\left(\frac{1}{y^5}\right)$ than does the interaction between two charged objects $\left(\frac{1}{y^2}\right)$.

2.9 Chapter Glossary

Charge density a scalar that is a measure of the amount of charge per unit of length, area, or volume on a one-, two-, or three-dimensional object, respectively.

- Linear λ (C/m)
- Surface σ (C/m²)
- Volume ρ (C/m³)

Dipole: A neutral charge configuration in which the center of positive charge is separated from the center of negative charge by a small distance.

- can be *permanent*, or *induced* by an external electric field

Dipole moment (electric) \vec{p} (C · m): a vector defined as the product of the *dipole charge* q_p (the positive charge of the dipole) and the vector \vec{r}_p that points from the center of negative charge to the center of positive charge:

$$\vec{p} = q_p \vec{r}_p$$

Electric field \vec{E} (N/C): a vector equal to the electric field exerted on a charged test particle divided by the charge on the test particle:

$$\vec{E} = \frac{\vec{F}_t^E}{q_t}$$

Induced dipole: a separation of the positive and negative charge centers in an electrically neutral object caused by an external electric field

Induced dipole moment \vec{p}_{ind} (C · m): a dipole moment induced by an external electric field in an electrically neutral object.

- for small electric fields, the induced dipole moment in an atom is proportional to the applied electric field:

$$\vec{p}_{ind} = \alpha \vec{E}$$

where α is the *polarizability* of the atom.

Interaction field / field: a physical quantity surrounding objects that mediates an interaction

- objects that have mass are surrounded by a **gravitational field**
- objects that carry an electrical charge are surrounded by an **electric field**
- both are **vector fields** specified by a direction and a magnitude at each position in space

Polarizability α (C² · m/N): a scalar measure of the amount of charge separation that occurs in an atom or molecule in the presence of an externally applied electric field

Superposition of electric fields: the electric field of a collection of charged particles is equal to the vector sum of the electric fields created by the individual charged particles:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots$$

Test particle: an idealized particle whose physical properties (mass or charge) are so small that the particle does not perturb the particles or objects generating the field we are measuring

Vector field diagram: a diagram that represents a vector field, obtained by plotting field vectors at a series of locations.

3 Gauss's Law

3.1 Electric Field Lines

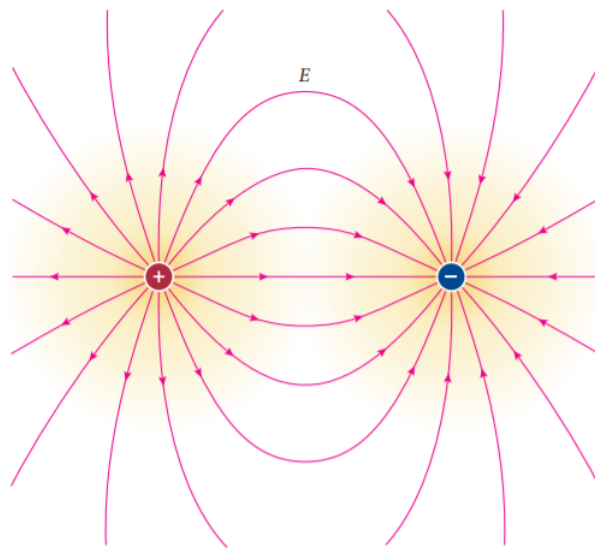
Gauss's Law is a relationship between an electric field and its source that can be used to determine the electric fields due to charge distributions that exhibit certain simple symmetries.

Electric field lines: a way to visualize electric fields in which lines are drawn so that at any location the electric field \vec{E} is tangent to them.

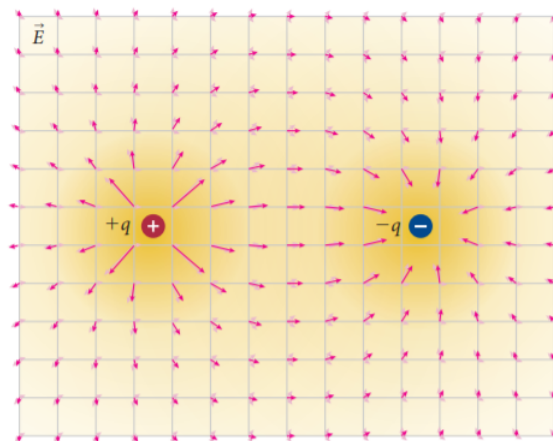
• are labeled with E to remind us that they represent an electrical field

• the number of field lines that emanate from a positively charged object or terminate on a negatively charged object is proportional to the charge carried by the object

Electric field lines:



A reminder of vector field diagrams:



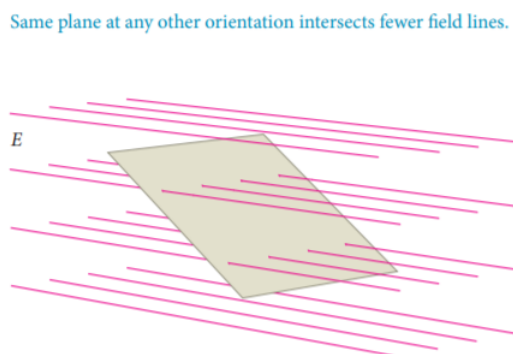
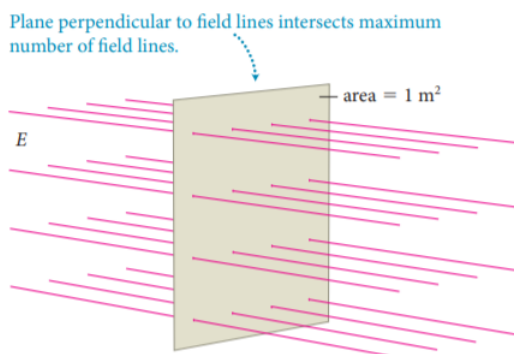
3.2 Field Line Density

The electric field and the number of field line crossings per unit area both decrease as $\frac{1}{r^2}$. This is expressed by the **field line density**, which is the number of field lines per unit area that cross a surface perpendicular to the field lines at a given position.

The below figure illustrates why the surface through which the field lines pass must be perpendicular to the field lines. The field represented by the field lines in the figure is uniform (magnitude and direction are same everywhere) and so the number of field lines that cross the surface depends on the orientation of the surface. We can then conclude that at every position in a field line diagram, the magnitude of the electric field is proportional to the field line density at that position.

Properties of Electric Field Lines:

1. Field lines emanate from positively charged objects and terminate on negatively charged objects
2. At every position, the direction of the electric field is given by the direction of the tangent to the electric field line through that position
3. Field lines never intersect or touch
4. The number of field lines emanating from or terminating on a charged object is proportional to the magnitude of the charge on the object
5. At every position, the magnitude of the electric field is proportional to the field line density



3.3 Closed Surfaces

Whenever a charged particle is placed inside a hollow spherical surface, the number of field lines that pierce the surface is the same *regardless of where inside the surface the particle is placed*. This is because so long as the charged particle is inside the surface, all the field lines emanating from the particle must go through the spherical surface. In fact, a sphere is not required and any other surface enclosing the charged particle will do, such as a cube.

Closed surface: a surface that completely encloses a volume

Enclosed charge: the sum of all charge enclosed by a closed surface

Field line flux: the number of outward field lines crossing a closed surface minus the number of inward field lines crossing the surface

- equal to the charge enclosed by the surface multiplied by the number of field lines per unit charge.
- is always zero when through a closed surface due to charged objects outside the volume enclosed by that surface.

Example: Consider the 3D dipole field line diagram shown below. Six field lines emanate from the positively charged end, and six terminate on the negatively charged end.

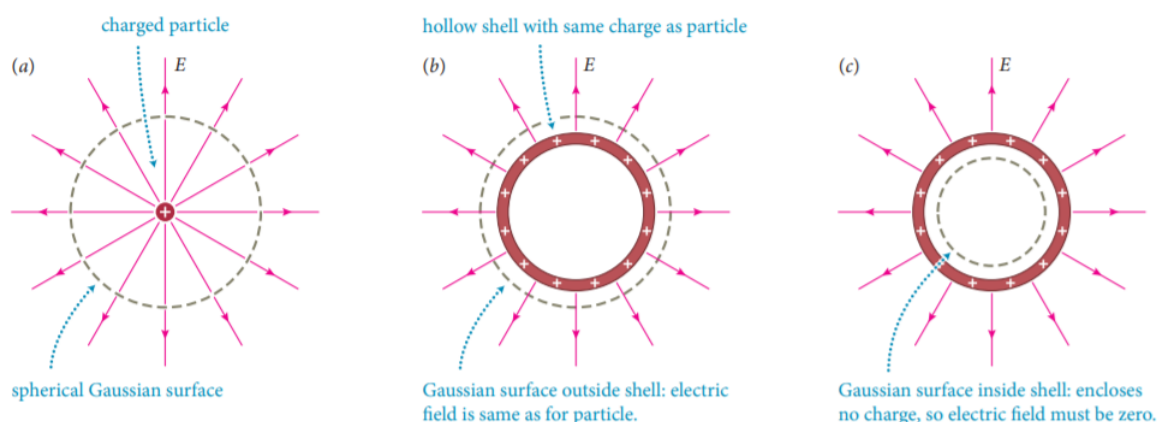
(a) Six field lines emanate from the positively charged particle. Each line crosses the surface of the cube in the outward direction and thus contributes a value of $+1$ to the field line flux. The field line flux is $+6$.

(b) Six field lines terminate on the negatively charged particle, so there are again six field line crossings. However, these field lines are directed inwards and so the field line flux is -5 .

3.4 Symmetry and Gaussian Surfaces

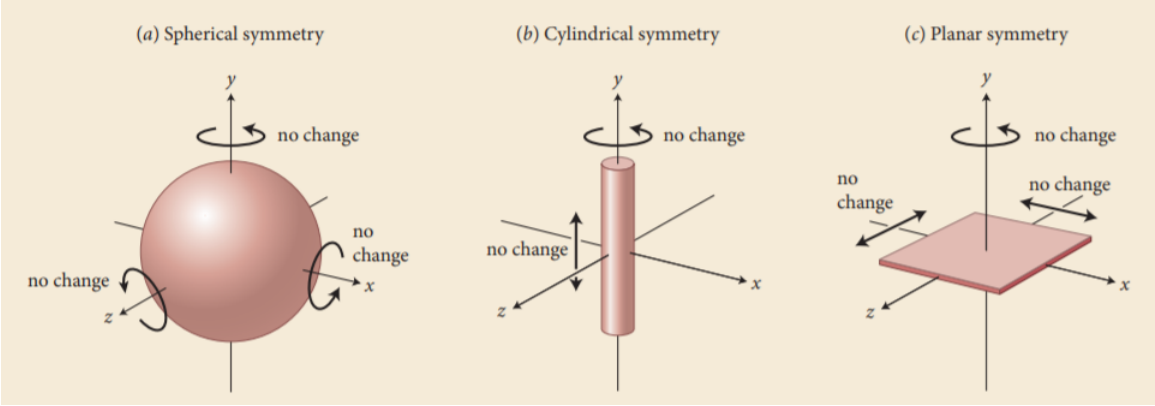
The relationship between field line flux and enclosed charge implies several important conclusions about charged objects and their electric fields without having to perform any calculations. To apply this relationship to a given situation, select a closed surface called a **Gaussian surface** that does not necessarily correspond to a real object, i.e. it can be any surface, real or imagined. The choice of surface is determined by the symmetry of the context. As a rule of thumb, choose a surface such that the electric field is the same (and possibly zero) everywhere along as many regions of the surface as possible, because such a choice makes it easy to determine the field line flux through the surface.

Consider the charged particle shown below. The field is symmetrical in all three dimensions - it has the same magnitude at the same distance from the center in any direction. Thus, if we draw a spherical Gaussian surface concentric with the particle, the magnitude of the electric field is the same at all locations on the sphere.

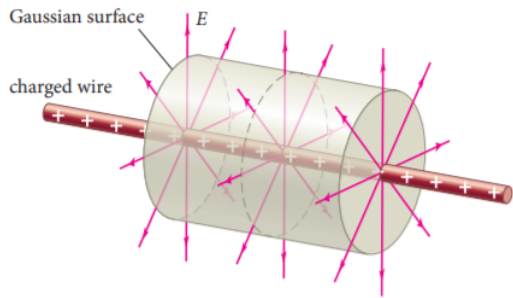


The electric field outside a uniformly charged spherical shell is the same as the electric field due to a particle that carries an equal charge located at the center of the shell. Because the electric field can only be radially outward by symmetry, the electric field must be zero everywhere on the Gaussian surface. Because the Gaussian surface's radius is arbitrary and can be changed from zero to the inner radius of the shell, it can be concluded that in the absence of other charged objects, the electric field in the space enclosed by a uniformly charged spherical shell is zero everywhere in the enclosed space.

Three Types of Symmetries Important for Applications of Gauss’s Law:



Example of Concentric Cylindrical Gaussian Surface



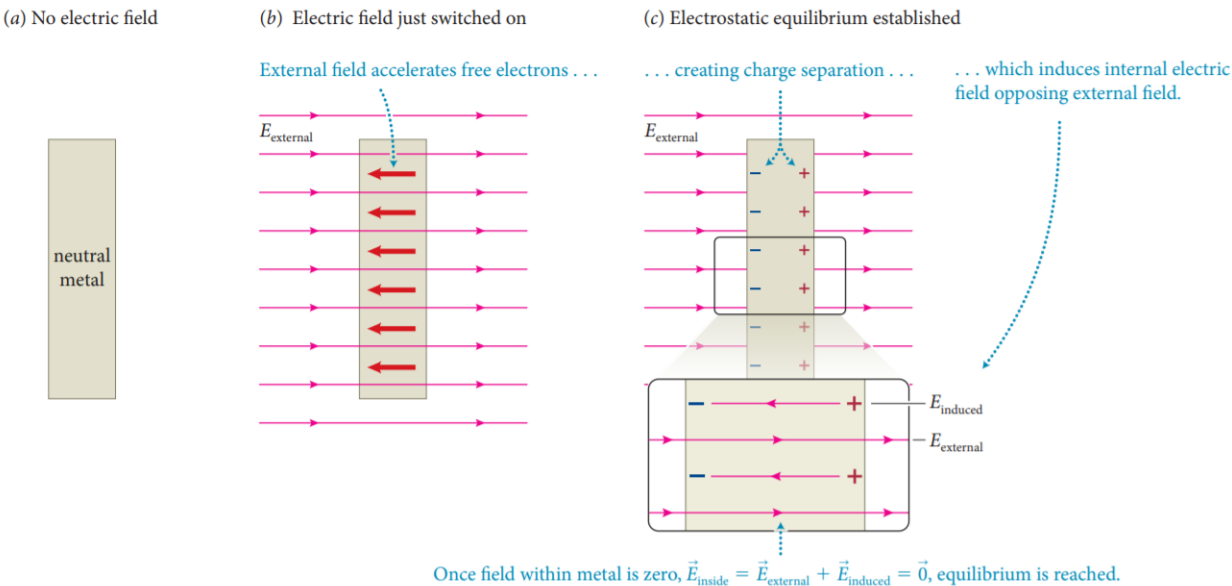
3.5 Charged Conducting Objects

Recall that conducting objects contain many charge carriers that are free to move, such as electrons (in a metal) or ions (in a liquid conductor). The material as a whole can still be electrically neutral. Due to this free motion of charged particles within a conducting object, the particles always arrange themselves in such a way as to make the electric field inside the bulk of the object zero.

Electrostatic equilibrium: the condition in which the distribution of charge in a system does not change

- time for a metal to reach electrostatic equilibrium is very short (about 10^{-16} s)
- the electric field inside a conducting object that is in electrostatic equilibrium is zero

Why electric field inside bulk of conducting object in electrostatic equilibrium is zero



Because the electric field inside a conducting object in electrostatic equilibrium is zero, we conclude that there cannot be any surplus charge inside the object. Hence, any surplus charge placed on an isolating conducting object arranges itself at the surface of the object. No surplus charge remains in the body of the conducting object once it has reached electrostatic equilibrium. In electrostatic equilibrium, the electric field at the surface of a conducting object is perpendicular to that surface.

3.6 Electric Flux

Electric flux: the arbitrary number of electric field lines that intersect a given area, represented by the symbol ϕ_E

- magnitude of electric flux through a surface with area A in a uniform electric field of magnitude E is defined as

$$\phi_E = EA \cos \theta, \text{ (uniform electric field),}$$

where θ is the angle between the electric field and the normal to the surface. If we define an area vector \vec{A} for a flat surface area as a vector whose magnitude is equal to the surface area A and whose direction is normal to the plane of the area. On closed surfaces \vec{A} is chosen to point outward. Using this definition, we can rewrite the equation for ϕ_E as

Electric flux for uniform electric fields and flat surfaces

$$\phi_E EA \cos \theta = \vec{E} \cdot \vec{A}, \text{ (uniform electric field)}$$

Electric flux general form

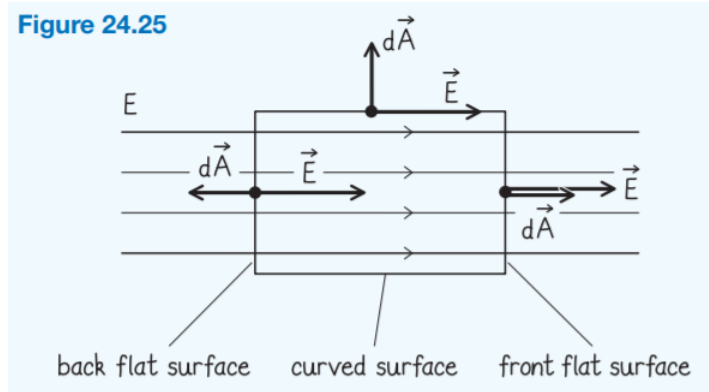
$$\phi_E = \int \vec{E} \cdot d\vec{A}$$

The integral above is a **surface integral** as $d\vec{A}$ is the area vector of an infinitesimally small surface segment. If the surface is closed, this surface integral is written like so:

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

where the circle indicates that integration is taken over the entire closed surface and $d\vec{A}$ is chosen to point outward.

For a cylindrical Gaussian surface, we have



$$\begin{aligned} \phi_E &= \oint \vec{E} \cdot d\vec{A} \\ &= (\text{back flat surface}) + (\text{curved surface}) + (\text{front flat surface}) \\ &= \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} + \int \vec{E} \cdot d\vec{A} \end{aligned}$$

From the sketch above, the angle between \vec{E} and $d\vec{A}$ is 180° , so $\vec{E} \cdot d\vec{A} = E(\cos 180^\circ)dA = -EdA$. Because the magnitude of the electric field is the same everywhere, E can be pulled out of the integral s.t.

$$\begin{aligned} \text{back flat surface} &= \int \vec{E} \cdot d\vec{A} \\ &= \int (-E)dA \\ &= -E \int dA \\ &= -E(\pi r^2) \end{aligned}$$

The integral over the curved region yields a value of zero because the angle between \vec{E} and $d\vec{A}$ is 90° everywhere on the curved region. For the front flat surface, $\vec{E} \cdot d\vec{A} = E(\cos 0^\circ)dA = EdA$, so

$$\int \vec{E} \cdot d\vec{A} = \int EdA = E \int dA = E(\pi r^2)$$

Summing the corresponding integrals for each of the three surfaces, it follows that

$$\phi_E = -E(\pi r^2) + 0 + E(\pi r^2) = 0$$

This result tells us that because there is no charge enclosed by the Gaussian surface, the field line flux through the surface must be zero.

3.7 Deriving Gauss's Law

The electric flux through a spherical Gaussian surface is equal to the charge q enclosed by the sphere times $4\pi k$, where $k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$ is the proportionality constant that appears in Coulomb's law. This relationship is written in the form

$$\Phi_E = 4\pi k q = \frac{q}{\epsilon_0},$$

where ϵ_0 is known as the **electric constant**:

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85418782 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$

The first equation in this subsection is a special case of **Gauss's Law**, which states that the electric flux through the closed surface of any arbitrary volume is

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0},$$

where q_{enc} is an enclosed charge.

Because Gauss's Law depends on $\frac{1}{r^2}$ and the superposition of electric fields, it can be derived from Coulomb's Law. It is used to greatly simplify calculations containing significant amounts of symmetry.

3.8 Applying Gauss's Law

To see the benefit of Gauss's Law, consider a charged spherical shell with radius r , carrying a uniformly distributed positive charge q . The electric field due to this charged shell can be calculated using a surface integral, but exploiting symmetry, the answer can be calculated in a much simpler manner. Draw a concentric Gaussian surface of radius $R > r$ around the shell. By Gauss's Law,

$$\Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

Because the Gaussian surface we drew has spherical symmetry, the electric field E will be constant at every point and perpendicular to the surface, and so we can pull it out of the integral:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA = E \oint dA = EA$$

where A is the area of the Gaussian surface:

$$A = 4\pi r^2$$

It follows then that

$$\begin{aligned} \frac{q}{\epsilon_0} &= 4\pi r^2 E \\ E &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = k \frac{q}{r^2} \end{aligned}$$

4 Work and Energy in Electrostatics

4.1 Electrical Potential Energy

4.2 Electrostatic Work

4.3 Equipotentials

4.4 Calculating Work and Energy in Electrostatics

4.5 Potential Difference

4.6 Electric Potentials of Continuous Charge Distributions

4.7 Obtaining the Electric Field from the Potential

