

1. Compute $f'(x)$ for

$$(a) f(x) = \int_x^{\sin(x)} e^{t^2} dt \quad \text{and} \quad (b) f(x) = x^x, x > 0.$$

(Simplify as much as possible.)

2. Evaluate the following integrals. (Simplify as much as possible).

$$(a) \int_0^{\pi/2} \frac{\sin(x) \cos(x)}{1 + \sin^2(x)} dx \quad \text{and} \quad (b) \int_e^{e^2} \frac{1}{x \ln x} dx$$

3. Let Ω be the region inside the triangle with vertices at the points $(0,0)$, $(1,1)$, and $(1,-1)$. Find the volume obtained by revolving Ω about the y -axis.
4. Find all points of inflection of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by the formula

$$f(x) = \int_0^x e^{-(t-3)^4} dt$$

(If there are no points of inflection, explain why; if there are points of inflection explain why they are points of inflection.)

5. Let $f : [0, 1] \rightarrow \mathbb{R}$ be an increasing function. For each positive integer n , let

$$P_n = \left\{ 0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n}{n} \right\},$$

(the uniform partition of the interval $[0, 1]$ into n subintervals).

Show that $U_f(P_n) - L_f(P_n) \leq \frac{f(1) - f(0)}{n}$.