Sudoku

章 宇 ZHANG Yu

y.zhang@swufe.edu.cn

Sudoku

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

- Developed by Howard Garns and first published in 1979
- Popularized in Japan since 1980's
- Rules: to fill a 9x9 grid with digits so that
  - Each column contains all the digits from 1 to 9
  - Each row contains all the digits from 1 to 9
  - Each 3x3 block contains all the digits from 1 to 9
- Many algorithms available

Let's play…

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Sudoku – optimization model

- Decision variables: $x_{ijk} \in \{0,1\}$
  - whether cell (i,j) contains digit k (binary)

- Objective ???
  - Nothing to maximize or minimize
  - Just looking for a feasible solution
  - *Feasibility Problems*

Sudoku – optimization model

- Constraints:
  - Only one k in each column
  - Only one k in each row
  - Only one k in each 3x3 block
  - All cells must be filled
  - The set of initially given numbers cannot be changed
  - Binary

Sudoku – optimization model

- Model

$$\min \quad \mathbf{0}'\boldsymbol{x}$$

$$\text{s.t.} \quad \sum_{i=1}^{9} x_{ijk} = 1, \qquad\qquad \forall j,k \in \{1,\ldots,9\} \qquad\qquad \text{: one k in each column}$$

$$\sum_{j=1}^{9} x_{ijk} = 1, \qquad\qquad \forall i,k \in \{1,\ldots,9\} \qquad\qquad \text{: one k in each row}$$

$$\sum_{j=3(q-1)+1}^{3q} \sum_{i=3(p-1)+1}^{3p} x_{ijk} = 1, \quad \forall k \in \{1,\ldots,9\}, p,q \in \{1,2,3\} \quad \text{: one k in each submatrix}$$

$$\sum_{k=1}^{9} x_{ijk} = 1, \qquad\qquad \forall i,j \in \{1,\ldots,9\} \qquad\qquad \text{: position must be filled}$$

$$x_{ijk} = 1 \quad \forall (i,j,k) \in G \qquad\qquad\qquad\qquad \text{: elements set to on}$$

$$x_{ijk} \in \{0,1\} \qquad\qquad \forall i,j,k \in \{1,\ldots,9\}$$

Exercise…