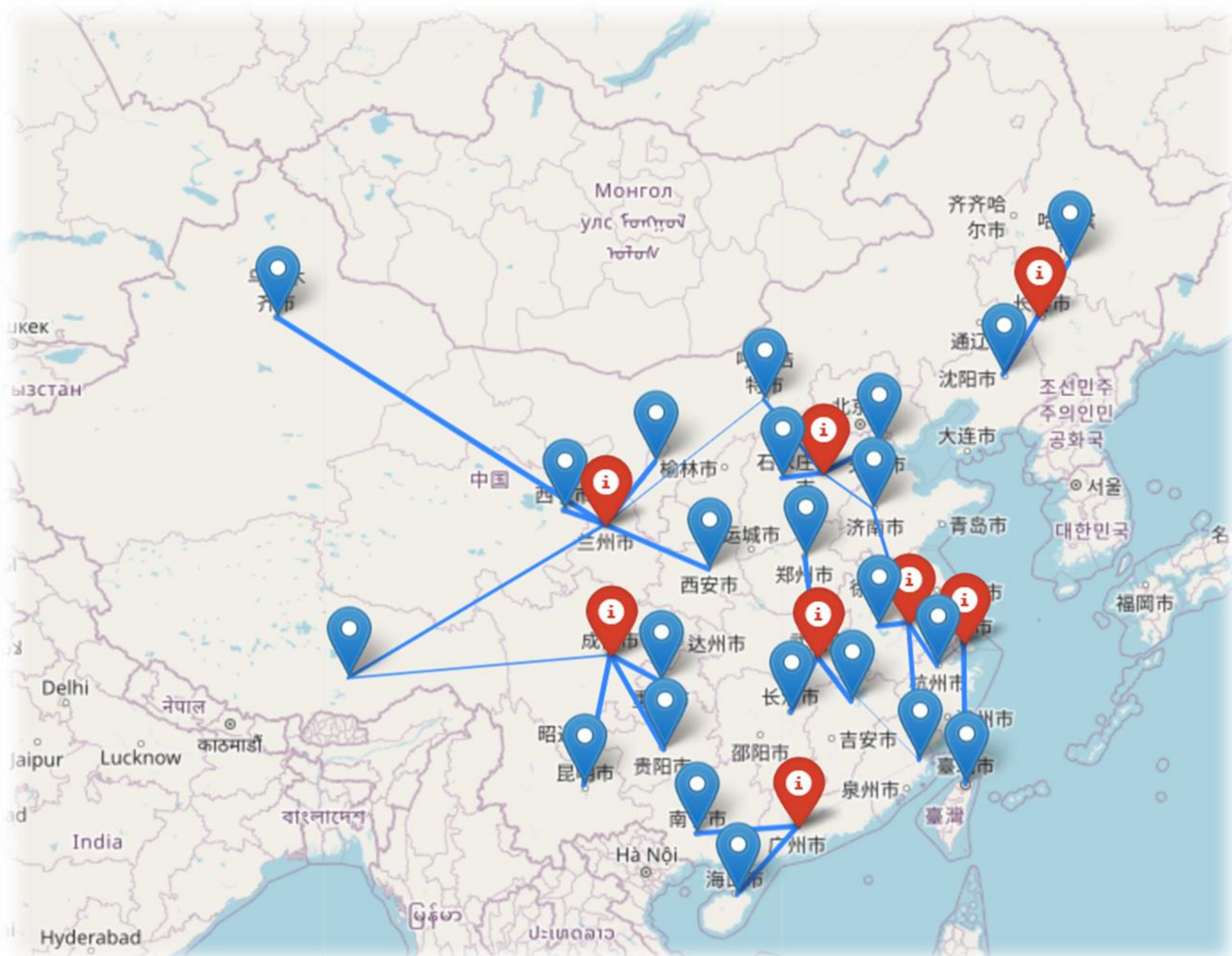
Facility Location

章 宇 ZHANG Yu
y.zhang@swufe.edu.cn

Facility Location



Facility Location

- Industrial applications
  - Location of chain stores
  - Location of fire stations
  - Location of chemical plants
  - ……
- Academic researches
  - Operations Research, Combinatorics ……
  - Logistics, Transportation, Communications……

Facility Location

- Many variants
  - p-median problem
  - p-center problem
  - Fixed-charge facility location
  - Covering location problem
  - Anti-covering problem
  - Facility location under uncertainty
  - ……

p-Median Problem

- Parameters
  - $\mathcal{I} = \{1,2,\ldots,n\}$: set of customers
  - $\mathcal{J} = \{1,2,\ldots,m\}$: set of candidate facility sites
  - c_{ij}: distance/cost to serve customer $i \in \mathcal{I}$ by facility $j \in \mathcal{J}$
  - p: number of facilities to open

p-Median Problem

- Decision Variables
  - $x_j \in \{0,1\}$: =1 iff facility $j \in \mathcal{J}$ will be open
    - "iff" = if and only if
  - $y_{ij} \in \{0,1\}$: =1 iff customer $i \in \mathcal{I}$ is assigned to facility $j \in \mathcal{J}$ to be served

- Problem
  - Determine p facilities to open and assign customers to facilities
  - Minimize total service distance/cost

p-Median Problem

- Model

$$\min$$

$$\text{s.t.}$$

$$x_j \in \{0,1\}, \qquad \forall j \in \mathcal{J},$$

$$y_{ij} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}.$$

- Minimize total distance/cost
- Each customer is served
- Open p facilities
- Can serve if open

p-Center Problem

- Problem
  - Similar to p-median problem
  - But minimizes the maximum (instead of total) customer-to-facility distance

- Application
  - Consider fairness
  - Usually applied in public facilities
    - Fire stations
    - Ambulance sites

p-Center Problem

- Model

$$\min$$

$$\text{s.t.} \sum_{j \in \mathcal{J}} y_{ij} = 1, \qquad \forall i \in \mathcal{I},$$

$$\sum_{j \in \mathcal{J}} x_j = p,$$

$$y_{ij} \le x_j, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J},$$

$$x_j \in \{0,1\}, \qquad \forall j \in \mathcal{J},$$

$$y_{ij} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}.$$

- Minimize the maximum distance

Discussion:
How to solve it?

p-Center Problem

- Model linearization

$$\min$$

$$\text{s.t.}$$

$$\sum_{j \in \mathcal{J}} y_{ij} = 1, \qquad \forall i \in \mathcal{I},$$

$$\sum_{j \in \mathcal{J}} x_j = p,$$

$$y_{ij} \le x_j, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J},$$

$$x_j \in \{0,1\}, \qquad \forall j \in \mathcal{J},$$

$$y_{ij} \in \{0,1\}, \qquad \forall i \in \mathcal{I}, j \in \mathcal{J}.$$

p-Center Problem

- Example
  - Open 6 warehouses to store relief materials

  - Can we improve the model?

Coding…

Fixed-Charge Facility Location

- Parameters
  - $\mathcal{I} = \{1,2,\ldots,n\}$: set of customers
  - $\mathcal{J} = \{1,2,\ldots,m\}$: set of candidate facility sites
  - d_i: demand of customer $i \in \mathcal{I}$
  - f_j: fixed-charge cost to open facility $j \in \mathcal{J}$
  - v_j: service capacity of facility $j \in \mathcal{J}$
  - c_{ij}: per-unit cost to serve customer $i \in \mathcal{I}$ from facility $j \in \mathcal{J}$

Fixed-Charge Facility Location

- Decision variables
  - $x_j \in \{0,1\}$: =1 iff facility $j \in \mathcal{J}$ will be open
  - $y_{ij} \in [0,1]$: fraction of demand from customer $i \in \mathcal{I}$ served by facility $j \in \mathcal{J}$
- Problem
  - Determine facilities to open and assign demands to facilities
  - Minimize the total cost (fixed and variable costs)

Fixed-Charge Facility Location

- Model

$\min$

$\mathrm{s.t.}$

- Minimize total cost
- Each customer is served
- Facility capacity constraint

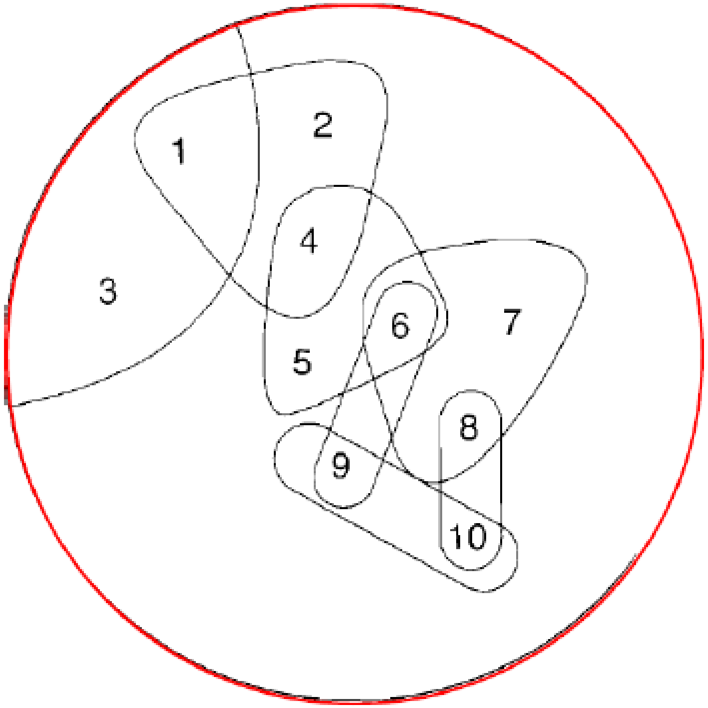
Exercise …

Homework

- A grocery store chain now has too many stores in close proximity to each other in certain cities.
- In a city the chain has 10 stores, and it does not want any stores closer than 2 miles to each other.
- Following are the monthly revenues ($1,000s) from each store and a map showing the general proximity of the stores. Stores within 2 miles of each other are circled. (next page)
- Develop and solve an integer programming model to determine which stores the grocery store chain should keep open while maximizing the revenue of the open stores.

Homework

Store	Monthly Revenue ($1,000s)
1	$127
2	83
3	165
4	96
5	112
6	88
7	135
8	141
9	117
10	94



Homework

- Submission
  - Model & results (.pdf)
    - Decision variable, objective, and constraints
    - Which stores to keep open
    - The revenue of the open stores
  - Source code (.py)
  - Submit to a link to be given in the QQ group

- Deadline: before next class
  - Tardiness: -5 points per day