CS 180, Fall 2015 Homework 5

The following homework is due on Wednesday, November 4th at the beginning of lecture.

When submitting your homework, please include your name at the top of each page. If you submit multiple pages, please staple them together. Late submissions are not accepted.

1. In a country called "no-greedy", they use coins denomination of k different positive integer values $v_1, \ldots, v_k$ in an increasing order and v_1 is 1.The goal is to generate change for a given integer value m using the minimum number of coins.

   (a) The greedy algorithm to generate coin changes works as follows. It starts with value m and deducts the value of the largest coin denomination v_i that is not larger than m. Then it repeats this greedy procedure on the remaining value. Show by counterexample that this greedy algorithm doesn't work for all choices of coin denominations. (Note: You may need to choose coin denominations carefully.)

   (b) Give an algorithm that given coin denominations $v_1, \ldots, v_n$ and target value m, generates change for m using the minimum number of coins. Explain why your algorithm works.

2. We have a number of n files, and file i has a length ℓ_i and probability p_i that whether it is accessed, and we want to write these files on a tape in some order. We assume that every time we need to access a file, we have to start from the beginning of tape until we reach the start of the requested file, so the time to access a file is proportional to the total length of all the files that are saved before it. Design an algorithm that finds the optimal order of files on the tape. The optimal order is an order that minimizes the average access time. The average access time is $\sum_{i=1}^{n} d_i p_i$ where d_i is distance of the beginning of file i from the beginning of the tape (the total length of all the files that is saved before file i).

3. Consider the problem of printing a paragraph with a monospaced font (all characters having the same width) on a printer. The input text is a sequence of n words of lengths $l_1, l_2, ..., l_n$, measured in characters. We want to print this paragraph neatly on a number of lines that hold a maximum of M characters each. If a given line contains words i through j, where $i \leq j$, and we leave exactly one space between words, the number of extra space characters at the end of the line is $M - j + i - \sum_{k=i}^{j} l_k$, which must be nonnegative so that the words fit on the line. We wish to minimize the sum, over all lines except the last, of the cube of sum of the numbers of extra space characters at the ends of lines. Give an algorithm to print a paragraph of n words on a printer in a way that minimizes the above sum. Analyze the running time and space requirements of your algorithm.

4. A palindrome is any string that is exactly the same as its reversal, like I, or DEED, or RACECAR, or AMANAPLANACATACANALPANAMA. Describe and analyze an algorithm to find the length of the longest subsequence of a given string that is also a palindrome. For example, the longest palindrome subsequence of MAHDYNAMICPROGRAMZLETMESHOWYOUTHEM is MHYMRORMYHM, so given that string as input, your algorithm should output the number 11.
   Give an algorithm or reduce the problem to another problem that you have seen.