CS 180, Fall 2015 Homework 8

The following homework is due on Wednesday, December 2nd at the beginning of lecture.

When submitting your homework, please include your name at the top of each page. If you submit multiple pages, please staple them together. Late submissions are not accepted.

We, the Professor and us the TAs, are still at a loss to reduce dominating-set to vertex-cover in an elegant way (in class we reduced vertex-cover to dominating-set), hence we cannot give this problem that we haven't solved for homework, as promised.

1. In a class, there are n boys and n women. For some positive integer k, every boy knows exactly k girls and every girl knows exactly k boys. Assume the "knowing" relationship is mutual. Show that there is a way to arrange k rounds of dances such that where every one is dancing with a partner that she/he knows, and no one dances with the same partner in two different rounds.

2. Given an undirected graph G and a pair of vertices s, t in G, *Hamiltonian Path* problem asks whether there is a simple path from s to t that visits every vertex of G exactly once. Given an undirected graph G, the *Hamiltonian Cycle* problem asks if there is a cycle in G that visits every vertex of G exactly once. Show that Hamiltonian Path and Hamiltonian Cycle problems are polynomial-time reducible to each other.

3. A boolean formula is in disjunctive normal form (or DNF) if it consists of a disjunction (Or) or several terms, each of which is the conjunction (And) of one or more literals. For example, the formula $(x \wedge y \wedge z) \vee (y \wedge z) \vee (x \wedge y \wedge z)$ is in disjunctive normal form. DNF-SAT asks, given a boolean formula in disjunctive normal form, whether that formula is satisfiable.

   (a) Describe a polynomial-time algorithm to solve DNF-SAT.

   (b) What is the error in the following argument that P=NP?

   Suppose we are given a boolean formula in conjunctive normal form with at most three literals per clause, and we want to know if it is satisfiable. We can use the distributive law to construct an equivalent formula in disjunctive normal form. For example,

   $$(x \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{y}) \iff (x \wedge \bar{y}) \vee (y \wedge \bar{x}) \vee (\bar{z} \wedge \bar{x}) \vee (\bar{z} \wedge \bar{y})$$

   Now we can use the algorithm from part (a) to determine, in polynomial time, whether the resulting DNF formula is satisfiable. We have just solved 3SAT in polynomial time. Since 3SAT is NP-Complete, we must conclude that P=NP and win the The Millennium Prize !

4. Given a directed graph G and a positive integer k, *Minimum-feedback Vertex Set* (MFV) problem asks if there is a subset of vertices of size $\leq k$ such that removing those vertices makes the graph G acyclic. Show a polynomial-time reduction from *Vertex Cover* problem to the Minimum-feedback Vertex Set problem (i.e. give a poly-time algorithm for vertex-cover when you can call MFV as a subroutine at the cost of 1).