Eric Yang

304263623

CS 180

Dis 1B

Homework 8

1. To arrange k rounds of dances such that every n boy and n girl is dancing with a partner they know, we can create a bipartite graph with n boys on one side and n girls on the other. There are edges directed from a boy to a girl if they know each other. Next we add a node s that has edges to every single boy. Then we add a node t with edges from every single girl. Each edge in this graph will have capacity 1. Using the Ford Fulkerson algorithm, we can mazimize the flow in this graph and the edges with flow will tell us the matching for the first round. We repeat this process k-1 times while removing the edges that have already been used. This should give k rounds of dances.
2. We can show that the Hamiltonian path and Hamiltonian cycle problems are polynomial time reducible to each other by proving Hamiltonian cycle ≤ Hamiltonian path and Hamiltonian cycle ≤ Hamiltonian path.

First, to prove Hamiltonian cycle ≤ Hamiltonian path, we construct from a graph G with vector set V and edge set E another graph G’ such that G’ has a Hamiltonian path if and only if G has a Hamiltonian cycle. To create G’, we choose an arbitrary vertex v in G and replace it with two nodes v’ and v’’. All edges out of v are now out of v’ and all edges into v are now into v’’. Suppose G has a Hamiltonian cycle. If we traverse the cycle starting at v, we can see that the same order of nodes in the cycle form a Hamiltonian path in G’ beginning at v’ and ending at v’’. Now suppose G’ has a Hamiltonian path that starts at v’ and ends at v’’. If replace v’ and v’’ with v, this ordering will form a hamiltonian graph in G. Thus, the Hamiltonian cycle can be reduced to Hamiltonian path.

Second, to prove Hamiltonian cycle ≤ Hamiltonian path, we construct from G(V,E) another graph G’ such that there is an extra node v’ in G’ that is connected to every other node. Suppose G has a Hamiltonian path from v0 to vn, then G’ must have a Hamiltonian cycle starting from v’ to v0, following the Hamiltonian path to vn and then returning to v’. Now suppose G’ has a Hamiltonian cycle v’ -> v0…->vn -> v’, then G must have a Hamiltonian path if we remove v’. Thus, the Hamiltonian path can be reduced to Hamiltonian cycle.

1. A. We can use an algorithm that will check each clause to make sure that a literal x and do not exist in the same clause. If they do exist, then the Boolean formula is no satisfiable. If they don’t exist, then the formula is satisfiable if we just assign 1 to every literal without - and 0 to every literal with – thus making one clause output 1.

B. The reduction shown is not polynomial time. We converted a conjunctive normal form formula (n clauses) into a disjunctive normal form formula with 3^n clauses.

1. First we look at an instance of vector cover with undirected graph G and an integer k. We create a new graph G’ from G with the same vertices but replace all edges with two directed edges going to and from each end. G’ will be the instance of minimum feedback vertex set.

First, we assume there is a vertex cover in G with size of k. Then we remove these vertices from G’ and their attached edges. This means that every directed edge in G’ must have had at least 1 vertex removed from it. Therefore, no vertices in G’ have an edge between them so there can also be no cycles.

Conversely, we assume there is a set of k vertices in G’ that we can remove to get rid of cycles. Since we constructed G’ from G, every pair of vertices in G must have a cycle in G’. Since we must have removed at least one of the nodes in each edge in G, the set of k vertices must be a vertex cover of G.