

ADVANCED UNDERGRADUATE LABORATORY

EXPERIMENT 19

ELECTRICAL RESISTIVITY OF METALS

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Theory

One of the prominent characteristics of metals compared with other solids such as insulators or semiconductors, (see Experiment 20), is their high electrical conductivity, σ . This results from the large number, n , of electrons per unit volume, typically $n \approx 5 \times 10^{22} \text{ cm}^{-3}$, corresponding to about 1 electron per atom which is free to carry electric current.

For an applied electric field, \mathbf{E} , the current density, \mathbf{J} , is determined by n , the electric charge, e , and the mass, m . We also need another quantity, the average time between collisions of the electron, with thermally vibrating atoms or with impurities, which restore the electron to its average velocity corresponding to zero applied field. We denote this time by $\Delta t = 2\tau$, and call τ the relaxation time. Thus the average drift velocity \bar{v} of the electrons in the field \mathbf{E} is given by:

$$\bar{v} = \frac{v_{\max} + 0}{2} = \frac{a\Delta t}{2} = a\tau$$

where the magnitude of a is given by Newton's 2nd Law:

$$a = \frac{F}{m} = \frac{eE}{m}$$

so that

$$\bar{v} = \frac{eE\tau}{m} \quad (1)$$

and the magnitude of the current density is

$$J = ne\bar{v} = \frac{ne^2\tau}{m} E \quad (2)$$

which, when compared with Ohm's Law, $\mathbf{J} = \sigma\mathbf{E}$, gives

$$\sigma = \frac{ne^2\tau}{m} \quad (3)$$

If we substitute the known values of n , e and m for a typical metal like copper, for which $\sigma = 0.6 \times 10^6 \Omega^{-1} \text{ cm}^{-1}$ at room temperature, we obtain $\tau \approx 2 \times 10^{-14} \text{ s}$.

The relaxation time, τ , depends upon the amplitude of the thermal vibrations of the atoms, which decreases with decreasing temperature, and also upon the concentration and nature of the impurities. The probability of scattering per unit time, which is inversely proportional to the relaxation time, τ , is the sum of probabilities of scattering by thermal vibrations and by impurities, so we can write

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{ideal}}} + \frac{1}{\tau_{\text{impurities}}} \quad (4)$$

Since, from equation (3), the resistivity $\rho = 1/\sigma \propto 1/\tau$, we have

$$\rho = \rho_{\text{ideal}} + \rho_{\text{impurities}} = \rho_i + \rho_o \quad (5)$$

where their ideal resistivity, ρ_i , is that of a perfect single crystal sample with no impurities while ρ_o is the resistivity contribution due to impurities.

One finds experimentally that ρ_i and ρ_o can be separated by measuring the temperature dependence of the resistivity. As shown schematically in Figure 1, ρ_o is independent of temperature, whereas ρ_i varies with temperature. θ_D is the Debye temperature, the characteristic temperature of the thermal vibrations of the crystal lattice. The value of θ_D for several metals is given in Table 1. For temperatures much lower than θ_D , the ideal resistivity should vary as $\rho_i(T) \propto T^5$, where T is measured in Kelvin. For temperatures much higher than θ_D , $\rho_i(T) \propto T$.

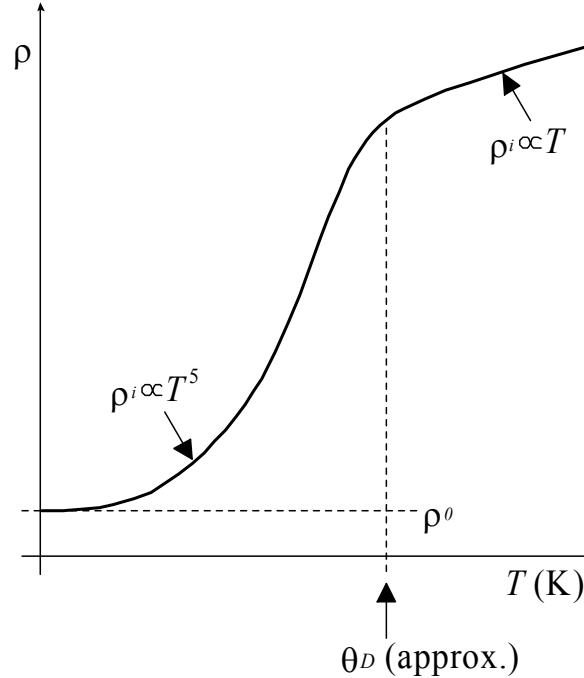


Figure 1. Resistivity versus Temperature for a Normal Metal.

	Debye Temperature θ_D (K)	Superconducting Transition temperature T_C (K)
Al	229	1.2
Cu	344	(does not superconduct)
In	109	3.4
Na	160	(does not superconduct)
Nb	252	9.2
Pb	96	7.2
Sn	195	3.7
W	270	0.01

Table 1. Debye temperatures and superconducting transition temperatures for some elements.

The normal boiling point of liquid helium, $T = 4.2$ K, is so far below θ_D in most metals that, unless the sample is extremely pure, the ideal resistivity is much less than the impurity resistivity. Conversely, unless the sample is very impure, the ideal resistivity at room temperature is usually much greater than the impurity resistivity. For this reason, the ratio of the resistivity at room temperature to its

value at 4.2 K, usually called the Residual Resistivity Ratio (R.R.R.) is often used as a measure of the purity of the metal sample.

$$R.R.R. = \frac{\rho_{RT}}{\rho_{4.2}} \quad (6)$$

The resistivity is related to the measured resistance by

$$\rho = \frac{RA}{l} \quad (7)$$

where l is the distance between potential leads and A is the cross sectional area. The temperature relation between resistance and resistivity may be complicated if A and l vary appreciably as the temperature changes. However, at low temperatures, thermal coefficients of expansion are low, so that the change in dimensions from 4 K to 300 K is much less than the change in dimensions from 304 K to 600 K, for example. Also, for impurity estimation using R.R.R. measurements, one is only interested in the order of magnitude of the R.R.R. value so that from equation (7) one can write

$$R.R.R. = \frac{R_{RT}}{R_{4.2}} \quad (8)$$

Many metals become superconducting at low temperatures, with the resistivity falling abruptly to zero at the superconducting temperature T_C as is shown in Figure 2. Values for T_C for several metals are given in Table 1. Lead (Pb) has one of the highest values of T_C for a metallic element, several degrees above the normal boiling point of helium, so that its superconductivity can be readily demonstrated. For a short explanation of the phenomenon of superconductivity, refer to Kittel's "Introduction to Solid State Physics", Chapter 12, or Chapter 34 of "Solid State Physics" by Ashcroft and Mermin.

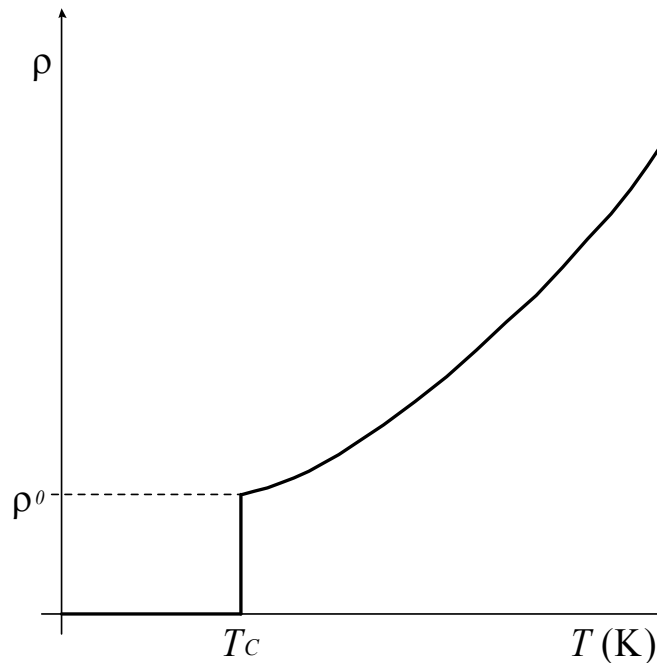


Figure 2. Resistivity versus Temperature for a Superconductor.

There are other metals which display a resistance minimum at low temperatures as is shown in Figure 3. Instead of flattening out or dropping to zero, the resistance increases strongly for temperatures less than about 20 K (see Ashcroft and Mermin, p 688).

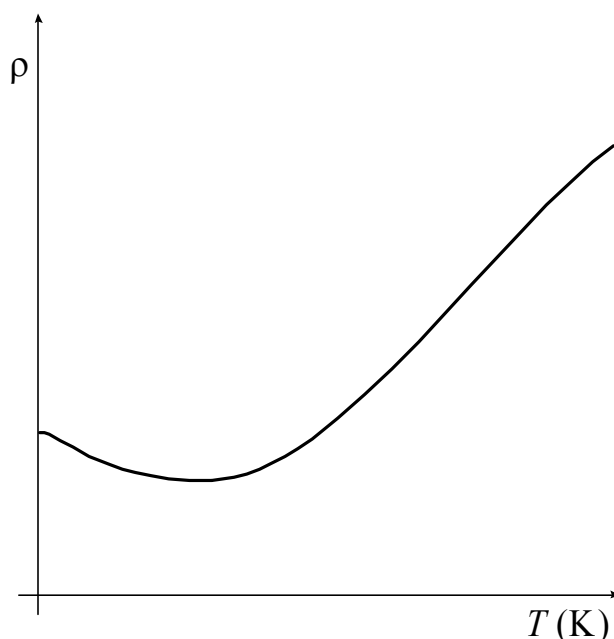


Figure 3. Resistivity versus Temperature for alloys with a resistance minimum.

Experiment

The resistivities of three metal samples are to be measured from about 4.2 K to about 300 K. There is a separate probe for each wire sample with the same pattern of connections in each case. The wire samples are quite long, and are therefore wound into coils in order to make them more compact. The different metal sample wires vary in length and wire thickness, so a comparison of their different resistances does not necessarily reflect differences in resistivities.

The samples of pure Pb, pure Cu, and Au + 0.07% Fe are arranged with current and potential leads as four terminal resistors for which the terminal connections are given in Figure 4. It is the circular plug on the probe which carries the current and potential leads for the experimental sample.

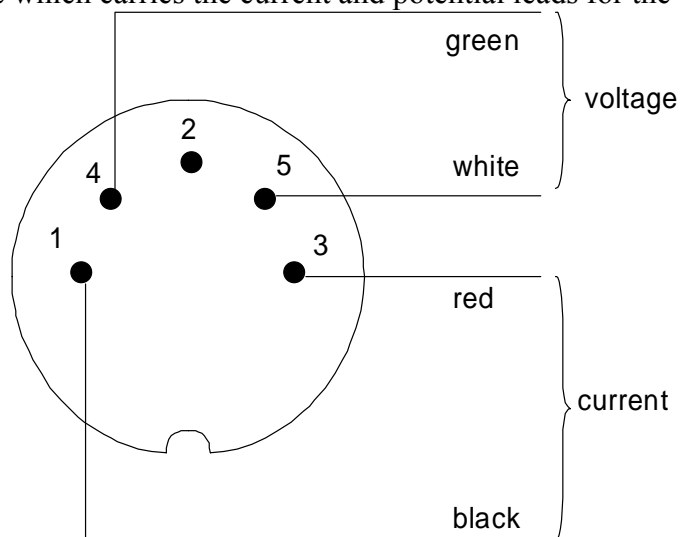


Figure 4. Connections on the probe viewed facing the circular plug.

An alternating current is sent through the wire sample via the red and black connections. The amplitude of this current can be made as constant as possible by connecting a large resistance in series

with the wire sample. You may use a decadal resistor and an ammeter to monitor the amplitude of the current through the wire sample. By adjusting the value of the resistance across the decadal resistor during the experiment, the amplitude of the current can be kept constant as the temperature changes. The voltage drop across the wire sample is monitored via the green and white connections, and this can be used to determine resistance of the wire sample. The voltage drop may be quite small, so a Lock-in amplifier is used to measure the voltage between the green and white wires. The voltage may be read directly off the dial on the Lock-in amplifier, or monitored with the computer by connecting the output of the Lock-in amplifier to the computer. A possible suggested circuit is shown in Figure 5.

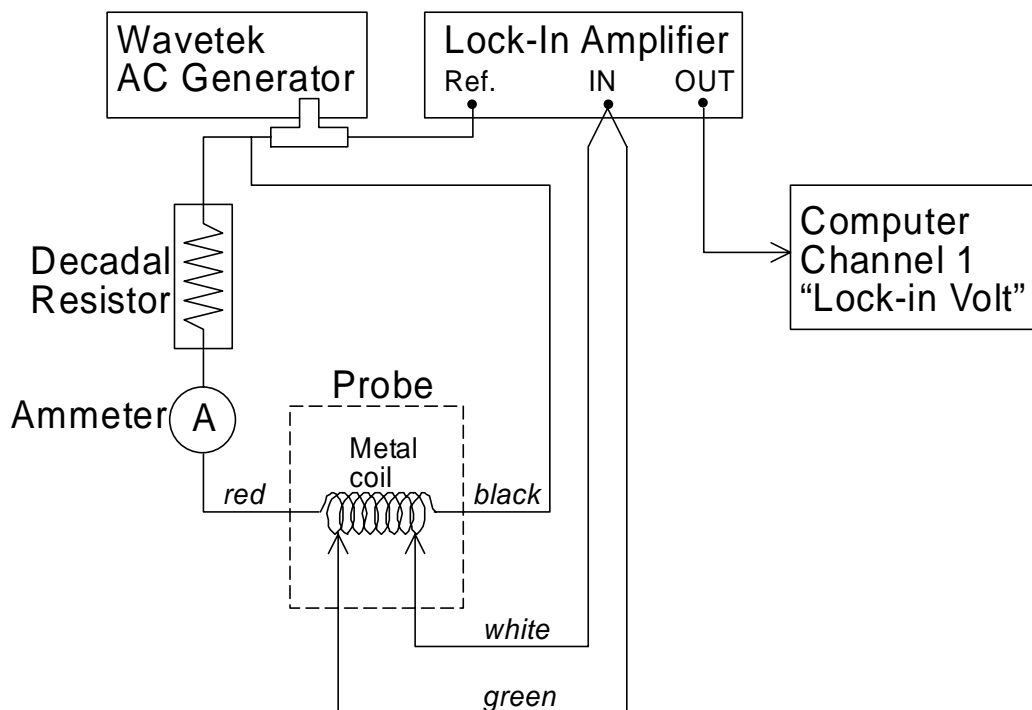


Figure 5. Suggested circuit diagram for the resistance measurements.

Temperature measurements are made using a silicon diode as a sensing element. Some technical information on silicon diodes as cryogenic sensors is given in Appendix I. The temperature is obtained by measuring the voltage developed across the diode for a given current supplied by the constant current source. There are red and white leads coming from the square Jones plug which are connected as voltage leads across the temperature sensing diode. Measuring the voltage across these leads and using the calibration tables provided in Appendix II will give the temperature of the wire sample. For each probe the resistance of the sample coil and the connections to the thermometer should be checked at room temperature at the start of the experiment. The output of the thermometer may be monitored directly by a computer. A suggested circuit diagram is shown in Figure 6.

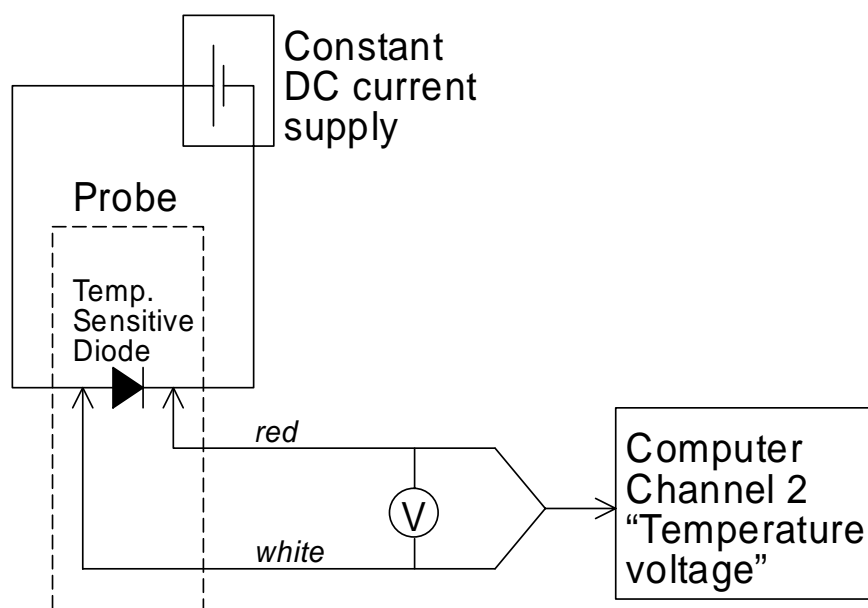


Figure 6. Suggested circuit diagram for temperature measurement.

The probe must be pre-cooled in a liquid nitrogen bath until all major bubbling has ceased before it is lowered into the liquid helium dewar. As a check, a measurement at 77 K should be made. When making this measurement, one should be careful not to use liquid nitrogen that has been sitting in a wide neck dewar exposed to the atmosphere for a long time. Liquid nitrogen can become contaminated with oxygen from the atmosphere and will not be at the boiling point of pure nitrogen but a little nearer to the eutectic for liquid air which is a few degrees higher than 77 K. Practically, one should expect a deviation from 77 K of no more than a fraction of a degree.

The lowest temperature is obtained by lowering the probe into the dewar containing liquid helium at its normal boiling point of 4.2 K. This operation should be done with the help of a laboratory supervisor or technician.

After making a measurement at 4.2 K, measurements may be made at higher temperatures by raising the bottom of the probe out of the liquid helium and having it cooled just in the helium gas. Greater height above the liquid will correspond to higher temperature. In a similar manner to that for helium, you may use the gas over liquid nitrogen to obtain temperatures above 77 K. Measurements up to room temperature may be made if the temperature can be raised slowly enough so that the coil and diode may be assumed to be at the same temperature.

Analysis

You will obtain three R versus T curves which will look, more or less, like Figures 1, 2 and 3.

For Cu it would be desirable to take the sample in the probe above room temperature to obtain resistivities for T ranging through the θ_D value and also the coefficient of the linear temperature above room temperature. However, heating up the probe can cause damage to the probes so we ask you to “make do” with a room temperature value for the Cu resistivity. You should be able to confirm the T^5 dependence at low temperatures. For a discussion of the linear and T^5 dependencies of the resistivity see Ashcroft and Mermin p. 525 and Dekker p. 292.

Determine the residual resistance ratio for two of the wire samples and estimate it for Pb. The values you obtain should show the effect of the iron impurity in the gold and other unspecified impurities in the lead.

It is possible, (see Dekker p. 294, and, in greater detail, Chapter 4 of Meaden which is given in Appendix III), to obtain a value of the Debye temperature, θ_D from electrical resistance measurements made in the low and high temperature regions. Calculate a value for θ_D for the copper sample and compare it with the value $\theta_D = 344$ K from specific heat measurements.

For the Au + 0.07% Fe sample you should obtain a clear indication of the resistance minimum and you could try to obtain confirmation of the $\ln(T)$ dependence of the Kondo contribution to the resistivity. For some explanation of the Kondo effect in increasing order of complexity read the following:

- Kittel, pp. 598-600
- Ashcroft and Mermin, pp. 687-9
- Hurd, pp. 181-191 (includes a simple analysis based on Feynman diagrams).

A discussion of the effect that magnetic exchange interaction has on the specific heat of dilute magnetic alloys is given in Appendix IV. This should provide some feeling for how the dilute magnetic impurity produces relatively large effects. In Appendix V is a reprint of a paper by Daybell and Steyert which gives an experimental review of how the Kondo effect appears in measurements of the magnetic susceptibility, resistivity, specific heat and thermoelectric power.

REFERENCES

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