Some Notes on Metropolis within Gibbs

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I was trying to show convergence of Metropolis within Gibbs (MwG) via detailed balance but failed. This makes me realize that I have several misunderstandings about Gibbs and Metropolis Hasting.

- Gibbs sampler does not necessarily satisfy detailed balance, see this post;
- It is easy to show that the Metropolis Hastings (MH) algorithm satisfy detailed balance. In practice, people usually use a generalized version of MH. That is, to update a subset (block) of variables each time. This is MH with alternate proposal and the proof of convergence of this generalized MH needs much more effort;
- "It would be more accurate to say that Gibbs sampling is a special case of a slightly generalized Metropolis-Hastings, where you alternate between multiple different proposals.", according to the same <u>post</u>; I think this generalized MH is called Metropolis-Hastings-Green algorithm, which is discussed in Section 3.3 in this document.

With this new concept of the Metropolis-Hastings-Green algorithm, I quickly realized that *Metropolis within Gibbs is a special case and hence its convergence is guaranteed by the same conditions required by the Metropolis-Hastings-Green algorithm*.